MATRICES METHODS IN THE DESIGN ANALYSIS OF MECHANISMS AND MULTIBODY SYSTEMS

This book is an integrated approach to kinematic and dynamic analysis. The matrix techniques presented are general and fully applicable to two- or three-dimensional systems. They lend themselves to programming and digital computation and can be the basis of a usable tool for designers. The techniques have broad applicability to the design analysis of all multibody mechanical systems. The more powerful and more flexible the approach, and the less specialization and reprogramming required for each application, the better. The matrix methods presented have been developed using these as primary goals. Although the matrix methods can be applied by hand to such problems as the slider-crank mechanism, this is not the intent of this text, and often the rigor required for such an attempt becomes quite burdensome in comparison with other techniques. The matrix methods have been extensively tested, both in the classroom and in the world of the engineering industry.

John J. Uicker is Professor Emeritus of mechanical engineering at the University of Wisconsin–Madison. Throughout his career, his teaching and research have focused on solid geometric modeling and the modeling of mechanical motion, and their application to computer-aided design and manufacture, including the kinematics, dynamics, and simulation of articulated rigid-body mechanical systems. He founded the UW Computer-Aided Engineering Center and served as its director for its initial ten years of operation. He has served on several national committees of the American Society of Mechanical Engineers (ASME) and the Society of Automotive Engineers (SAE), and he received the ASME Mechanisms Committee Award in 2004 and the ASME Fellow Award in 2007. He is a founding member of the U.S. Council for the Theory of Mechanism and Machine Science (USCToMM), and of the International Federation of Mechanism and Machine Science (IFToMM). He is a registered mechanical engineer in Wisconsin and has served for many years as an active consultant to industry.

Bahram Ravani is Professor of mechanical and aerospace engineering at the University of California, Davis. He has served as the chair of the department as well as the interim chair of electrical and computer engineering. He is also a member of the graduate programs in biomedical engineering and in forensic science and engineering. Among his honors are the Young Manufacturing Engineer Award from the Society of Manufacturing Engineers, the Gustus L. Larson Memorial Award from ASME for outstanding achievements in mechanical engineering, the Design Automation Award from the Design Engineering Division of ASME for his lifetime of sustained contributions, and the Machine Design Award for eminent achievements. He is currently the technical editor of the ASME Journal of Computers and Information Science in Engineering and he was a technical editor of the Journal of Mechanical Design. He is a Fellow of ASME and is the former chair of its Design Engineering Division. He is also a member of the Society of Automotive Engineers, the International Society of Biomechanics, and the Association for Advancement of Automotive Medicine.

Pradip N. Sheth (1944–2009) was born in Vadodara, India. He earned his BE (1965) and MS (1968) in mechanical engineering at Maharaja Sayajirao University, Baroda, India, and his PhD in 1972 at the University of Wisconsin–Madison, where John Uicker served as his advisor. In his research, he developed the Integrated Mechanisms Program for the computer-aided design and analysis of multibody mechanical systems. More than 200 industrial and educational organizations worldwide have used the system, which was the first of its kind.
This textbook is dedicated to the memory of the third author, the late Associate Professor Pradip N. Sheth, Department of Mechanical Engineering, University of Virginia, Charlottesville, who passed away during his writing of this book after several years of testing it in his classes. His doctoral dissertation included the original development of the Integrated Mechanisms Program (IMP), the first general software system for the simulation of articulated multibody mechanical systems. Much of this text can be traced to that seminal work. His intention was to dedicate his writings to his loving wife, Diane C. Sheth, who provided her encouragement and support throughout his foreshortened career.

This work is also dedicated to the memory of my father, John J. Uicker, Sr., emeritus dean of engineering, University of Detroit; to my mother, Elizabeth F. Uicker; and to my six children, Theresa A. Zenchenko, John J. Uicker III, Joseph M. Uicker, Dorothy J. Winger, Barbara A. Peterson, and Joan E. Horne, and their families.

– John J. Uicker

This work is also dedicated first and foremost to my father, Abraham Ravani, who inspired me from early childhood to pursue science and provided the opportunity for my U.S. education; to my children, Sarah and Samuel Ravani, and Paris Kent; and finally to my wife, Sara Kent, who endured while I spent time working on this book.

