Convex Optimization in Signal Processing and Communications

Over the past two decades there have been significant advances in the field of optimization. In particular, convex optimization has emerged as a powerful signal-processing tool, and the range of applications continues to grow rapidly. This book, written by a team of leading experts, sets out the theoretical underpinnings of the subject and provides tutorials on a wide range of convex-optimization applications. Emphasis throughout is placed on cutting-edge research and on formulating problems in convex form, making this an ideal textbook for advanced graduate courses and a useful self-study guide.

Topics covered:

- automatic code generation
- graphical models
- gradient-based algorithms for signal recovery
- semidefinite programming (SDP) relaxation
- radar waveform design via SDP
- blind source separation for image processing
- modern sampling theory
- robust broadband beamforming
- distributed multiagent optimization for networked systems
- cognitive radio systems via game theory
- the variational-inequality approach for Nash-equilibrium solutions

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Preface

The past two decades have witnessed the onset of a surge of research in optimization. This includes theoretical aspects, as well as algorithmic developments such as generalizations of interior-point methods to a rich class of convex-optimization problems. The development of general-purpose software tools together with insight generated by the underlying theory have substantially enlarged the set of engineering-design problems that can be reliably solved in an efficient manner. The engineering community has greatly benefited from these recent advances to the point where convex optimization has now emerged as a major signal-processing technique. On the other hand, innovative applications of convex optimization in signal processing combined with the need for robust and efficient methods that can operate in real time have motivated the optimization community to develop additional needed results and methods. The combined efforts in both the optimization and signal-processing communities have led to technical breakthroughs in a wide variety of topics due to the use of convex optimization. This includes solutions to numerous problems previously considered intractable; recognizing and solving convex-optimization problems that arise in applications of interest; utilizing the theory of convex optimization to characterize and gain insight into the optimal-solution structure and to derive performance bounds; formulating convex relaxations of difficult problems; and developing general purpose or application-driven specific algorithms, including those that enable large-scale optimization by exploiting the problem structure.

This book aims at providing the reader with a series of tutorials on a wide variety of convex-optimization applications in signal processing and communications, written by worldwide leading experts, and contributing to the diffusion of these new developments within the signal-processing community. The goal is to introduce convex optimization to a broad signal-processing community, provide insights into how convex optimization can be used in a variety of different contexts, and showcase some notable successes. The topics included are automatic code generation for real-time solvers, graphical models for autoregressive processes, gradient-based algorithms for signal-recovery applications, semidefinite programming (SDP) relaxation with worst-case approximation performance, radar waveform design via SDP, blind non-negative source separation for image processing, modern sampling theory, robust broadband beamforming techniques, distributed multiagent optimization for networked systems, cognitive radio systems via game theory, and the variational-inequality approach for Nash-equilibrium solutions.

There are excellent textbooks that introduce nonlinear and convex optimization, providing the reader with all the basics on convex analysis, reformulation of optimization problems, algorithms, and a number of insightful engineering applications. This book is targeted at advanced graduate students, or advanced researchers that are already familiar with the basics of convex optimization. It can be used as a textbook for an advanced graduate course emphasizing applications, or as a complement to an introductory textbook that provides up-to-date applications in engineering. It can also be used for self-study to become acquainted with the state-of-the-art in a wide variety of engineering topics.

This book contains 12 diverse chapters written by recognized leading experts worldwide, covering a large variety of topics. Due to the diverse nature of the book chapters, it is not possible to organize the book into thematic areas and each chapter should be treated independently of the others. A brief account of each chapter is given next.

In Chapter 1, Mattingley and Boyd elaborate on the concept of convex optimization in real-time embedded systems and automatic code generation. As opposed to generic solvers that work for general classes of problems, in real-time embedded optimization the same optimization problem is solved many times, with different data, often with a hard real-time deadline. Within this setup, the authors propose an automatic code-generation system that can then be compiled to yield an extremely efficient custom solver for the problem family.

In Chapter 2, Beck and Teboulle provide a unified view of gradient-based algorithms for possibly nonconvex and non-differentiable problems, with applications to signal recovery. They start by rederiving the gradient method from several different perspectives and suggest a modification that overcomes the slow convergence of the algorithm. They then apply the developed framework to different image-processing problems such as ℓ_1-based regularization, TV-based denoising, and TV-based deblurring, as well as communication applications like source localization.

In Chapter 3, Songsiri, Dahl, and Vandenberghe consider graphical models for autoregressive processes. They take a parametric approach for maximum-likelihood and maximum-entropy estimation of autoregressive models with conditional independence constraints, which translates into a sparsity pattern on the inverse of the spectral-density matrix. These constraints turn out to be nonconvex. To treat them, the authors propose a relaxation which in some cases is an exact reformulation of the original problem. The proposed methodology allows the selection of graphical models by fitting autoregressive processes to different topologies and is illustrated in different applications.

The following three chapters deal with optimization problems closely related to SDP and relaxation techniques.

