

STOCHASTIC PHYSICS AND CLIMATE MODELLING

This is the first book to promote the use of stochastic, or random, processes to understand, model and predict our climate system.

One of the most important applications of this technique is in the representation in comprehensive climate models of processes that, although crucial, are too small or fast to be explicitly modelled. The book shows how stochastic methods can lead to improvements in climate simulation and prediction, compared with more conventional bulk-formula parameterisation procedures.

Beginning with expositions of the relevant mathematical theory, the book moves on to describe numerous practical applications. It covers the complete range of time scales of climate variability, from seasonal to decadal, centennial and millennial.

With contributions from leading experts in climate physics, this book is invaluable to anyone working on climate models, including graduate students and researchers in the atmospheric and oceanic sciences, numerical weather forecasting, climate prediction, climate modelling and climate change.

TIM PALMER is a Royal Society Research Professor at the University of Oxford and a Senior Scientist at the European Centre for Medium-Range Weather Forecasts (ECMWF). He has won the Royal Society Esso Energy Award, the Royal Meteorological Society Adrian Gill Prize and the American Meteorological Society Jule Charney Award. He is a fellow of the Royal Society, the Royal Meteorological Society, the American Meteorological Society and Academia Europaea. He is a lead author of the Intergovernmental Panel on Climate Change (IPCC), co-chair of the Scientific Steering Group of the UN World Meteorological Organisation's Climate Variability and Predictability (CLIVAR) project and coordinator of two European Union climate prediction projects (PROVOST and DEMETER). He has had numerous appearances on radio and TV, in relation to weather, climate and chaos theory, and has co-edited another book with Cambridge University Press – *Predictability of Weather and Climate* – in 2006.

PAUL WILLIAMS is a Royal Society Research Fellow at the National Centre for Atmospheric Science (NCAS) and the Department of Meteorology, University of Reading. He has won the Royal Astronomical Society Blackwell Prize (2004) and the Royal Meteorological Society Rupert Ford Travel Award (2005) and has received a prestigious Crucible Fellowship from the National Endowment for Science, Technology and the Arts (2007). He was the lead author of a climate change report commissioned and published by the European Parliament (2004). He is a Fellow of the Royal Meteorological Society, the Institute of Physics, and the Royal Astronomical Society. His research findings have been reported widely in the media, including feature articles in *New Scientist* and the *Financial Times*, and a panel discussion on BBC Radio 4.

STOCHASTIC PHYSICS AND CLIMATE MODELLING

TIM PALMER

European Centre for Medium-Range Weather Forecasts, UK and the University of Oxford, UK

PAUL WILLIAMS

National Centre for Atmospheric Science and the University of Reading, UK

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 477 Williamstown Road, Port Melbourne, VIC 3207, Australia
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Contributors

JULIE ALEXANDER, School of Earth and Ocean Sciences, University of Victoria, PO Box 3055 STN CSC, Victoria BC, V8W 3P6, Canada

THOMAS ALLEN, Met Office, FitzRoy Road, Exeter, EX1 3PB, UK

MICHAEL A. W. BALL, Department of Meteorology, University of Reading, PO Box 243, Earley Gate, Reading, RG6 6BB, UK

JUDITH BERNER, ECMWF, Shinfield Park, Reading, RG2 9AX, UK

PETER C. CHU, Department of Oceanography, Code OC/CU, Naval Postgraduate School, 833 Dyer Road, Monterey, CA 93943, USA

TIMOTHY DELSOLE, Center for Ocean-Land-Atmosphere Sciences, 4041 Powder Mill Rd, Suite 302, Calverton, MD 20705, USA

FRANCESCA DI GIUSEPPE, ARPA-SIM, viale Silvani, 6, 40122 Bologna, Italy

HENK A. DIJKSTRA, Institute for Marine and Atmospheric Research, Department of Physics and Astronomy, Utrecht University, Princetonplein 5, 3584 CC Utrecht, The Netherlands

