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978-0-521-76144-4 - Kurt Gödel and the Foundations of Mathematics: Horizons of Truth

Edited by Matthias Baaz, Christos H. Papadimitriou, Hilary W. Putnam, Dana S. Scott and Charles L. Harper
Frontmatter

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Kurt Gödel and the Foundations of Mathematics

Horizons of Truth

This volume commemorates the life, work, and foundational views of Kurt Gödel (1906–1978), most famous for his hallmark works on the completeness of first-order logic, the incompleteness of number theory, and the consistency – with the other widely accepted axioms of set theory – of the axiom of choice and of the generalized continuum hypothesis. It explores current research, advances, and ideas for future directions not only in the foundations of mathematics and logic but also in the fields of computer science, artificial intelligence, physics, cosmology, philosophy, theology, and the history of science. The discussion is supplemented by personal reflections from several scholars who knew Gödel personally, providing some interesting insights into his life. By putting his ideas and life's work into the context of current thinking and perceptions, this book will extend the impact of Gödel's fundamental work in mathematics, logic, philosophy, and other disciplines for future generations of researchers.

Matthias Baaz is currently University Professor and Head of the Group for Computational Logic at the Institute of Discrete Mathematics and Geometry at the Vienna University of Technology.

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To every ω -consistent recursive class κ of *formulae*, there correspond recursive *class signs* r , such that neither $(v \text{ Gen } r)$ nor $\text{Neg}(v \text{ Gen } r)$ belongs to $\text{Flg}(\kappa)$, where v is the *free variable* of r .¹

(Any not-too-weak consistent formal theory, in particular any reasonable formalization of number theory, cannot prove everything that is true; i.e., such a theory is necessarily incomplete.)

– Gödel's first incompleteness (undecidability) theorem, 1931

In any not-too-weak formal theory, the formalization of consistency implies the Gödel sentence, which is unprovable if the formal theory is consistent.

(If the formal theory is consistent, then its consistency cannot be proved within the formal theory.)

– Gödel's second incompleteness theorem, 1931

¹ See: <http://mathworld.wolfram.com/GoedelsIncompletenessTheorem.html>.

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Foreword

While I was writing some words to say about Professor Kurt Gödel's major works for his 2006 centenary celebration at the University of Vienna, it suddenly came to me that for everyone who gathered in his honor, Gödel's extraordinary contributions to and tremendous influence on mathematics would be something of which we were already deeply aware. Thinking that perhaps a repeat of Gödel's results would be unnecessary with this group, I decided to share some of my own personal memories that are recalled when I remember Professor Gödel.

I met Gödel for the first time at the Institute for Advanced Study in Princeton in January 1959, when he was fifty-two years old. At the time, I was a very young thirty-two-year-old whose only interest was my own problem within logic; I knew little of logic as a whole. Throughout my first stay in Princeton, Gödel taught me many new ideas, specifically about nonstandard models and large cardinals. On certain occasions, he would lead me to the library and show me the precise page of a book on which a pertinent theorem was presented, and he advised me on which books I should be reading. He even counseled me that I needed to improve my English to communicate with other mathematicians.

Gödel showed a keen interest in the problem on which I was working then: my fundamental conjecture, that is, the cut elimination theorem on the generalized logic calculus, which is the higher type extension of Gentzen's *logistischer klassischer Kalkül* sequent calculus, as introduced in 1934. At first, Gödel thought that one could find a counterexample using his incompleteness theorem or a nonstandard model. He thought that there must be a counterexample in every impredicative case, that is, a similar situation to my problem in the way that the incompleteness theorem holds. Interestingly, my fundamental conjecture trivially holds in the very impredicative cases. Professor Gödel was surprised to find this and became intrigued with my conjecture. He thought it would help my work if I could meet with Professor Kurt Schütte, whom he immediately invited to the institute. Professor Schütte found the model-theoretic formulation of my fundamental conjecture. Dr. Takahashi's and Dr. Pravitza's later works, which proved my conjecture, were based on Professor Schütte's result. Professor Gödel's insight was correct.

