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978-0-521-74868-1 - Prospect Theory: For Risk and Ambiguity
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Excerpt
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Introduction

This book has been written and organized especially for readers who do not want to read all of its contents, but want to skip parts and select the material of their own interest. This has been achieved by an organization of exercises explained later, and by an Appendix K that describes the interdependencies between sections. Because of this organization, this book can be used by readers with different backgrounds.

We will examine theories of individual decision making under uncertainty. Many of our decisions are made without complete information about all relevant aspects. This happens for instance if we want to gamble on a horse race and have to decide which horse to bet on, or if we are in a casino and have to decide how to play roulette, if at all. Then we are uncertain about which horse will win or how the roulette wheel will be spun. More serious examples include investments, insurance, the uncertain results of medical treatments, and the next move of your opponent in a conflict. In financial crises, catastrophes can result from the irrational attitudes of individuals and institutions towards risks and uncertainties.

Two central theories in this book are expected utility theory and prospect theory. For all theories considered, we will present ways to empirically test their validity and their properties. In many applications we require more than just qualitative information. We may want to know exactly that spending 1 percent more on a new medicine will generate a 3 percent increase in quality of life for the patient group affected, rather than just knowing that spending more money improves the quality of life. Similarly, we may want to know that a person is willing to pay a maximum of $350 extra tax so as to avoid a 1:100 risk of losing savings to the value of $30,000 in case a bank goes bankrupt. Hence, for all the theories presented in this book, methods will be provided for obtaining precise quantitative measurements concerning those theories and their parameters. Thus precise quantitative predictions can be made. The possibility of obtaining tractable quantitative measurements was a selection criterion for the theories presented in this book.

Typical for the analyses in this book is the interaction between, on the one hand, theoretical and algebraic tools, and, on the other hand:

- prescriptive considerations as relevant for consultancies, policy decisions, and your own decision making;
- descriptive considerations as relevant in psychology and other empirical disciplines.

Prospect theory

Until the end of the 1970s, irrational behavior was believed to be chaotic and unsuited for modeling. The normative expected utility model was taken to be the best approximation of descriptive behavior (Arrow 1951a p. 406; Tversky & Kahneman 1981 opening sentence). Kahneman & Tversky's (1979) prospect theory provided a major breakaway. It was the first descriptive theory that explicitly incorporated irrational behavior in an empirically realistic manner (Kahneman 2003 p. 1456), while at the same time being systematic and tractable. It was the first rational theory of irrational behavior, so to say.

Tversky & Kahneman (1992) introduced an improved version of prospect theory. First, they used Quiggin's (1982) rank dependence to correct a theoretical problem in probability weighting. Second, and more importantly, they extended the theory from risk (known probabilities) to uncertainty and ambiguity (unknown probabilities), using Schmeidler's (1989) rank dependence. In this manner, for the first time a theory has resulted that combines empirical realism with theoretical soundness and tractability. Prospect theory comprises the happy marriage between the empirical insights of Kahneman & Tversky (1979) and the theoretical insights of Gilboa (1987) and Schmeidler (1989).

At this moment of writing, 30 years after its invention, prospect theory is still the only theory that can deliver the full spectrum of what is required for decision under uncertainty, with a natural integration of risk and ambiguity. Therefore, a textbook on the theory is useful. The main purpose of this book is to make this theory accessible to a wide audience by presenting it in a manner as tractable as possible.