FRANCISCO J. DOBLAS-REYES, ECMWF, Shinfield Park, Reading, RG2 9AX, UK

JINQIAO DUAN, Laboratory for Stochastics and Dynamics, Illinois Institute of Technology, Department of Applied Mathematics, Chicago, IL 60616, USA

BRIAN D. EWALD, Department of Mathematics, Florida State University, Room 205C, 1017 Academic Way, Tallahassee, FL 32306–4510, USA

LEELA M. FRANKCOMBE, Institute for Marine and Atmospheric Research, Department of Physics and Astronomy, Utrecht University, Princetonplein 5, 3584 CC Utrecht, The Netherlands

CHRISTIAN FRANZKE, British Antarctic Survey, Natural Environment
Research Council, High Cross, Madingley Road, Cambridge, CB3 0ET, UK

MICHAEL GHIL, Geosciences Department & Laboratoire de Météorologie
Dynamique (CNRS and IPSL), Ecole Normale Supérieure, F-75231 Paris Cedex
05, France

KATRINA HALES, University of California, Los Angeles, Department of
Atmospheric and Oceanic Sciences, Institute of Geophysics and Planetary
Physics, 405 Hilgard Ave, Los Angeles, CA 90095-1565, USA

CHRISTOPHER E. HOLLOWAY, Department of Meteorology, University of
Reading, PO Box 243, Earley Gate, Reading, RG6 6BB, UK

JOCHEM I. JONGMA, Faculty of Earth and Life Sciences, Vrije Universiteit
Amsterdam, De Boelelaan 1085, NL-1081 Amsterdam, The Netherlands

BOUALEM KHOUIDER, Department of Mathematics and Statistics, University
of Victoria, PO BOX 3045 STN CSC, Victoria, BC, V8W 3P4, Canada

RICHARD KLEEMAN, Courant Institute of Mathematical Sciences, New York
University, 251 Mercer Street, New York, NY 10012, USA

DMITRI KONDRASHOV, Department of Atmospheric and Oceanic Sciences,
Institute of Geophysics and Planetary Physics, University of California at Los
Angeles, Los Angeles, CA 90095, USA

SERGEY KRAVTSOV, Department of Mathematical Sciences, Atmospheric
Science Group, University of Wisconsin-Milwaukee, PO Box 413, Milwaukee,
WI 53201, USA

JOHNNY W.-B. LIN, North Park University, Physics Department-Box 30,
3225 W. Foster Ave, Chicago, IL 60625, USA

ANDREW J. MAJDA, Department of Mathematics and Climate, Atmosphere,
Ocean Science (CAOS), Courant Institute of Mathematical Sciences, New York
University, 251 Mercer Street, New York, NY 10012, USA

ADAM H. MONAHAN, School of Earth and Ocean Sciences, University of
Victoria, PO Box 3055 STN CSC, Victoria BC, V8W 3P6, Canada

BALASUBRAMANYA T. NADIGA, MS-B296, Los Alamos National Laboratory,
Los Alamos, NM 87544, USA

List of contributors

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J. DAVID NEELIN, University of California, Los Angeles, Department of Atmospheric and Oceanic Sciences, Institute of Geophysics and Planetary Physics, 405 Hilgard Ave, Los Angeles, CA 90095-1565, USA

TIM N. PALMER, ECMWF, Shinfield Park, Reading, RG2 9AX, UK

CÉCILE PENLAND, NOAA/ESRL/Physical Sciences Division, 325 Broadway, Boulder, CO 80305, USA

OLE PETERS, University of California, Los Angeles, Department of Atmospheric and Oceanic Sciences, Institute of Geophysics and Planetary Physics, 405 Hilgard Ave, Los Angeles, CA 90095-1565, USA

ROBERT S. PLANT, Department of Meteorology, University of Reading, PO Box 243, Earley Gate, Reading, RG6 6BB, UK

MATTHIAS PRANGE, MARUM (Center for Marine Environmental Sciences) and Faculty of Geosciences, Universität Bremen, Fachbereich 5, Klagenfurter Str., D-28334 Bremen, Germany

