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Excerpt
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1 Concepts and Definitions

1.1 Mechanical Design: Synthesis versus Analysis

There are two completely different aspects of the study of mechanical systems: *design* and *analysis*. The concept embodied in the word *design* might be more properly termed *synthesis*, the process of contriving a scheme or a device for accomplishing a given purpose. Design is the process of developing the sizes, shapes, material compositions, types and arrangements of parts, and manufacturing processes so that the final system will perform a prescribed task. Although there are many phases of the design process that can be approached in a well-ordered scientific manner, the process is, by its very nature, as much an art as a science. It calls for imagination, intuition, creativity, judgment, and experience. The role of science in the design process can be viewed as providing tools to be used as the designer practices this art. Computer programs and computations that allow a designer to simulate a system and evaluate its potential performance play an important role in helping the designer practice the art. This is why scientific techniques such as the matrix methods discussed in this text play such an important role in dealing with the design of three-dimensional mechanisms and multibody systems.

In the *synthesis* of a mechanical system, from a functional point of view, there are three basic stages that correspond approximately to three basic steps in the design process. The first stage is designated *type synthesis*; it deals with the fundamental decisions a designer makes regarding the style of machine, device, or system to be used. Initially, for example, such decisions include whether a mechanical device should be used at all, or whether an electronic circuit or hydraulic appliance should be chosen instead. After deciding on the use of a mechanism or multibody system, for example, we must then ponder the relative merits of linkages as compared with gear trains or perhaps belts and pulleys.

Once the type synthesis has been accomplished, we have established some general boundaries for the overall system; further study must then go into specifying its basic internal characteristics. The numbers of parts and the types and numbers of joints connecting them must be decided. This process is called *number synthesis*. At this stage, we do not concern ourselves with the detailed shapes of the parts or their strength or wear characteristics, but we are concerned with their

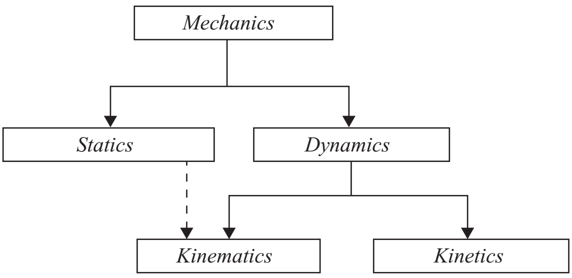


Figure 1.1. The science of mechanics.

overall arrangement. Typical questions considered at this stage include "Will this configuration have the desired degrees of freedom, and can it provide the functionality that is intended?"

Given at least tentative answers to these questions, we are in a position to attempt the third step, *dimensional synthesis*. It is here that we assign dimensions, materials, weights, strengths, and other properties to each of the members or parts of the design. Either by calculation, by experiment, or by intuition and experience, we make all of the detailed decisions that are necessary before the product or system can be manufactured. It is during the process of evaluating the various interacting alternatives and choosing among them that we find the need for a collection of mathematical and scientific methods in the hope of finding at least a valid – and perhaps even an optimal – selection for the given task. These scientific tools do not make decisions for us; we have every right to exert our imagination and creative abilities, even to the extent of overriding mathematical recommendations. Science-based techniques are useful, however, in generating, comparing, and judging various alternatives.

Probably the largest collection of scientific methods at our disposal falls into a category called *analysis*. These are the techniques that allow us to critically examine an already existing or proposed design in order to judge its suitability for a given task. Thus, in itself, analysis is not a creative science, but rather is used for evaluating and rating things already conceived. In fact, it can be used to help the creative process by allowing a formal evaluation of a design and allowing the designer to accept or dismiss a concept or to find ways to improve it. Therefore, analysis is a useful tool in redesign or design improvement, and can be integrated with the creative process. We should always bear in mind, however, that although the majority of our efforts may be spent on analysis, the real goal is synthesis – the design of a product or system. Analysis is simply a tool. It is, however, a vital tool and will invariably be used during the design process. This is particularly true when the analysis techniques lend themselves to computer software and programmed computations because this allows a designer to simulate different concepts and compare the performance of competing design alternatives.