– Bahram Ravani
Song of the Screw

A moving form or rigid mass,
Under whate’er conditions
Along successive screws must pass
Between each two positions.
It turns around and slides along –
This is the burden of my song.
The pitch of screw, if multiplied
By angle of rotation,
Will give the distance it must glide
In motion of translation.
Infinite pitch means pure translation,
And zero pitch means pure rotation.
Two motions on two given screws,
With amplitudes at pleasure,
Into a third screw-motion fuse,
Whose amplitude we measure
By parallelogram construction
(A very obvious deduction).
Its axis cuts the nodal line,
Which to both screws is normal,
And generates a form divine
Whose name, in language formal,
Is “surface-ruled of third degree.”
Cylindroid is the name for me.
Rotation round a given line
Is like a force along,
If to say couple you decline,
you’re clearly in the wrong –
Tis obvious upon reflection,
A line is not a mere direction.
So couples with translations too
In all respects agree;
And thus there centers in the screw
A wondrous harmony
Of Kinematics and of Statics –
Sweetest thing in mathematics.
The forces in one given screw,
With motion on a second,
In general some work will do,
Whose magnitude is reckoned
By angle, force, and what we call
The coefficient virtual.
Rotation now to force convert,
And force into rotation;
Unchanged the work, we can assert,
In spite of transformation.
And if two screws no work can claim,
Reciprocal will be their name.
Five numbers will a screw define,
A screwing motion, six;
For four will give the axial line,
One more the pitch will fix;
And hence we always can contrive
One screw reciprocal to five.
Screws – two, three, four, or five, combined
(No question here of six),
Yield other screws which are confined
Within one screw complex.
Thus we obtain the clearest notion
Of freedom and constraint of motion.
In complex III, three several screws
At every point you find,
Or, if you one direction choose,
One screw is to your mind;
And complexes of order III
Their own reciprocals may be.
In IV, wherever you arrive,
You find of screws a cone,
On every line of complex V
There is precisely one;
At each point of this complex rich,
A plane of screws has given pitch.
But time would fail me to discourse
Of Order and Degree;
Of Impulse, Energy, and Force,
And Reciprocity.
All these and more, for motions small,
Have been discussed by Dr. Ball.

Anonymous

Published anonymously in *Nature*, 14, 30–30 (11 May 1876). This poem accurately captures in verse the main points of the mathematical theory of screws which forms a common thread of the theory behind this book.
# Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preface</td>
<td></td>
<td>xiii</td>
</tr>
<tr>
<td>About the Authors</td>
<td></td>
<td>xvii</td>
</tr>
<tr>
<td>1</td>
<td>Concepts and Definitions</td>
<td>1</td>
</tr>
<tr>
<td>1.1</td>
<td>Mechanical Design: Synthesis versus Analysis</td>
<td>1</td>
</tr>
<tr>
<td>1.2</td>
<td>Multibody Systems and Mechanisms</td>
<td>3</td>
</tr>
<tr>
<td>1.3</td>
<td>Planar, Spherical, and Spatial Mechanisms</td>
<td>7</td>
</tr>
<tr>
<td>1.4</td>
<td>Mechanical Body</td>
<td>10</td>
</tr>
<tr>
<td>1.5</td>
<td>Mechanical Chain and Kinematic Inversion</td>
<td>11</td>
</tr>
<tr>
<td>1.6</td>
<td>Joints and Joint Elements</td>
<td>12</td>
</tr>
<tr>
<td>1.7</td>
<td>The Six Lower-Pairs</td>
<td>14</td>
</tr>
<tr>
<td>1.8</td>
<td>Higher-Pairs and Kinematic Equivalence</td>
<td>19</td>
</tr>
<tr>
<td>1.9</td>
<td>Restraints versus Constraints</td>
<td>20</td>
</tr>
<tr>
<td>REFERENCES</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>Topology and Kinematic Architecture</td>
<td>22</td>
</tr>
<tr>
<td>2.1</td>
<td>Introduction</td>
<td>22</td>
</tr>
<tr>
<td>2.2</td>
<td>The Incidence Matrix</td>
<td>24</td>
</tr>
<tr>
<td>2.3</td>
<td>Connectedness and Assemblies</td>
<td>27</td>
</tr>
<tr>
<td>2.4</td>
<td>Kinematic Loops</td>
<td>27</td>
</tr>
<tr>
<td>2.5</td>
<td>Kinematic Paths</td>
<td>32</td>
</tr>
<tr>
<td>REFERENCES</td>
<td></td>
<td>37</td>
</tr>
<tr>
<td>PROBLEMS</td>
<td></td>
<td>37</td>
</tr>
<tr>
<td>3</td>
<td>Transformation Matrices in Kinematics</td>
<td>42</td>
</tr>
<tr>
<td>3.1</td>
<td>Introduction</td>
<td>42</td>
</tr>
<tr>
<td>3.2</td>
<td>Homogeneous Coordinates of a Point</td>
<td>42</td>
</tr>
<tr>
<td>3.3</td>
<td>Line Coordinates and Plücker Vectors</td>
<td>45</td>
</tr>
<tr>
<td>3.4</td>
<td>Three-dimensional Orientation</td>
<td>47</td>
</tr>
<tr>
<td>3.5</td>
<td>Transformation of Coordinates</td>
<td>51</td>
</tr>
<tr>
<td>3.6</td>
<td>Positions, Postures, and Displacements</td>
<td>55</td>
</tr>
<tr>
<td>3.7</td>
<td>Euler’s and Chasles’ Theorems</td>
<td>60</td>
</tr>
</tbody>
</table>
# Contents