MICHAEL SCHULZ, MARUM (Center for Marine Environmental Sciences) and Faculty of Geosciences, Universität Bremen, Fachbereich 5, Klagenfurter Str., D-28334 Bremen, Germany

GLENN J. SHUTTS, Met Office, FitzRoy Road, Exeter, EX1 3PB, UK

CHRISTOPHER J. SMITH, Met Office, FitzRoy Road, Exeter, EX1 3PB, UK

ADRIAN M. TOMPKINS, Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, 34014 Trieste, Italy

GEOFFREY VALLIS, AOS Program, Princeton University, Princeton, NJ 08544, USA

ANNA S. VON DER HEYDT, Institute for Marine and Atmospheric Research, Department of Physics and Astronomy, Utrecht University, Princetonplein 5, 3584 CC Utrecht, The Netherlands

ANDREW J. WEAVER, School of Earth and Ocean Sciences, University of Victoria, PO Box 3055 STN CSC, Victoria BC, V8W 3P6, Canada

ANTJE WEISHEIMER, ECMWF, Shinfield Park, Reading, RG2 9AX, UK

DANIEL S. WILKS, Department of Earth & Atmospheric Sciences, Cornell University, Ithaca, NY 14853, USA

Preface

Eleven chapters of this book were originally published as an issue of the *Philosophical Transactions of the Royal Society A: Mathematical, Physical & Engineering Sciences* (Volume 366; Issue 1875). Several chapters have been materially changed and updated. Seven new chapters have been added, which were commissioned specially for this book. We are grateful for the assistance of our Senior Commissioning Editors at CUP, Dr Matt Lloyd and Dr Susan Francis; our Production Editor at CUP, Anna-Marie Lovett; and our copy-editor, Zoë Lewin.

The dynamical evolution equations for weather and climate are formally deterministic. As such, one might expect that solutions of these dynamical evolution equations are uniquely determined by the imposed initial condition. A key purpose of this book is to suggest otherwise.

Before expanding on this seemingly paradoxical claim, let us first outline the reason why the theme of this book is of enormous practical importance. As discussed below, we could legitimately call it a trillion-dollar topic.

While weather forecasting has a long and perhaps chequered history, the present era, whereby predictions are made from numerical solutions of the underlying dynamic and thermodynamic equations, can be traced back to the pioneering work of L. F. Richardson in the early years of the twentieth century. As is well known, the notion that detailed weather forecasts could be made arbitrarily far into the future was dealt a practical blow through the discovery that weather was chaotic, i.e. that weather forecasts are sensitive to small errors in their initial conditions. To some people, the fact that the weather is chaotic seemed to imply that it is hopeless to try to forecast it. However, a fundamental property of any chaotic system is that the degree to which it is predictable is itself a function of the initial state; forecasts from some initial states can be very predictable, even though the system as a whole is chaotic.

To exploit this property of weather as a chaotic dynamical system, methods based on ensemble forecasting have been developed to try to predict when the

weather is predictable and when it is unpredictable. The method is conceptually simple: an ensemble is a collection of forecasts made from almost, but not quite, identical initial conditions. The spread among members of the ensemble gives an estimate of flow-dependent predictability.

In recent years, the ensemble method has become a backbone of numerical weather prediction and is used not only by weather forecasters but also by commercial traders whose activities depend on weather. For example, weather is a dominant driver of many commodities traded in liberalised markets (electricity, gas, coal, oil, crops). Having an estimate of flow-dependent uncertainty in forecasts of weather is critical to the success of such trading, and ensemble weather forecasting is the tool used by the traders to determine this.

Developing practical tools for estimating the uncertainty of a forecast requires a detailed knowledge of the sources of forecast uncertainty. The simple chaotic paradigm discussed above suggests that the only relevant uncertainty lies in the weather observations that determine the initial state of the forecast, e.g. that the measuring instruments are never perfectly accurate or never sufficiently dense in space to determine every small fluctuation in the initial atmospheric state. However, the problem is not nearly as simple as this. Another key source of uncertainty in any weather forecast is the numerical model used to make the predictions.

