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Edited by Matthias Baaz, Christos H. Papadimitriou, Hilary W. Putnam, Dana S. Scott and Charles L. Harper

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