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In my later visits with Gödel, we discussed the more philosophical aspects of logic. He seemed to believe that the cardinality of the continuum is small. His theory was that if one assumes the existence of a “beautiful scale” in the real numbers, this conclusion is inevitable. Although I had a hard time understanding this idea, our discussions were stimulating and gave me tremendous pleasure. I wish now that I had pursued his ideas further. My hope is that future generations will take up his many interesting concepts and develop them.

It would be Kurt Gödel’s greatest delight to see how his ideas are alive and remain the cornerstones of modern logic more than thirty years after his death. Going further back in time, in a letter to Robert Hooke dated February 5, 1675, Sir Isaac Newton wrote of his own discoveries, “If I have seen further it is by standing on the shoulders of giants.”¹ I believe this applies to all who gathered in 2006 and to those who have contributed their work to this volume – for Professor Gödel, although very shy, was truly a giant.

It was in the spirit of acknowledging Professor Gödel’s ever-searching imagination and philosophical open-mindedness that the historic 2006 meeting took place and that this book was written. No doubt the legacy of the centenary and this volume will serve as an inspiration for yet other generations of mathematicians (and philosophers). Although I very much regretted that for reasons of health, I could not attend the 2006 celebration, I enthusiastically shared from afar the timeless spirit of Kurt Gödel, which lives on in this book and in our minds.

Gaisi Takeuti
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¹ http://www.quotationspage.com/quotes/Isaac_Newton/.

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Preface

Kurt Gödel and the Foundations of Mathematics: Horizons of Truth is the culmination of a creative research initiative coorganized by the Kurt Gödel Society, Vienna; the Institute for Experimental Physics; the Kurt Gödel Research Center; the Institute Vienna Circle; the Vienna University of Technology; the Austrian Academy of Sciences; and the Anton Zeilinger Group at the University of Vienna, where the Gödel centenary celebratory symposium “Horizons of Truth: Logics, Foundations of Mathematics, and the Quest for Understanding the Nature of Knowledge” was held from April 27 to April 29, 2006.¹

More than twenty invited world-renowned researchers in the fields of mathematics, logic, computer science, physics, philosophy, theology, and the history of science attended the symposium, giving the participants the remarkable opportunity to present their ideas about Gödel’s work and its influence on various areas of intellectual endeavor. These fascinating interdisciplinary lectures provided new insights into Gödel’s life and work and their implications for future generations of researchers.

The interaction among international scholars who only rarely, if ever, have the opportunity to hold discussions in the same room – and some of whom almost never write articles – has produced a book that contains chapters expanded and developed to take advantage of the rich intellectual exchange that took place in Vienna. Written by some of the most renowned figures of the scientific and academic world, the resulting volume is an opus of current research and thinking that is built on the work and inspiration of Gödel.

Several of the contributors were colleagues of or studied with Gödel: Gaisi Takeuti, who contributed the foreword and offers warm remembrances of Gödel’s impact on his work; Georg Kreisel, who contributed a detailed chapter on logic and foundations; and Fields Medal winner Paul J. Cohen, who, sadly, died shortly after completing his personal reflections for this volume.

¹ See the symposium Web site for further information: <http://www.logic.at/goedel2006/>.

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PREFACE

Also, a number of other chapters in this volume contain extensive biographical details about various aspects of Gödel's life and work to supplement the technical discussions. In addition, we include a short biography of Gödel's life that contains additional biographical resources and a select bibliography of his seminal works, which are frequently cited throughout this book.