The branch of scientific analysis that deals with motions and forces in a mechanical system is called *mechanics*. As shown in Figure 1.1, it is made up of two parts, called *statics* and *dynamics*. Statics deals with the analysis of stationary systems, that is, those in which time is not a factor. Dynamics, on the other hand, deals with systems that change with time.

Dynamics is also made up of two major disciplines. The great Swiss mathematician, Leonhard Euler (1707–83), was the first to distinguish these [2]:

> The investigation of the motion of a rigid body may be conveniently separated into two parts, the one geometrical, the other mechanical. In the first part, the transference of the body from a given position to any other position must be investigated without respect to the causes of the motion, and must be represented by analytical formulae which will define the position of each point of the body after the transference with respect to its initial placement. This investigation will therefore be referable solely to geometry, or rather to stereotomy [the art of stone-cutting].
>
> It is clear that by the separation of this part of the question from the other, which belongs properly to Mechanics, the determination of the motion from dynamic principles will be made much easier than if the two parts were undertaken conjointly.

These two aspects of dynamics were later recognized as the distinct sciences of *kinematics* and *kinetics*, which treat the motion and the forces producing it, respectively. Kinematics was first defined as a separate study by the French mathematician and physicist, André Marie Ampère (1775–1836). He chose the French name *cinématique* from the Greek word $\kappa\iota\nu\eta\mu\alpha$ (*kinema*), meaning motion [1]. An interesting narrative on the history of kinematics is found in [3, pp. 1–27].

The field of kinematics, however, has grown to include not only the geometric part of dynamics but also those aspects of statics that deal with the geometry, but not the magnitudes, of the system of forces acting on the bodies. For this reason, Figure 1.1 shows a dashed line indicating the interaction of kinematics with statics. This should not be surprising because there is a well-established duality between the geometry of a system of forces and a set of velocities in kinematics.

The predominant problem in multibody system analysis, as will become evident, is often one of kinematics – a topic of major emphasis in this book. Statics and kinetics, however, are also important parts of any complete design analysis, and these topics are also covered in detail.

1.2 Multibody Systems and Mechanisms

A multibody system can be defined as a collection of bodies (mechanical parts) in which some or all of the bodies may be interconnected by joints that constrain the relative motions between the joined bodies. However, the presence of joints or connections is not an absolute requirement for a multibody system; the bodies may be "restrained," rather than constrained, by interconnections with other bodies by elements such as springs or dampers. There are a number of abstract concepts that must be further considered for a rigorous understanding and for purposes of modeling a multibody system; these include (1) body, (2) joint, (3) constraint, (4) restraint, (5) spring, and (6) damper.

The general definition of a multibody system covers a very large variety and many different kinds of mechanical systems. The radio-controlled model car shown in Figure 1.2 is one example of a multibody system.

The NASA Mars Exploration Rover, Figure 1.3, is another example of a multibody system. Biomechanical models of the human body, as shown in Figure 1.4, and

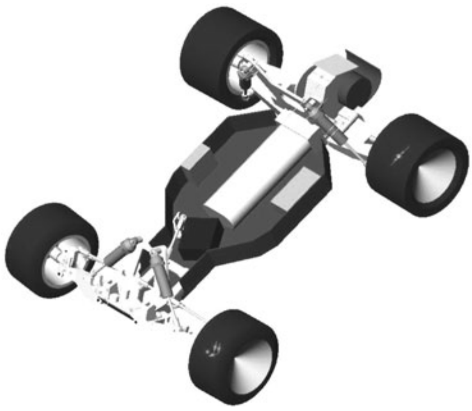
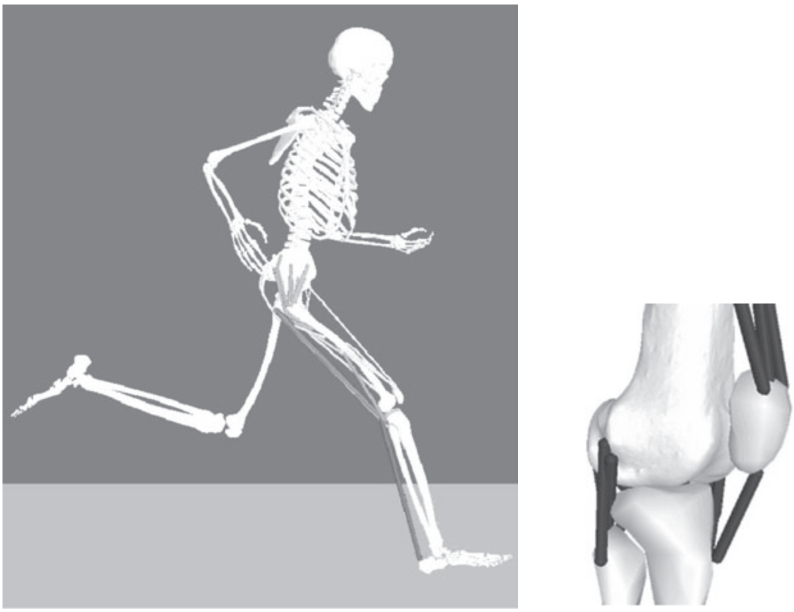