3.8 Euler-Rodrigues Parameters 69  
3.9 Displacement of Lines 74  
3.10 Quaternions 74  
REFERENCES 75  
PROBLEMS 77  

4 Modeling Mechanisms and Multibody Systems with Transformation Matrices 80  
4.1 Introduction 80  
4.2 Body Coordinate Systems 80  
4.3 Joint and Auxiliary Coordinate Systems 81  
4.4 Specifying Data for a Coordinate System 82  
4.5 Modeling Dimensional Characteristics of a Body 85  
4.6 Modeling Joint Characteristics 87  
4.6.1 Helical Joint 88  
4.6.2 Revolute Joint 90  
4.6.3 Prismatic Joint 91  
4.6.4 Cylindric Joint 92  
4.6.5 Spheric Joint 93  
4.6.6 Flat Joint 94  
4.6.7 Rigid Joint 96  
4.6.8 Open Joint 96  
4.6.9 Parallel-Axis Gear Joint 98  
4.6.10 Involute Rack-and-Pinion Joint 100  
4.6.11 Straight-Tooth Bevel-Gear Joint 102  
4.6.12 Point on a Planar-Curve Joint 104  
4.6.13 Line Tangent to a Planar-Curve Joint 106  
PROBLEMS 108  

5 Posture Analysis by Kinematic Equations 111  
5.1 Introduction 111  
5.2 Consecutive Transformations 112  
5.3 Denavit-Hartenberg Transformations 116  
5.4 Absolute Position 118  
5.5 The Loop-closure Equation (Kinematic Equation for Position Analysis) 119  
5.6 Closed-form Solution of Kinematic Equations for Joint-variable Positions 121  
5.7 General Styles for Closed-Form Solutions of Kinematic Equations 140  
REFERENCES 145  
PROBLEMS 145  

6 Differential Kinematics and Numeric Solution of Posture Equations 148  
6.1 Introduction 148  
6.2 Differential Kinematics of a Helical Joint 149
6.3 Derivative Operator Matrices 153
6.3.1 Helical Joint 155
6.3.2 Revolute Joint 155
6.3.3 Prismatic Joint 155
6.3.4 Cylindric Joint 155
6.3.5 Spheric Joint 156
6.3.6 Flat Joint 156
6.3.7 Rigid Joint 156
6.3.8 Open Joint 156
6.3.9 Parallel-axis Gear Joint 157
6.3.10 Involute Rack-and-Pinion Joint 157
6.3.11 Straight-tooth Bevel-gear Joint 158
6.3.12 Point on a Planar-Curve Joint 158
6.3.13 Line Tangent to a Planar-Curve Joint 158
6.4 Screw Axes and Ball Vectors for Differential Displacements 159
6.5 Numeric Solution of Kinematic Posture Equations 163
6.5.1 Solution for a Nearby Posture 164
6.5.2 Avoiding Convergence to a False Solution 168
6.5.3 Numeric Solution of the Loop-closure Equation 169
6.6 Identification of Generalized Coordinates 173
6.7 Scaling Internal Length Units 175
6.8 Quality Index 176
6.9 Convergence and Robustness 177
REFERENCES 181
PROBLEMS 182