So let us return to the beginning of this preface. The dynamic and thermodynamic equations are given as deterministic partial differential equations, but are solved by discretisation onto some sort of grid (or spectral or other equivalent representation). Since there are inevitably scales of motion and indeed key processes that are not resolved by this discretisation, methods must be found to represent approximately the subgrid features of the flow. For example, if a global numerical weather prediction problem has a typical grid spacing of 50 km, then all individual cloud systems will be unresolved. For this reason, the numerical equations are ‘closed’ by adding empirically based subgrid parameterisation formulae to represent the effects of the unresolved scales. Hence, for example, convective clouds (e.g. associated with thunderstorms) are represented by convective subgrid parameterisation formulae. Other subgrid parameterisation formulae represent the effects of flow over and around small-scale topography, boundary-layer turbulence and the absorption and emission of radiation in various relevant parts of the electromagnetic spectrum by radiatively active constituents in our atmosphere.

The formulation of these parameterisation formulae is motivated by notions in statistical mechanics. So, just as the momentum transfer by the bulk effects of molecular motions is represented by a diffusive formula, so a similar type of formula might represent the bulk effects of cumulus clouds on vertical temperature, humidity and momentum transfer on the grid scale. However, there is a problem with such an approach. Within a typical 50 km square grid box, there often exist

sufficiently few individual cumulus clouds for the parameterised bulk formula to be an accurate estimate of the subgrid effects.

How can we represent this source of error in ensemble forecasts? This is where the concept of stochastic modelling of the subgrid scales is relevant. By representing model uncertainty through stochastic equations (or more generally by stochastic–dynamic models) the resulting ensemble forecasts can sample the effects of both initial observation uncertainties and forecast model uncertainties. The resulting ensemble weather forecasts are more reliable (in a precise statistical sense) than those associated with only a sampling of initial observation error, and this has made the whole process of predicting uncertainty more valuable to the real-world customers of weather forecasts.

But this is only half the story! Although weather forecasting has a long history, it is only in recent years that the world has become aware of the threat of climate change. Many regard this as the most serious threat facing humanity – a threat literally to our civilisation. Others, while perhaps acknowledging that the world has warmed in recent years and that some of this could be due to human activities, believe that the climate-change problem is not as important as other problems facing society. To some extent, extreme views about climate change, the cataclysmic and the dismissive, arise because there remains considerable uncertainty in the magnitude of future global warming, e.g. as reflected in the Intergovernmental Panel on Climate Change (IPCC) assessment reports. Certainly the IPCC assessment reports show that among the range of model predictions, there is a quantifiable risk of dangerous climate change in the coming century; most sensible observers deduce from this that the world needs to take action, first to reduce emissions of greenhouse gases and second to start preparing to adapt to inevitable climate change.

Climate-change predictions will play a key role in both mitigation and adaptation policies in years to come. For mitigation, policy makers need more precise predictions about how much more likely dangerous climate change will occur, as a function of anticipated atmospheric greenhouse-gas concentrations. For adaptation, predictions are needed to guide decisions on infrastructure investment. For example, how will patterns of precipitation change; what parts of the world need to be prepared for water shortages and what parts of the world need to be prepared for more frequent and devastating flooding?

Reducing uncertainty in climate prediction, both global and regional, requires improvements in the models used to predict climate. These models are similar in many respects to the types of weather forecast model discussed above, but differ in two key respects. First, because climate models have to be run over century time scales, rather than days, they must include processes like dynamic sea ice and biogeochemistry, processes that are not especially relevant for weather

prediction. This makes the climate models intrinsically more complex than weather prediction models. Owing to this additional complexity and the need to simulate climate on longer time scales than numerical weather prediction models, climate models typically have much coarser grid resolution than weather prediction models: hundreds of kilometres rather than tens of kilometres.