Figure 1.2. Multibody model of a radio-controlled car showing the front and rear suspension systems.

Figure 1.3. NASA Mars Exploration Rover.



(a) (b)

Figure 1.4. (*a*) A biomechanical model for studying human gait, (*b*) Detailed model of a human knee. (Courtesy Prof. Darryl Thelen, University of Wisconsin, Madison, WI).

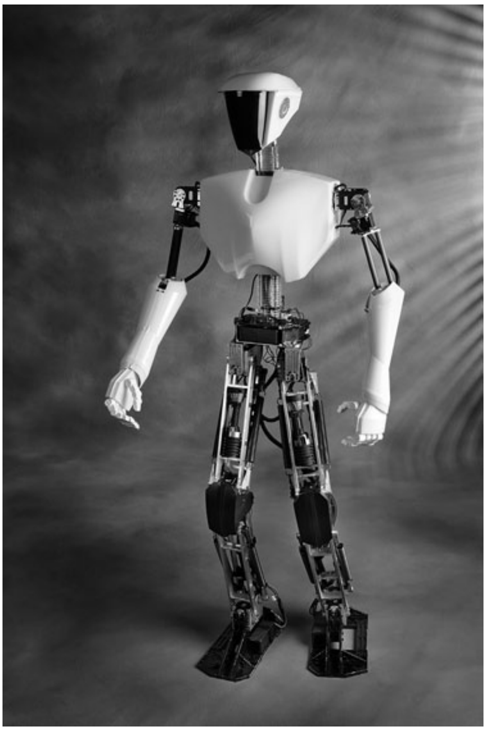


Figure 1.5. Humanoid robot CHARLI-2, winner of the RoboCup 2011 World Soccer Competition (Courtesy John McCormick and Prof. Dennis Hong, Robotics and Mechanisms Lab, Virginia Polytechnic Institute, Blacksburg, VA).

also bipedal walking robots, such as that in Figure 1.5, represent additional examples of multibody systems.

The Gough/Stewart platform, shown in Figure 1.6, has been a popular system for a number of applications since the 1960s, including many recent adaptations in parallel robotic systems.

Parallel (Figure 1.7) and serial (Figure 1.8) manipulators are also examples of multibody systems.

Mechanisms constitute an important category of multibody systems. Of course, the variety of possible systems is unlimited. One example of a mechanism is the automotive suspension system shown in Figure 1.9.

Speaking rigorously, a *mechanism* is defined as an assemblage of mechanical bodies, movably connected by joints to form a mechanical system with one body fixed and having the purpose of transforming motion. Whereas a mechanism is

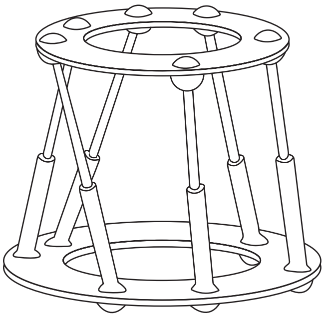


Figure 1.6. The Gough/Stewart platform. Parallel (Figure 1.7) and serial (Figure 1.8) manipulators are also examples of multibody systems.

Figure 1.7. Parallel manipulators.