The main content of the volume is divided into the following three major parts, broken down further into subparts to highlight the multidimensional impact of Gödel's contributions to academic advancement:

I Historical Context: Gödel's Contributions and Accomplishments

Gödel's Historical, Philosophical, and Scientific Work

Gödel's Legacy: A Historical Perspective

The Past and Future of Computation

Gödelian Cosmology

II A Wider Vision: The Interdisciplinary, Philosophical, and Theological

Implications of Gödel's Work

On the Unknowables

Gödel and the Mathematics of Philosophy

Gödel and Philosophical Theology

Gödel and the Human Mind

III New Frontiers: Beyond Gödel's Work in Mathematics and Symbolic Logic

Extending Gödel's Work

The Realm of Set Theory

Gödel and the Higher Infinite

Gödel and Computer Science

These topics cover not only the technical aspects of Gödel's work and its legacy but also the profoundly reflective aspects of his thinking, augmenting the appeal of the book and ensuring its interest to both a specialized and a multidisciplinary readership.

Because of the stature and diverse research areas of the contributors, we believe this book will appeal not only to mathematicians and logicians but also to computer scientists, physicists, astrophysicists, cosmologists, philosophers, theologians, historians of science, and postdoctoral and graduate students working in these areas as well as to educated and informed others interested in foundational questions.

We hope we have achieved our goal of creating a lasting impact on the academic community by further advancing the legacy of a man without whose stunning contributions to mathematical logic our world's intellectual culture certainly would have been diminished.

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The editors wish to thank the Kurt Gödel Society, Vienna; the Institute for Experimental Physics; the Kurt Gödel Research Center; the Institute Vienna Circle; the Vienna University of Technology; the Austrian Academy of Sciences; and the Anton Zeilinger Group at the University of Vienna for coorganizing the Gödel centenary celebratory symposium in April 2006.

The symposium was sponsored by the John Templeton Foundation (JTF), United States, which also provided the funding for this book.¹ Additional funding for the centenary symposium was provided in Austria by the Federation of Austrian Industry; the Federal Ministry of Infrastructure; the Federal Ministry of Education, Science, and Culture; the city of Vienna; and the Austrian Mathematical Society. We also thank the Microsoft Corporation for its contribution.

The editors wish to acknowledge the contributions of a number of individuals who contributed to the Gödel research initiative:

Norbert Preining, associate professor at the Research Center for Integrated Science of the Japan Advanced Institute of Science and Technology, former research assistant at the Vienna University of Technology, and Marie Curie Fellow at the University of Siena, as well as publicity chair of the Kurt Gödel Society, codeveloped and cohosted the symposium at the University of Vienna in 2006, in conjunction with Matthias Baaz.

Hyung S. Choi, director of mathematical and physical sciences at JTF, assumed an integral role in developing the academic program for the symposium, in conjunction with Charles L. Harper, Jr. (in his former role as senior vice president and chief strategist of JTF).

Pamela M. Contractor, president and director of Ellipsis Enterprises Inc., working in conjunction with JTF and the volume editors, served as developmental editor of this

¹ “Supporting science, investing in the big questions”: <http://www.templeton.org/>.

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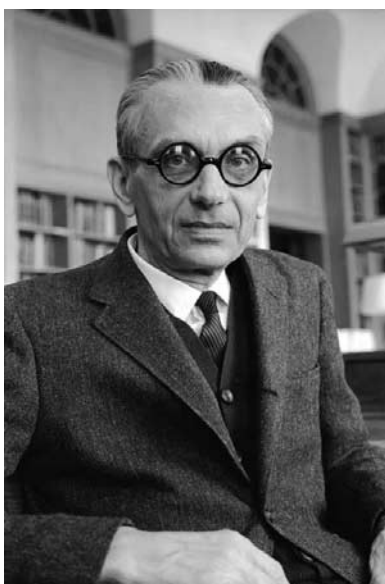
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Short Biography of Kurt Gödel



Gödel seated in the Mathematics–Natural Sciences Library at Fuld Hall, Institute for Advanced Study, Princeton, New Jersey, 1963. Photograph by Alfred Eisenstaedt from the Time and Life Pictures collection. Reproduced with permission from Getty Images.