Behavioral foundations

Behavioral foundations will play a central role in this book. For a particular decision model, a behavioral foundation gives a list of conditions, stated directly in terms of observable preferences, that hold if and only if the decision model holds. Preference foundations translate the meaning of quantitative decision models and their subjective parameters ("theoretical constructs"), such as subjective probabilities or utilities, into observables. Descriptively, they show how to verify or falsify decision models. Normatively, they provide the terms to justify or criticize models. When de Finetti (1931a), von Neumann & Morgenstern (1944), and Savage (1954) provided behavioral foundations for expected utility, this gave a big boost to the popularity of this theory in many fields. Those fields include economics and game theory (Mas-Colell, Whinston, & Green 1995), management science under the name decision analysis

(Keeney & Raiffa 1976), medicine (Weinstein *et al*. 1980) where utilities are often referred to as QALYs, and statistics (reviving the Bayesian approach; DeGroot 1970). Behavioral foundations ensure the intrinsic soundness of a decision model, preventing historical accidents such as happened for what is known as the separate-probability-transformation model (details in Chapter 5).

Homeomorphic versus paramorphic modeling

A model is *paramorphic* if it describes the empirical phenomena of interest correctly, but the processes underlying the empirical phenomena are not matched by processes in the model (Harré 1970). For example, as emphasized by Milton Friedman (1953; see Bardsley *et al*. 2010 Box 2.4), market models can make correct predictions even if their assumptions about consumers do not match actual consumers' behavior. A model is *homeomorphic* if not only its empirical phenomena match reality, but also its underlying processes do so. We will seek homeomorphic models of decision making. Not only do the decisions predicted by the model match the decisions observed, but also we want the theoretical parameters in the model to have plausible psychological interpretations.

Friedman's arguments in favor of paramorphic models are legitimate if all that is desired is to explain and predict a prespecified and limited domain of phenomena. It is, however, usually desirable if concepts are broadly applicable, also for future and as yet unforeseen developments in research. Homeomorphic models are best suited for this purpose. In recent years, economics has been opening up to introspective and neuro-imaging data. It is to be expected that the concepts of prospect theory, in view of their sound psychological basis, will be well suited for such future developments and for connections with other domains of research. Behavioral foundations with plausible preference conditions support the homeomorphism of a model.

Intended audience

No particular mathematical background knowledge is required, besides a basic knowledge of probability theory and calculus. A willingness to work with formal models and to follow abstract trains of thought is needed for this book though. The measurement methods and behavioral foundations presented in this book will be as simple and transparent as possible, so as to be accessible to as many readers as possible.

Mathematically sophisticated readers may be interested in this book, and will perhaps be surprised by it, from a didactic perspective. For example, Gilboa's (1987) and Schmeidler's (1989) rank-dependent utility theory, and Tversky & Kahneman's (1992) new prospect theory have often been considered to be complex, with presentations based on a comonotonicity concept. These theories can, however, be presented and derived in an elementary manner if we use ranks instead of comonotonicity, as will be done in this book.

Decisions under uncertainty are relevant in many fields, including finance, economics, psychology, management science, medicine, computer science, Bayesian statistics, and engineering. Readers from any of these fields can rest assured that no advanced concepts will appear from any of the other fields because the author does not have a bachelor's degree in any of the fields mentioned.

Attractive feature of decision theory

An attractive feature of decision theory is that the reader can always imagine that he or she is the decision maker. For each preference condition presented in the text, you can ask yourself whether you would want to satisfy this condition in your own decisions. It is easiest to read this book with this question in mind. Hence, the decision maker and the readers will usually be referred to as "you."

Structure

The material in this book has been structured so as to be accessible to readers with different backgrounds, levels, and interests. Many results that will be relevant to some readers but not to all have been stated in exercises, whose elaborations are in Appendix J. This structure gives different readers the chance to skip and select different parts. Italicized superscripts *a*, *b*, *c* indicate which exercises are suited for which readers. The superscript *a* refers to exercises that are easiest, and the superscript *c* refers to exercises that are most difficult and that will be of interest only to the most theoretically oriented readers. Many readers, especially empirically oriented readers who are not used to working with formal models, will want to skip almost all exercises. Typically, psychology students interested in formal models will be *a*-students who will study the empirical parts of this book; mathematical students are *c*-students who are required to study the theoretical parts; and economics students are somewhere in between, so that they are usually *b* students.