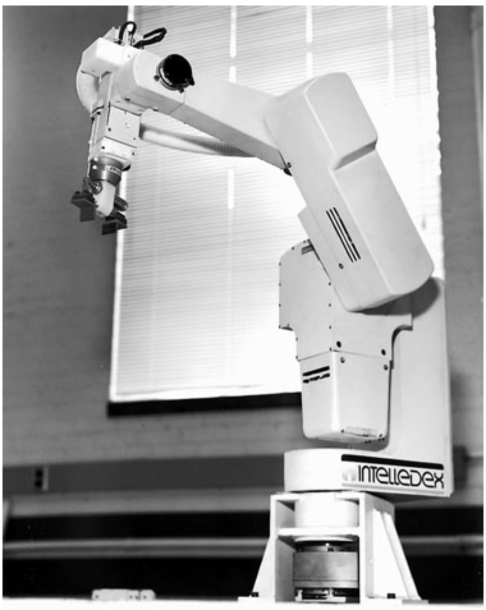


Figure 1.8. Serial manipulator.

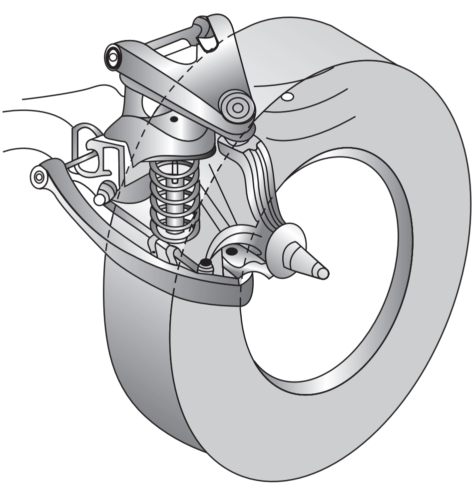


Figure 1.9. Automotive independent front suspension mechanism.

considered to have one of its bodies fixed, a general multibody system, in contrast, may be an unrooted, free-floating system. This definition of a mechanism includes several terms that must themselves be more precisely defined, which is the inherent pitfall of any first definition. However, a start must be made somewhere, and as such, this definition is perhaps as good as any.

Much of the material in this chapter is based on definitions originally established by Professor Franz Reuleaux (1829–1905), a German kinematician whose work [6] marked the beginning of a systematic treatment of kinematics. For an English translation, including additional reading, see British engineer and academic, Alexander Blackie William Kennedy (1847–1928) [5]. Reuleaux's second book [7] also made a lasting impression but, unfortunately, has not been translated into English.

Some light is shed on the meaning of the word "mechanism" by discussing what is *not* meant. Let us distinguish first between the words "mechanism" and "structure." A *structure* is also an assemblage of mechanical bodies connected by joints, but its purpose is definitely not to transform motion. A structure, such as a truss, is intended to be rigid. It can, perhaps, be mobile in the sense of being movable from place to place. However, it has no internal mobility; no relative motion takes place between its parts or members. A mechanism, on the other hand, does have this freedom among its various members to move relative to one another. Indeed, the whole purpose of a mechanism is to utilize these relative motions for transforming or modifying a given input motion to produce a different output motion. For example, a shaft set in a pair of bearings is not a mechanism, because the intent is to transmit the input motion to the output, rather than to transform it, but it can be viewed as a multibody system. A speed-reducing set of gears between input and output shafts, on the other hand, does form a mechanism.

This brings us to distinguishing between the words "machine" and "mechanism." A *machine* is an assemblage of fixed and moving bodies for doing work, a device for applying power or changing its direction. It differs from a mechanism in its purpose. In a machine, force, torque, work, and power are the predominant concepts. In a mechanism, even though it may transmit power or force, the predominant concept is one of altering motion. Both machines and mechanisms are multibody systems with multiple masses and may contain elements such as springs and frictional damping elements.

1.3 Planar, Spherical, and Spatial Mechanisms

Mechanisms, like many other things, may be categorized in several different ways in order to emphasize their similarities and differences. One such grouping divides mechanisms into planar, spherical, and spatial categories. Of course, all three groups have many things in common, but there must also be some criterion to distinguish them. In this instance, the criterion is found in the characteristics of the motions of the individual bodies.