A handwritten signature of Kurt Gödel in black ink. The signature is written in a cursive, slightly slanted style, with the first letters of 'Kurt' and 'Gödel' being capitalized and prominent.

Gödel's signature. The electronic signature is used with permission from the Institute for Advanced Study.

Kurt Friedrich Gödel is considered one of the most outstanding mathematical logicians of the twentieth century and is thought by many to be the greatest logician since

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SHORT BIOGRAPHY OF KURT GÖDEL

Aristotle. He was born on April 28, 1906, in what was then Brünn in the Austro-Hungarian Monarchy and today is Brno in the Czech Republic. After attending school in Brünn and graduating with honors, he enrolled at the University of Vienna in 1923 with the original intention of studying physics. He attended lectures on number theory by professor Philipp Furtwängler, who, paralyzed from the neck down, lectured from his wheelchair. Thereafter Gödel became interested in mathematical logic, the field to which he would make his major contributions. As a student, he also attended meetings of what would later become the Vienna Circle (Wiener Kreis), a group composed mainly of philosophers that met to discuss foundational problems, inspired by Ludwig Wittgenstein's *Tractatus Logico-Philosophicus*. The group focused on questions of language and meaning and logical relations such as entailment, originating logical positivism (logical empiricism). Led by Moritz Schlick, who was later murdered by a deranged former student in 1936 at the University of Vienna, its members included Rudolf Carnap, Otto Neurath, Carl Hempel, Hans Reichenbach, Hans Hahn, Karl Menger, and others. A Platonist from an early age, Gödel disagreed with many of his colleagues' views, yet the Vienna Circle had a major influence on his thinking. In his doctoral dissertation, written under the supervision of Hans Hahn, he proved the completeness of first-order predicate logic with identity, which states that any sentence that holds in every model of the logic is derivable in the logic. His dissertation was finished in 1929, and the result was published in 1930.

Also in 1930, at a meeting in Königsberg (David Hilbert's hometown) on September 7 that was attended by, among others, John von Neumann, Gödel, still in his mid-twenties, announced his work demonstrating that systems of mathematics have limits. In particular, he showed that any not-too-weak consistent formal theory (say, any reasonable formalization of number theory) cannot prove everything that is true; that is, such a theory is necessarily incomplete.

Gödel's startling results in formal logic, considered landmarks of twentieth-century logic, were published as the now-famous incompleteness theorems the following year, in 1931, ending many years of attempts to find a set of axioms sufficient for all mathematics and implying that not all mathematical questions are formally solvable in a fixed system. Gödel demonstrated, in effect, that hopes of reducing mathematics to an axiomatic system, as envisioned by mathematicians and philosophers at the turn of the twentieth century, were in vain. His findings put an end to the logicist efforts of Bertrand Russell and Alfred North Whitehead and demonstrated the severe limitations of David Hilbert's program for arithmetic. In the introduction to his 1931 paper, Gödel stated:

It is well known that the development of mathematics in the direction of greater precision has led to the formalization of extensive mathematical domains, in the sense that proofs can be carried out according to a few mechanical rules. . . . It is reasonable therefore to make the conjecture that these axioms and rules of inference are also sufficient to decide all mathematical questions, which can be formally expressed in the given systems. In what follows it will be shown that this is not the case.¹

In addition to his proof of the incompleteness of formal number theory, Gödel (1938, 1939, 1953, 1990) published proofs of the relative consistency of the axiom of choice

¹ See Gödel (1931) and <http://www.ias.edu/people/godel>.

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Gödel with Einstein at the Institute for Advanced Study, early 1950s. Photograph by Richard F. Arens. From the Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, New Jersey. Reproduced with permission.

and of the generalized continuum hypothesis. His findings strongly influenced the (later) discovery that a computer can never be programmed to answer all mathematical questions.