7 Velocity Analysis .......................................................... 183
7.1 Introduction 183
7.2 Definition of Velocity 184
7.3 First Geometric Derivatives of Joint Variables 186
7.4 Velocities of Joint Variables 189
7.5 First Geometric Derivatives of Body Postures 191
7.6 Velocities of Bodies 194
7.7 First Geometric Derivatives of Point Positions 195
7.8 Velocities of Points 196
REFERENCE 196
PROBLEMS 196

8 Acceleration Analysis ......................................................... 197
8.1 Definition of Acceleration 197
8.2 Derivatives of the $Q_h$ Operator Matrices 198
8.2.1 Helical (Screw) Joint 199
8.2.2 Revolute Joint 199
8.2.3 Prismatic Joint 199
8.2.4 Cylindric Joint 200
8.2.5 Spheric Joint 200
8.2.6 Flat Joint 201
x

Contents

8.2.7 Rigid Joint 201
8.2.8 Open Joint 201
8.2.9 Parallel-Axis Gear Joint 202
8.2.10 Involute Rack-and-Pinion Joint 204
8.2.11 Straight-Tooth Bevel-Gear Joint 204
8.2.12 Point on a Planar-Curve Joint 205
8.2.13 Line Tangent to a Planar-Curve Joint 205
8.3 Derivatives of the \(D_h\) Operator Matrices 205
8.4 Second Geometric Derivatives of Joint Variables 207
8.5 Accelerations of Joint Variables 214
8.6 Second Geometric Derivatives of Body Postures 216
8.7 Second Geometric Derivatives of Point Positions 220
8.8 Accelerations of Bodies 220
8.9 Accelerations of Points 223

REFERENCE 223

PROBLEMS 223

9 Modeling Dynamic Aspects of Mechanisms and Multibody Systems 225

9.1 Introduction 225
9.2 Modeling Kinetic Energy 226
9.3 The Inertia Matrix 227
9.4 Systems of Units 230
9.5 Modeling Gravitational Effects 230
9.6 Modeling Joint Stiffness 232
9.7 Modeling Joint Damping 232
9.8 Modeling Point-to-Point Springs 233
9.9 Modeling Point-to-Point Dampers 234
9.10 Modeling External Forces and Torques Applied with Joint Variables 235
9.11 Modeling External Forces and Torques Applied to Bodies 236

REFERENCES 241

PROBLEMS 242

10 Dynamic Equations of Motion 244

10.1 Introduction 244
10.2 Lagrange’s Equation 244
10.3 Generalized Momentum 245
10.4 D’Alembert Inertia Forces 246
10.5 Generalized Restoring Forces 249
10.6 Generalized Applied Forces 250
10.7 Complete Equations of Motion 250