In Chapter 4, Luo and Chang consider the SDP relaxation for several classes of quadratic-optimization problems such as separable quadratically constrained quadratic programs (QCQPs) and fractional QCQPs, with applications in communications and signal processing. They identify cases for which the relaxation is tight as well as classes of quadratic-optimization problems whose relaxation provides a guaranteed, finite worst-case approximation performance. Numerical simulations are carried out to assess the efficacy of the SDP-relaxation approach.

In Chapter 5, So and Ye perform a probabilistic analysis of SDP relaxations. They consider the problem of maximum-likelihood detection for multiple-input–multiple-output systems via SDP relaxation plus a randomization rounding procedure and study its loss in performance. In particular, the authors derive an approximation guarantee based on SDP weak-duality and concentration inequalities for the largest singular value of the channel matrix. For example, for MPSK constellations, the relaxed SDP detector is shown to yield a constant factor approximation to the ML detector in the low signal-to-noise ratio region.

In Chapter 6, Huang, De Maio, and Zhang treat the problem of radar design based on convex optimization. The design problem is formulated as a nonconvex QCQP. Using matrix rank-1 decompositions they show that nonetheless strong duality holds for the nonconvex QCQP radar code-design problem. Therefore, it can be solved in polynomial time by SDP relaxation. This allows the design of optimal coded waveforms in the presence of colored Gaussian disturbance that maximize the detection performance under a control both on the region of achievable values for the Doppler-estimation accuracy and on the similarity with a given radar code.

The next three chapters consider very different problems, namely, blind source separation, modern sampling theory, and robust broadband beamforming.

In Chapter 7, Ma, Chan, Chi, and Wang consider blind non-negative source separation with applications in imaging. They approach the problem from a convex-analysis perspective using convex-geometry concepts. It turns out that solving the blind separation problem boils down to finding the extreme points of a polyhedral set, which can be efficiently solved by a series of linear programs. The method is based on a deterministic property of the sources called local dominance which is satisfied in many applications with sparse or high-contrast images. A robust method is then developed to relax the assumption. A number of numerical simulations show the effectiveness of the method in practice.

In Chapter 8, Michaeli and Eldar provide a modern perspective on sampling theory from an optimization point of view. Traditionally, sampling theories have addressed the problem of perfect reconstruction of a given class of signals from their samples. During the last two decades, it has been recognized that these theories can be viewed in a broader sense of projections onto appropriate subspaces. The authors introduce a complementary viewpoint on sampling based on optimization theory. They provide extensions and generalizations of known sampling algorithms by constructing optimization problems that take into account the goodness of fit of the recovery to the samples as well as any other prior information on the signal. A variety of formulations are considered including aspects such as noiseless/noisy samples, different signal priors, and different least-squares/minimax objectives.

In Chapter 9, Rübsamen, El-Keyi, Gershman, and Kirubarajan develop several worst-case broadband beamforming techniques with improved robustness against array manifold errors. The methods show a robustness matched to the presumed amount of uncertainty, each of them offering a different trade-off in terms of interference suppression capability, robustness against signal self-nulling, and computational complexity.

The authors obtain convex second-order cone programming and SDP reformulations of the proposed beamformer designs which lead to efficient implementation.

The last three chapters deal with optimization of systems with multiple nodes. Chapter 10 takes an optimization approach with cooperative agents, whereas Chapters 11 and 12 follow a game-theoretic perspective with noncooperative nodes.

In Chapter 10, Nedic and Ozdaglar study the problem of distributed optimization and control of multiagent networked systems. Within this setup, a network of agents has to cooperatively optimize in a distributed way a global-objective function, which is a combination of local-objective functions, subject to local and possibly global constraints. The authors present both classical results as well as recent advances on design and analysis of distributed-optimization algorithms, with recent applications. Two main approaches are considered depending on whether the global objective is separable or not; in the former case, the classical Lagrange dual decompositions can be employed, whereas in the latter case consensus algorithms are the fundamental building block. Practical issues associated with the implementation of the optimization algorithms over networked systems are also considered such as delays, asynchronism, and quantization effects in the network implementation.

In Chapter 11, Scutari, Palomar, and Barbarossa apply the framework of game theory to different communication systems, namely, ad-hoc networks and cognitive radio systems. Game theory describes and analyzes scenarios with interactive decisions among different players, with possibly conflicting goals, and is very suitable for multiuser systems where users compete for the resources. For some problem formulations, however, game theory may fall short, and it is then necessary to use the more general framework of variational-inequality (VI) theory. The authors show how many resource-allocation problems in ad-hoc networks and in the emerging field of cognitive radio networks fit naturally either in the game-theoretical paradigm or in the more general theory of VI (further elaborated in the following chapter). This allows the study of existence/uniqueness of Nash-equilibrium points as well as the design of practical algorithms with provable converge to an equilibrium.

In Chapter 12, Facchinei and Pang present a comprehensive mathematical treatment of the Nash-equilibrium problem based on the variational-inequality and complementarity approach. They develop new results on existence of equilibria based on degree theory, global uniqueness, local-sensitivity analysis to data variation, and iterative algorithms with convergence conditions. The results are then illustrated with an application in communication systems.