After obtaining his *Habilitation*, Gödel joined the faculty of the University of Vienna in 1930, becoming a *Privatdozent* (unsalaried lecturer) in 1933. He would remain there until the Anschluss in 1938, when Austria became part of Nazi Germany. During the 1930s, he made several visits abroad: to the Institute of Advanced Study in Princeton (1933–1934, 1935, and 1938), where he would eventually settle; to the University of Göttingen (1938), where he gave lectures on set theory; and to the University of Notre Dame (1939), where he worked with the newly emigrated Karl Menger. By 1938, Gödel saw that his position as *Privatdozent* would not be continued, and he feared that he would be drafted into the army. He left Europe with his wife via the Trans-Siberian Railway in January 1940; they arrived in San Francisco by ship on March 4. They would never return to Austria.

In Princeton, Gödel joined the Institute for Advanced Study, where he was professor in the School of Mathematics from 1953 until 1976, when he became professor emeritus, holding the mathematics chair until his death from malnutrition on January 14, 1978. There Gödel's interests turned increasingly to philosophy and physics. In the 1940s, he was able to demonstrate the existence of paradoxical solutions to Einstein's field equations in the theory of general relativity, which allowed for the possibility of time travel into the past. Gödel's theorems and other theoretical explorations in physics and philosophy helped usher in the age of computer technology, influencing the innovative work of John von Neumann, Alan Turing, and others in computer science that has so profoundly influenced the world and our attempts to understand and manage it.² Gödel's last published paper appeared in 1958.

² In fact, the most prestigious award for a research contribution in theoretical computer science is called the "Gödel Prize."

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SHORT BIOGRAPHY OF KURT GÖDEL

During his life, Gödel received several prizes and honorary doctorates and memberships (and rejected some others). Among them were the Institute for Advanced Study's Einstein Award (1951) as well as the National Medal of Science (in the disciplines of mathematics and computer science) from President Ford (1974) for "laying the foundation for today's flourishing study of mathematical logic." Gödel received honorary doctorates from Yale, Harvard, and Rockefeller universities and from Amherst College. He was a member of the National Academy of Sciences of the United States, a foreign member of the Royal Society of London, a corresponding member of the Institute of France, a corresponding Fellow of the British Academy, and an honorary member of the London Mathematical Society. In 2000, *Time* magazine included Gödel among its top one hundred most influential thinkers of the twentieth century.

As noted in the editors' preface, a number of chapters in this volume contain extensive biographical details about various aspects of Gödel's life and work to supplement the technical discussions. In the following, we provide additional biographical resources and a select bibliography of Gödel's seminal works, which are frequently cited throughout this book.

Additional Biographical Resources

Dawson, John W. Jr. *Logical Dilemmas: The Life and Work of Kurt Gödel*. Wellesley, MA: A K Peters, 1997.

Institute for Advanced Study. <http://www.ias.edu/people/godel>.

Kurt Gödel Papers. Princeton University Library Manuscripts Division. <http://diglib.princeton.edu/ead/getEad?id=ark:/88435/v979v310g#bioghist>.

Kurt Gödel Society. <http://kgs.logic.at/index.php?id=23>.

Sigmund, Karl, and John Dawson. *Gödel's Jahrhundert Ausstellung (Gödel's Centenary Exhibition)*. <http://www.goedelexhibition.at/start/>.

Sigmund, Karl, John Dawson, and Kurt Mühlberger. *Kurt Gödel: The Album*. Wiesbaden, Germany: Vieweg, 2006. (Available in German and English)

Select Bibliography of Gödel's Seminal Works

Gödel, K. (1930). Die Vollständigkeit der Axiome des logischen Funktionenkalküls. *Monatshefte für Mathematik und Physik*, **37**, 349–60. [Published PhD diss.]