REFERENCES 253

PROBLEMS 253
## Contents

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>Linearized Equations of Motion</td>
<td>254</td>
</tr>
<tr>
<td>11.1</td>
<td>Introduction</td>
<td>254</td>
</tr>
<tr>
<td>11.2</td>
<td>Linearization Assumptions</td>
<td>254</td>
</tr>
<tr>
<td>11.3</td>
<td>Linearization</td>
<td>255</td>
</tr>
<tr>
<td>11.4</td>
<td>Linearized Equations of Motion</td>
<td>258</td>
</tr>
<tr>
<td>11.5</td>
<td>Dynamic Equations with Specified Input Motions</td>
<td>260</td>
</tr>
<tr>
<td>PROBLEMS</td>
<td></td>
<td>261</td>
</tr>
<tr>
<td>12</td>
<td>Equilibrium Posture Analysis</td>
<td>262</td>
</tr>
<tr>
<td>12.1</td>
<td>Introduction</td>
<td>262</td>
</tr>
<tr>
<td>12.2</td>
<td>Seeking a Nearby Posture of Equilibrium</td>
<td>263</td>
</tr>
<tr>
<td>12.3</td>
<td>Seeking Equilibrium with Some Generalized Coordinates Specified</td>
<td>266</td>
</tr>
<tr>
<td>12.4</td>
<td>Large Increments of the Generalized Coordinates</td>
<td>266</td>
</tr>
<tr>
<td>12.5</td>
<td>Stable versus Unstable Equilibrium</td>
<td>267</td>
</tr>
<tr>
<td>12.6</td>
<td>Postures of Neutral Equilibrium</td>
<td>269</td>
</tr>
<tr>
<td>REFERENCE</td>
<td></td>
<td>270</td>
</tr>
<tr>
<td>PROBLEM</td>
<td></td>
<td>270</td>
</tr>
<tr>
<td>13</td>
<td>Frequency Response of Mechanisms and Multibody Systems</td>
<td>271</td>
</tr>
<tr>
<td>13.1</td>
<td>Introduction</td>
<td>271</td>
</tr>
<tr>
<td>13.2</td>
<td>Homogeneous First-order Equations of Motion</td>
<td>271</td>
</tr>
<tr>
<td>13.3</td>
<td>Modal Coordinates</td>
<td>274</td>
</tr>
<tr>
<td>13.4</td>
<td>Laplace Transformed Equations of Motion</td>
<td>276</td>
</tr>
<tr>
<td>13.5</td>
<td>Frequency Response</td>
<td>277</td>
</tr>
<tr>
<td>REFERENCES</td>
<td></td>
<td>278</td>
</tr>
<tr>
<td>PROBLEMS</td>
<td></td>
<td>279</td>
</tr>
<tr>
<td>14</td>
<td>Time Response of Mechanisms and Multibody Systems</td>
<td>280</td>
</tr>
<tr>
<td>14.1</td>
<td>Inverse Laplace Transform</td>
<td>280</td>
</tr>
<tr>
<td>14.2</td>
<td>Cauchy’s Residue Theorem</td>
<td>282</td>
</tr>
<tr>
<td>14.3</td>
<td>Systems with Repeated Eigenvalues</td>
<td>284</td>
</tr>
<tr>
<td>14.4</td>
<td>Time Integration Algorithm</td>
<td>288</td>
</tr>
<tr>
<td>14.5</td>
<td>Adaptive Time-step Control</td>
<td>291</td>
</tr>
<tr>
<td>REFERENCES</td>
<td></td>
<td>292</td>
</tr>
<tr>
<td>PROBLEM</td>
<td></td>
<td>293</td>
</tr>
<tr>
<td>15</td>
<td>Collision Detection</td>
<td>294</td>
</tr>
<tr>
<td>15.1</td>
<td>Introduction</td>
<td>294</td>
</tr>
<tr>
<td>15.2</td>
<td>Vertex-Face Contact</td>
<td>295</td>
</tr>
<tr>
<td>15.3</td>
<td>Edge-Edge Contact</td>
<td>296</td>
</tr>
<tr>
<td>15.4</td>
<td>Finding the Time Increment until Contact</td>
<td>297</td>
</tr>
<tr>
<td>REFERENCES</td>
<td></td>
<td>299</td>
</tr>
</tbody>
</table>
## Contents

### 16 Impact Analysis

16.1 Applied Impulsive Loads ........................................ 300
16.2 Location and Type of Contact ................................. 303
16.3 Simple Impact Model ............................................. 303
16.4 Impact Model with Tangential Impulse ........................ 305
16.5 Impact Model with Normal Torsional Impulse ............... 306
16.6 Impact Model with Moment Impulse ........................... 307
16.7 Integrated Model of Impact ..................................... 307
16.8 Impact Analysis with SGCs ..................................... 308
REFERENCES .................................................................. 309
PROBLEM ..................................................................... 309

### 17 Constraint Force Analysis

17.1 Introduction ............................................................. 310
17.2 Fictitious Displacements .......................................... 311
17.3 Fictitious Derivatives ............................................... 313
17.4 Lagrange Equation for Constraint Force .................... 316
REFERENCES .................................................................. 320
PROBLEMS ................................................................. 320

Index ............................................................................. 321
This text presents a uniform and comprehensive treatment of the theory and use of homogeneous coordinates and transformation matrices in the kinematic and dynamic design analysis and the numeric simulation of mechanisms and multibody systems.

The following observations, originally set down by Reuleaux in 1875,1 are every bit as true today, and it would be difficult to state them better.