A *planar* mechanism is one in which all moving points describe planar curves and in which all of these curves lie in parallel planes. That is, the loci of all points are planar curves, all parallel to a common plane. Owing to this characteristic, it is possible to represent the locus of any chosen point in its true size and shape in a single drawing or figure. The motion transformation of any such mechanism is

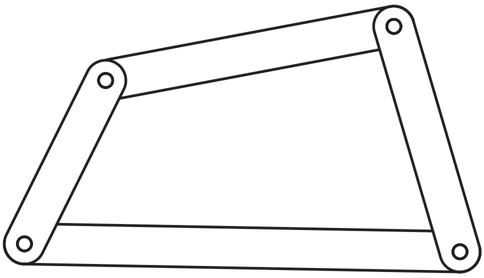


Figure 1.10. Planar four-bar linkage.

called *coplanar*. The planar four-bar linkage (Figure 1.10), the disk-cam and follower (Figure 1.11), and the slider-crank mechanism (Figure 1.12) are familiar examples of planar mechanisms.

A *spherical* mechanism is one in which each moving body (or its extension) has one point that remains stationary as the system moves, and in which the stationary points of all bodies lie at a common location. That is, the locus of any point is a curve contained in a spherical surface and the spherical surfaces defined by arbitrarily chosen points are all concentric. The motions of all particles, therefore, can be completely described by their radial projections on the surface of a sphere with a properly chosen center. The Cardan/Hooke universal joint (Figure 1.13) is perhaps a familiar example of a spherical mechanism.

Spatial mechanisms, on the other hand, include no restrictions on the relative motions of their bodies. The motion transformation is not necessarily coplanar, nor must it be concentric. A spatial mechanism may have particles with loci of double curvature. Any linkage that contains a screw joint, for example, is a spatial mechanism, because the relative motion within a screw joint is helical. Examples

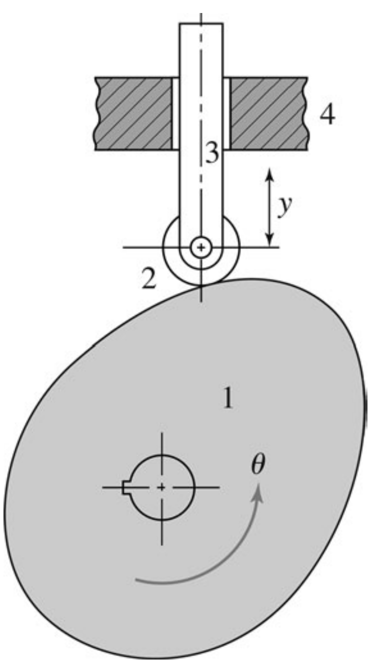


Figure 1.11. Disk-cam and follower.

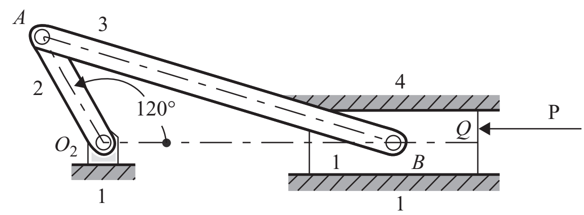


Figure 1.12. Slider-crank mechanism.

of spatial mechanisms are industrial robots (Figure 1.14) and the human skeletal system (Figure 1.4).

It should be pointed out that the overwhelmingly large category of planar mechanisms as well as the category of spherical mechanisms are only special cases or subsets of the all-inclusive category – *spatial* mechanisms. They occur as a consequence of special geometry in the locations and orientations of their joint axes. Unique geometric situations yield their own particular mechanisms.

If planar and spherical mechanisms are only special cases of spatial mechanisms, why is it desirable to identify them separately? Because of the particular geometric conditions that identify these types, simplifications are possible in their design and analysis. As previously mentioned, it is possible to observe the motions of all points of a planar linkage in true size and shape from a single direction. In other words, all motions can be represented graphically in a single view. Thus, graphic techniques are well suited to their analysis, as demonstrated by the abundance of texts such as [9] on the kinematics of mechanisms. Because spatial mechanisms do not enjoy this special geometry, visualization can become difficult, and more powerful techniques are needed for their analysis.