_____. (1931). Über formal unentscheidbare Sätze der *Principia Mathematica* und verwandter Systeme I. *Monatshefte für Mathematik und Physik*, **38**, 173–98. [English trans. J. van Heijenoort, ed. (1967). *From Frege to Gödel: A Source Book on Mathematical Logic*. Cambridge, MA: Harvard University Press, pp. 596–616. Repr. with facing English trans. On formally undecidable propositions of *Principia Mathematica* and related systems. I. In *Collected Works*, vol. 1 (1986), pp. 145–95.]

_____. (1938). The consistency of the axiom of choice and of the generalized continuum-hypothesis. *Proceedings of the National Academy of Sciences of the United States of America*, **24**, 556–7. [Also in *Collected Works*, vol. 2 (1990), pp. 26–7.]

_____. (1939). Consistency-proof for the Generalized Continuum Hypothesis. *Proceedings of the National Academy of Sciences, USA*, **25**, 220–4. [Also in *Collected Works*, vol. 2 (1990), pp. 27–32.]

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- _____. (1949a). A remark about the relation between relativity theory and idealistic philosophy. In *Albert Einstein: Philosopher-Scientist*, ed. P. A. Schilpp, pp. 557–62. Library of Living Philosophers 7. Evanston, IL: MJF Books. [Also in *Collected Works*, vol. 2 (1990), pp. 202–7.]
- _____. (1949b). An example of a new type of cosmological solution of Einstein's field equations of gravitation. *Reviews of Modern Physics*, **21**, 447–50. [Also in *Collected Works*, vol. 2 (1990), pp. 190–8.]
- _____. (1952). Rotating universes in general relativity theory. In *Proceedings of the International Congress of Mathematicians*, vol. 1, ed. L. M. Graves et al., pp. 175–81. Cambridge, MA: American Mathematical Society. [Also in *Collected Works*, vol. 2 (1990), pp. 208–16.]
- _____. (1953 [1940]). The consistency of the axiom of choice and of the generalized continuum-hypothesis with the axioms of set theory. In *Annals of Mathematics Studies*, vol. 3, rev. ed. Princeton, NJ: Princeton University Press. [Also in *Collected Works*, vol. 2 (1990), pp. 33–101.]
- _____. (1958). Über eine bisher noch nicht benützte Erweiterung des finiten Standpunktes. *Dialectica*, **12**, 280–87. [Repr. English trans. On a hitherto unutilized extension of the finitary standpoint. In *Collected Works*, vol. 2 (1990), pp. 241–51.]
- _____. (1964 [1947]). What is Cantor's continuum problem? *American Mathematical Monthly*, **54**, 515–25. [Rev. version in P. Benacerraf and H. Putnam, eds. (1984 [1964]). *Philosophy of Mathematics*. Englewood Cliffs, NJ: Prentice Hall, p. 483. Also in *Collected Works*, vol. 2 (1990), pp. 176–87 (1947 version); pp. 254–70 (1964 version).]
- _____. (1986 [1929]). On the completeness of the calculus of logic. PhD diss. In *Collected Works*, vol. 1, pp. 61–101.
- _____. (1986–2003). *Collected Works*. 5 vols. Edited by S. Feferman et al. Vols. 1–3, New York: Oxford University Press. Vols. 4 and 5, Oxford: Clarendon Press. [Throughout, referenced as *Collected Works* by vol. number, year, and page.]
- _____. (1990 [1939]). The consistency of the generalized continuum hypothesis. In *Collected Works*, vol. 2, p. 27.
- _____. (1995a [1949]). Lecture on rotating universes; given at the Institute for Advanced Study, Princeton, May 7, 1949. In *Collected Works*, vol. 3, pp. 269–87.
- _____. (1995b [1970]). Ontological proof. In *Collected Works*, vol. 3, pp. 403–4. [Introductory note by R. M. Adams, pp. 388–402. Appendix B: Texts relating to the ontological proof, including Gödel's first version, 1941, pp. 429–37.]