The whole study of the constitution of machines – the Kinematics of Machinery – naturally divides itself into two parts, the one comprehending the theoretical and the other the applied or practical side of the subject; of these the former alone forms the subject of this work. It deals chiefly with the establishment of those ideas which form the foundation of the applied part of the science, and in its treatment of these its method differs in great part essentially from those heretofore employed.

As I have here to do chiefly with theoretical questions, it might seem that I could hardly expect to interest other than those concerned only with the theoretical side of this special study. But Theory and Practice are not antagonists, as is so often tacitly assumed. Theory is not necessarily unpractical, nor Practice unscientific, although both of these things may occur. Indeed in any department thoroughly elucidated by Science the truly practical coincides with the theoretical, if the theory be right. The popular antithesis should rather be between Theory and Empiricism. This will always remain, and the more Theory is extended the greater will be the drawback of the empirical, as compared with the theoretical methods. The latter can never be indifferent, therefore, to any who are able to use them, even if their work be entirely “practical,” and although they may be able for a while longer to get on without them. The theoretical questions, however, which are here to be treated, are of so deep-reaching a nature that I entertain the hope that those who are practically, as well as those who are theoretically concerned with the subject, may obtain help from the new method of treating them.

Certainly, the science of kinematics has grown a great deal and today rests on a much firmer foundation than it did in Reuleaux’s time. However, to a great extent, the gulf between theory and empiricism still exists. On the one hand, we find that

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academics have developed a vast body of science, steeped in the elegance and sophistication traditional to their views. However, their efforts, almost to an individual, are still directed toward further understanding of the four-bar linkage, the slider-crank mechanism, and, more recently, the robotic manipulator, and rather simple multibody systems. On the other hand, even today, we find that the inventor – the completely practical person who develops a working machine or performs an analysis of a complex multibody system, despite the richness of modern theoretical developments – finds very little of modern theory truly usable as a practical design technique or as a broadly applicable computational analysis tool.

Among the several reasons for this paradox is the fact that modern theoretic approaches are difficult for the novice to comprehend and, by the very nature of the problem, are quite tedious to apply. A thorough understanding of these methods takes years of specialized study and, very likely, we find in the end that they do not really solve the complex problems encountered in the design or analysis of present-day equipment. Thus, to be of value, the methods presented in the following chapters must accomplish two apparently conflicting goals. First, they must be applicable to an extremely broad category of problems, including the large multifaceted problems represented in the design of modern machinery and analysis of complex multibody systems. Secondly, they must be put into a form that is useful to the practicing engineer without years of advanced study.

It is our firm belief that the sole hope for accomplishing both of these purposes lies with the development of a unified and powerful analytic method that can be programmed for solution by computer. Only in this way can the more sophisticated methods be made usable without requiring significant specialized training of every user. Also, this is the only apparent method of dealing with some of the more complex mechanisms and multibody systems, if only because the number of calculations involved would be prohibitive by any other means. If sufficiently general software can be written, however, the application of even the most sophisticated theoretical approach to very complex multibody systems becomes a feasible goal. In presenting such a general approach, however, we will be careful, from time to time, to also present alternative – less general, but perhaps more intuitive – approaches. This is intended to provide a balanced and better understanding of the methods presented, and to illustrate the power of the more general techniques.

Furthermore, much of the more recent trends toward miniaturization and high performance for mechanisms necessitate the inclusion of dynamic analysis along with kinematics. In the broader category of multibody systems, dynamic analysis has always played a key role. However, this book deals with an integrated approach to both kinematic and dynamic analyses. The transformation matrix techniques presented are general and fully applicable to systems in either two or three dimensions. In addition, they lend themselves to programming and digital computation and can, therefore, be the basis of a usable tool for the designer. This book may appear to place more emphasis on mechanisms because much of the techniques have their roots in the kinematics and design literature. However, the techniques have broad applicability to the design analysis of all multibody mechanical systems.

Another pitfall one must avoid when taking a general approach is that of replacing the effort a designer or an engineer must spend in learning and applying the analysis procedures with an equal or worse task of writing and testing complex
computer programs. Whatever methods proposed for real design and analysis use in the future – it seems to the authors – must include the generality and flexibility to handle a very broad class of problems and give a thorough analysis, without requiring separate programming for each new problem. Only in this way can real usability be achieved. The more powerful and more flexible the approach, and the less specialization and reprogramming required for each application, the better.