Because the vast majority of mechanisms in use today are planar, we may question the need for the more complicated techniques developed in later chapters. There are several reasons why more powerful methods are of value for such systems, even though the "simpler" graphic techniques may have been mastered. First, they provide new, alternative methods that solve problems in a different way. Thus, they provide a means for checking results. Certain problems by their nature may be more amenable to one method than to another. Second, methods that are analytic in nature are better suited to solution by digital computation than are graphic techniques and, therefore, can be analyzed with higher accuracy. Third, even though

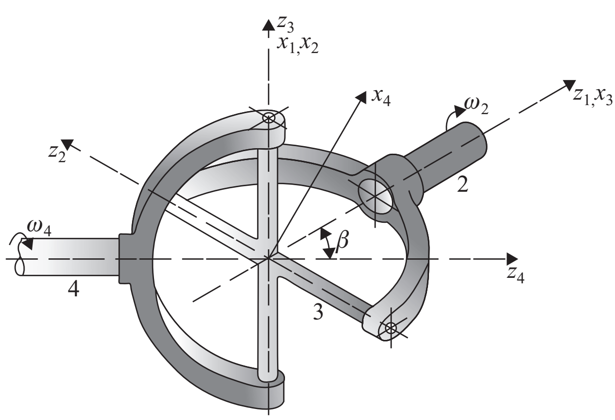


Figure 1.13. Cardan/Hooke universal joint.

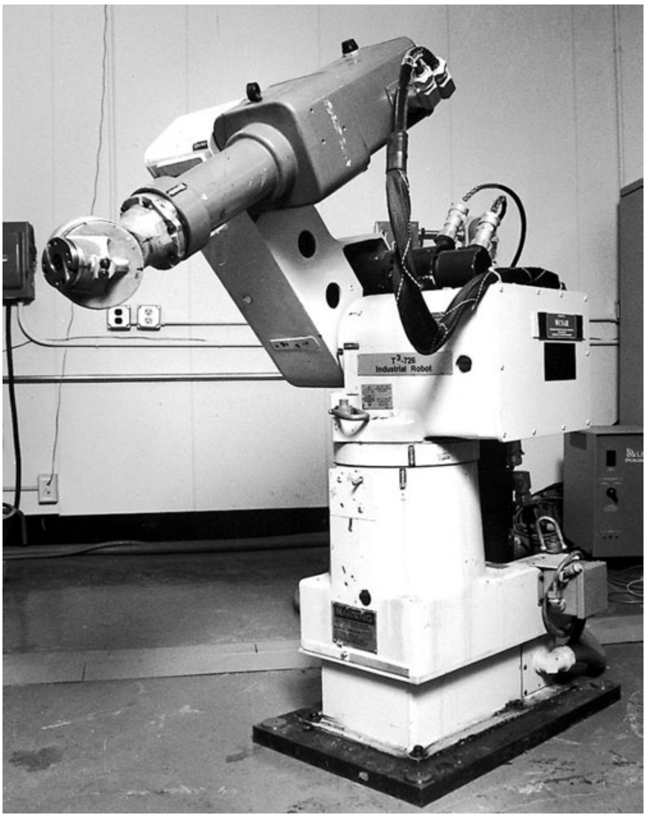


Figure 1.14. An industrial robot.

the majority of useful linkages are planar and well suited to graphic solution, the few remaining must also be analyzed, and techniques should be known for analyzing them. Fourth, a possible reason that planar linkages are so common is that good methods for analysis of the more general spatial systems have not been readily available until recent years. Therefore, their design and use have not been common, even though they may be inherently better suited in certain applications.

Finally, spatial mechanisms are much more common in practice than their formal description indicates. Consider a "planar" four-bar linkage (Figure 1.10). It has four bodies connected by four pin joints whose axes are "parallel." This parallelism is a mathematical hypothesis; it is not a reality. The joint axes, as produced in a shop – in any shop, no matter how good – are only approximately parallel. If the axes are nearly parallel, the system operates because of looseness in the bearings or flexibility of the bodies. If the joint axes are far out of parallel, there is binding in no uncertain terms, and the system only moves because the bodies flex and twist, producing loads in the bearings. A common way of compensating for non-parallelism is to connect the bodies with self-aligning bearings, actually spheric joints allowing three-dimensional rotation. Such a "planar" linkage is, thus, really a low-grade spatial mechanism.

1.4 Mechanical Body

Let us now look more closely at a term that has been used frequently in previous sections. The term is "body," or more precisely, "mechanical body." In this text, a