On the other hand, as with weather prediction, neglecting the small-scale motions causes problems. For climate models, it causes the models to drift compared with reality, even for variables that, in principle, are well resolved in terms of the model's grid spacing. The problem of systematic error is an endemic problem in climate modelling. One of the primary goals of any climate-modelling centre is to eliminate, or at least minimise, this systematic drift. To give one example, many climate models have difficulty simulating the atmospheric phenomenon known as persistent anticyclonic blocking. However, such persistent anticyclonic blocks are the primary cause of drought in many locations; a persistent block causes rain-bearing weather systems to be diverted away from the region of interest. Hence, in order to know whether such a region is likely to be more prone to drought under climate change, it is necessary to know whether the frequency of occurrence of persistent blocking anticyclones will increase in that region as a result of increases in greenhouse-gas concentrations. However, if the models have difficulty simulating the blocking phenomenon in the first place, due to systematic drift, they are not well placed to answer this key question.

Clearly a potential solution to the problem of model drift is to reduce the grid spacing, e.g. to that of contemporary numerical weather prediction models. However, to do this would require computing resources beyond the means of most climate institutes. For example, to run century-long integrations with a 10 km grid would require sustained multi-petaflop computing capability.

This raises a fundamental theoretical question. How can we expect uncertainty in our predictions of climate change to reduce as the grid spacing reduces? If we look to our knowledge of the mathematical properties of the Navier–Stokes equations for guidance, we are left with a potential dilemma: a simple scaling argument based on the Kolmogorov turbulence suggests that any systematic truncation error, no matter how small scale it may be, can infect the large-scale systematic error of the model in finite time. Whether the Navier–Stokes equations really have this property is the topic of one of the unsolved million-dollar Clay Mathematics Millennium Prize problems.

This analysis suggests that, effectively, solutions of the dynamic and thermodynamic equations may have some irreducible uncertainty. In this case, it makes sense to try to treat at least the small-scale components of the flow by computationally simple stochastic processes, rather than by the conventional deterministic bulk formula.

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This should not be seen as a council of despair, but as a way forward for a problem, climate prediction, that is arguably the most challenging of problems in computational science. For example, let us return to the problem of simulating persistent blocking anticyclones. One way of thinking of the persistent blocking anticyclone is as a preferred regime in the state space of our climate. However, it is secondary to the normal westerly flow that could be viewed as defining the dominant flow regime. Hence think of a double-well potential, the deeper of which represents normal westerly flow, the shallower representing blocking anticyclonic flow. With a highly resolved model, it should be possible not only to represent this potential well but also the right transition frequency between regimes. With a lower resolution model, perhaps the potential well structure is resolved, but the model is sufficiently damped and inactive that the state resides too frequently in the dominant, deeper, westerly flow regime. As a result, this low-resolution model will exhibit a westerly systematic bias, and be poor at simulating spells of persistent anticyclonic weather. However, if this is the case, then injecting stochastic noise into the near-grid scale may be sufficient to lead to a significant improvement in simulating the correct regime statistics.

Hence, as well as exploring the benefits of high resolution (and this work must certainly be done), climate modellers should additionally explore the benefits of improving the representation of near and subgrid flow in lower-resolution models by stochastic processes. In practice, it is quite probable that these pursuits are not mutually exclusive: as explicit resolution approaches that associated with individual convective cloud systems, the unresolved sub-cloud dynamics will then be represented stochastically.

In his study of the economics of climate change, Lord Stern has shown that the climate problem is, globally, a trillion-dollar problem. Reliable global and regional climate predictions with accurate error bars are an essential element in trying to combat the threat of climate change. This is the reason why, at the beginning of this preface, we suggested that the theme of this book is itself a trillion-dollar theme!

We believe we are at the beginning of a new era in weather and climate modelling – an era that recognises that although the equations of motion are formally deterministic, the best predictions, whether of weather on time scales of days, or climate on time scales of a century or more, may be based on models that are at least partially stochastic.

Tim Palmer and Paul Williams
Reading, UK