The transformation matrix methods presented in the following chapters have been developed using these as primary goals. The reader must keep these firmly in mind throughout the book; they are essential to the appreciation and perhaps even to the comprehension of the methods. Although the transformation matrix methods can be applied by hand to such problems as the slider-crank mechanism, this is not the intent of this text, and often the rigor required for such an attempt becomes quite burdensome in comparison with other techniques.

The transformation matrix methods have been extensively tested, both in the classroom and in engineering industry. In the classroom, the authors have tested the drafts of this text in senior/graduate-level courses at the University of Wisconsin–Madison and the University of Virginia, and more recently at the University of California, Davis, and we are indebted to all of those students for their trials and suggestions for improvements. As for use in engineering industry, the methods presented herein have been the basis for the software system known as the Integrated Mechanisms Program (IMP). First released in 1972, IMP has been extensively used in many companies and academic institutions to analyze many different kinds of mechanical systems. Although it is still not a perfect tool, IMP continues to be used, and its many users also deserve much credit for the authors’ insights and the experience reflected in the methods described herein.

Developing methods for computer solution requires several radical alterations in the approach taken from those of more traditional methods. It requires simplicity and precision, almost to a fault. Because the computer has no reasoning capability, any possible conflict in interpretation of the user’s intent will result in disaster. Definitions of terms must be extremely precise; identification of parts must be unique; sign conventions must be established, once and for all, in a clear understandable manner; and the sequencing of the solution process must take every possible eventuality into account, even those cases that seem trivial in the rational world of humans.

Again, Reuleaux expresses our thoughts very well:

> The remodeling which has become necessary requires undisturbed adherence to clear, simple, logical principles. What, however, is to be drawn from our criticism of the system heretofore used – what I have endeavored to illustrate and develop by single instances – what the philosophical sentences I have quoted bring before us in a condensed form – we may contract into one word. So far as our special problem is concerned, the question is to make the science of machinery deductive. The study must be so formed that it rests upon a few fundamental truths peculiar to itself. The whole fabric must be reducible to their

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3 For an up-to-date version of IMP in open-source form (GNUPL, version 3), the reader should see [http://code.google.com/p/impsim/](http://code.google.com/p/impsim/+).

4 Reuleaux, *op. cit.*
strictness and simplicity, and from them again we must be able, conversely, to develop it. Here again is a point from which the weakness of the method heretofore employed can be surveyed at a glance. Its difference from the ideal method is not that it employs the inductive instead of the deductive method; that indeed would be no advantage but it might still be defensible. No, it has been entirely unmethodical. It has chosen no fixed method of investigation, or rather, it has not found any in spite of zealous search; indeed it has so often cried “Eureka” that it now rests quietly in the impression that such fixed standpoint has really been found.
About the Authors

John J. Uicker is Professor Emeritus of mechanical engineering at the University of Wisconsin–Madison. He received his BME degree from the University of Detroit, and his MS and PhD degrees in mechanical engineering from Northwestern University with Professor J. Denavit as his advisor. He joined the University of Wisconsin faculty in 1967, where he served until his retirement in 2007. Throughout his career, his teaching and research have been in solid geometric modeling and the modeling of mechanical motion, and their application to computer-aided design and manufacture; these include the kinematics, dynamics, and simulation of articulated rigid-body mechanical systems. He was the founder of the UW Computer-Aided Engineering Center and served as its director for its initial ten years of operation. He is a member of the American Society of Mechanical Engineers (ASME); the American Society for Engineering Education (ASEE); Society of Automotive Engineers (SAE); and the Tau Beta Pi, Sigma Xi, and Pi Tau Sigma professional engineering honor societies. He has served on several national committees of ASME and SAE, and he received the ASME Mechanisms Committee Award in 2004 and ASME Fellow Award in 2007. He is one of the founding members of the U.S. Council for the Theory of Mechanism and Machine Science (USCToMM) and of IFToMM, the International Federation of Mechanism and Machine Science. He served for several years as editor-in-chief of the Mechanism and Machine Theory journal of the federation. He is a registered mechanical engineer in Wisconsin and has served for many years as an active consultant to industry.