The best way to completely master the material in this book – if there are no time restrictions – is to stop reading after every exercise and then first do that exercise. Readers who are satisfied with a less thorough and time-consuming study can use the exercises flexibly. *Sometimes an exercise contains results that are needed to understand the rest of the text. This is indicated by an exclamation! as superscript. Then every reader, even those not doing the exercise, should read its results.*

Exercises are interspersed throughout the text, and are located where they are most relevant. Some sections conclude with assignments. These are further exercises that serve to grade students and/or to practice. Their results play no role in the theory development in the main text, and no elaborations of assignments are given in this book. On the author's homepage, further exercises and assignments are provided. This serves teachers who wish to have more exercises without solutions available to the students. Teachers can obtain solutions to assignments from the publisher. Proofs of theorems are collected in appendices at the end of chapters.

For the use of this book, with comprehensive theoretical discussions and comprehensive discussions of empirical implications, Appendix K is instrumental. It illustrates how sections depend on preceding sections. By using this appendix, you need not read the book from start to finish. In a backward approach, you pick out any topic of interest, and then use Appendix K to see which preceding material you need to read for it. In a forward approach, you skip whatever you do not like. If needed later after all, then Appendix K will show you so. If you are interested in only part of the book, this organization allows you to use the book efficiently. In particular, teachers can easily select the material targeted at the interests of specific students.

If you want to know the definition of prospect theory for unknown probabilities in §12.1, then you can select the texts depicted in Figure K.1 in Appendix K. The sections listed there comprise about 46 pages to be read. If you are not interested in the tradeoff technique of §4.1 and §4.5, then you can skip all of Chapter 4 except for §4.2 and §4.9, and then skip §§6.5, 9.4, 10.5, and 12.3. If you are interested only in decision under risk, then you can learn about the definition of prospect theory in §9.2, using the same method and the same figure, skipping virtually all sections on decision under uncertainty, and reading approximately 34 pages. If you want to learn about a pragmatic index of ambiguity aversion under prospect theory, then you can similarly use Figure K.2. If you want to understand as quickly as possible how the popular value at risk (VaR) for measuring the reliability of banks is a special case of prospect theory and rank dependence (Exercise 6.4.4), then you can find the shortest path: §§6.4, 6.3, 6.1, 3.2, 2.5, 2.4, 2.2, 2.1, 1.3, 1.2, 1.1.

For 10 meetings of three hours each, a typical timetable may be: *meeting 1*: §1.1–§1.8; *meeting 2*: §2.1–§2.9; *meeting 3*: §3.1–§3.6, §4.1; *meeting 4*: §4.2–§4.7, §4.9.1, §4.11, §4.12; *meeting 5*: §5.1–§5.7, §6.1, §6.3–§6.5; *meeting 6*: §7.1–§7.4, §7.6–§7.11; *meeting 7*: §8.1–§8.5, §9.1–§9.5; *meeting 8*: §10.1–§10.6, §10.7.1, §10.8; *meeting 9*: §11.1, §11.4–§11.8; and *meeting 10*: §12.1–§12.3, §12.7. I have used this book in teaching advanced master's students in economics who had digested large parts of Mas-Colell, Whinston, & Green (1995). I would then cover the material allocated to the first four meetings above in about two meetings, after which I would follow the above timetable. The total workload of this selection for students is about 120 hours of full-time work.

A nice way to teach only part of this book is by restricting all models only to binary (two-outcome) prospects. This domain is rich enough to measure and define all components of risk attitude, utility, probability- or event-weighting, and loss aversion. Rank dependence and prospect theory are considerably simplified on this domain. This is how I taught this course to business students. They are particularly interested in prescriptive applications of decision theory.