As an ASEE Resident Fellow, he spent 1972–73 at Ford Motor Company. He was also awarded a Fulbright-Hayes Senior Lectureship and became a visiting professor to Cranfield Institute of Technology in Cranfield, England, in 1978–79. After graduate study under the originators, Professors Denavit and Hartenberg, he became the pioneering researcher on transformation matrix methods of linkage analysis, and he was the first to advance their use into the dynamics of mechanical systems. He has been awarded twice for outstanding teaching, three times for outstanding research publications, and twice for historically significant publications.
About the Authors

Bahram Ravani is a professor of mechanical engineering at University of California, Davis. He received his BS degree Magna Cum Laude from Louisiana State University in Baton Rouge; his MS degree, with distinction, from the College of Engineering, Columbia University in New York; and his PhD degree from Stanford University, in Stanford, California, all in mechanical engineering. From 1982–87, he was on the faculty of mechanical engineering at the University of Wisconsin–Madison, first as an assistant professor and later as a tenured associate professor. He then joined the University of California, Davis, where he has been performing teaching and research in the areas of kinematics and dynamics, mechanical design, robotics and mechatronics, collision mechanics, and biomechanics. He served as the chair of the department of Mechanical and Aeronautical Engineering, as well as the interim chair of Electrical and Computer Engineering. At Davis, he is also a member of the graduate program in Biomedical Engineering and the graduate program in Forensic Science and Engineering. In 1985, he was on leave from the University of Wisconsin and worked for the Manufacturing Systems Product Division of IBM Corporation in Boca Raton, Florida. He was also a visiting professor in the Department of Mechanical and Production Engineering at the Katholieke Universiteit of Leuven in Belgium during the summer of 1987. In 1987, he was the recipient of the Young Manufacturing Engineer award from the Society of Manufacturing Engineers, and in 1993 he was the recipient of the Gustus L. Larson Memorial award from ASME for outstanding achievements in mechanical engineering within 10 to 20 years following graduation. In 1997, he received the Design Automation award from the Design Engineering Division of ASME for his lifetime of sustained contributions to the field of design automation. In 2005, he was the recipient of the Machine Design Award of ASME for eminent achievements in mechanical design. He was the technical editor of the ASME Journal of Mechanical Design from 1993–97 and is presently the technical editor of ASME’s Journal of Computers and Information Science in Engineering. He is a Fellow of ASME and is the former chair of the Design Engineering Division of ASME. He is also a member of the Society of Automotive Engineers, the International Society of Biomechanics, and the Association for Advancement of Automotive Medicine.

Pradip N. Sheth (deceased, January 2009) was born in Baroda/Vadodara, India, in 1944. He earned his BE degree in mechanical engineering at Maharaja Sayajirao University, Baroda, India, in 1965, and earned his MS in 1968 and PhD in 1972, in mechanical engineering at the University of Wisconsin–Madison where Professor John Uicker served as his advisor. In his research, he developed the Integrated Mechanisms Program (IMP) for the computer-aided design and analysis of multibody mechanical systems. More than 200 industrial and educational organizations worldwide have used this system, which was the first of its kind.
Pradip was a Postdoctoral Fellow at the University of Michigan from 1972–74 and developed a vehicle analysis program as a research project for Ford Motor Company. From 1974–85, he was manager of Engineering Development at Allis-Chalmers Corporation. From 1985, Pradip served on the faculty of the Department of Mechanical and Aerospace Engineering at the University of Virginia (UVA) where he came to help establish a Master’s degree program in Manufacturing Systems Engineering. This program was offered locally as well as through the educational television system, and it has produced hundreds of Master of Engineering graduates.

Pradip worked with industry throughout his academic career. He served as regional coordinator for the Manufacturing Action Program of the Virginia Center for Innovative Technology, where he worked with 75 Virginia manufacturing companies. He also helped to establish a Manufacturing Extension Program in Martinsville, Virginia. The Philpott Manufacturing Center is now a statewide resource, partially funded by the National Institute of Standards and Technology. He was also an active participant in UVA’s Rotating Machinery and Controls Consortium and the Kluge Rehabilitation Center. He designed consumer products and specialized systems that vary from lawn mowers and tractors to pole-climbing robots, and an array of assistive devices for physically impaired individuals. He held several patents.

Pradip was a member of the American Society of Mechanical Engineers, the American Society for Engineering Education, the Society of Manufacturing Engineers (SME), and the Sigma Xi and Pi Tau Sigma engineering honor societies.