Preview

The book consists of three parts. Part I deals with the classical expected utility theory, and Parts II and III consider deviations. In Part I, §1.1 and §1.2 present the basics of decision under uncertainty. The rest of Chapter 1 presents the famous bookmaking condition of de Finetti, developed in the 1930s to justify the use of subjective probabilities for one-shot events. This condition is equivalent to the no-arbitrage condition in finance, which implies that market prices of financial derivatives have to be based on what are called as-if risk neutral evaluations. That is, these conditions imply expected utility when probabilities are unknown but utility is known (linear). Chapter 2 deals with expected utility when probabilities are known ("decision under risk") but utilities are unknown. There are so many applications of this long-existing theory that they are presented separately in Chapter 3. Chapter 4 turns to the more complex topic of expected utility when both probabilities and utilities are unknown, using tradeoffs between outcomes as a tool to measure utility differences. It ends with some empirical violations of expected utility, preparing for the parts to follow.

Part II deals with deviations from expected utility for decision under risk, where probabilities are known. We present rank-dependent utility, which generalizes expected utility by adding a new dimension of risk attitude: probabilistic sensitivity – i.e., the nonlinear ways in which people may process probabilities. This dimension is descriptively as relevant for risk attitudes as the nonlinear ways in which people process outcomes (utility), and had been sorely missing in the models used before. In 1982 John Quiggin introduced a correct theoretical manner of modeling such nonlinear processing, the rank-dependent formula. It was only then that a serious descriptive analysis of risk attitudes could begin.

Chapter 5 presents mathematical and psychological arguments to show that the rank-dependent model naturally captures probabilistic sensitivity. Chapter 6 defines the theory formally, and shows how it can be used to tractably capture prevalent phenomena regarding risk attitude. We use ranks, introduced by Abdellaoui & Wakker (2005), as a tool to measure probability weight differences. We can then define ranked probabilities, which are the analogs in the probability dimension of the tradeoffs in the outcome dimension used in Chapter 4. Ranked probabilities facilitate the analyses of the rank-dependent model and are more tractable than the comonotonicity concepts that have been used in the literature. Chapter 7 presents empirical findings and special cases of rank dependence.

In Chapters 8 and 9 we turn to prospect theory. In 1992, Tversky and Kahneman incorporated Quiggin's idea of rank dependence to solve a theoretical problem of their original prospect theory of 1979. It led to the present version of prospect theory, also called cumulative prospect theory. To prepare for prospect theory, Chapter 8 introduces another generalization of expected utility beyond rank dependence: reference dependence. Outcomes are reinterpreted as changes with respect to a reference point (often the status quo). With reference dependence introduced, all ingredients are now available to define and analyze prospect theory for risk (Chapter 9).

Part III concerns decision under uncertainty, where probabilities need not be known. Ambiguity attitudes, which deviate from expected utility in fundamental ways and may not even admit the existence of (subjective) probabilities, are analyzed. Chapter 10 starts by extending Quiggin's definition of rank dependence from risk to the more subtle context of uncertainty, for which Schmeidler (1989, first version

1982) conceived it independently. Chapter 11 presents the main novelties of uncertainty, namely source dependence, which includes ambiguity aversion. We show how rank dependence can be used to analyze uncertainty, and to provide tractable measures of ambiguity aversion and sensitivity to ambiguity. These measures encompass the currently popular α-maxmin model.

Chapter 12 presents the most important model of this book, namely prospect theory for uncertainty. This model entails a common generalization of all the models presented up till then. Relative to Chapters 10 and 11, it allows ambiguity attitudes for losses to be different than for gains. This generalization is desirable because empirical studies have shown that such differences are pronounced. Prospect theory is the first theory for decision under uncertainty that is both theoretically sound and empirically realistic. It means that only since 1992 do we have a satisfactory theory that can deliver the full spectrum of what is needed for decision under risk and uncertainty. Chapter 13 concludes the main text.

Appendices A–K complete the book. I will only discuss a few here. §A.1 in Appendix A contains a general methodological discussion of models being imperfect with inconsistencies in data, and of the nonparametric measurements that are central to this book. Appendix B presents some general issues of the revealed-preference paradigm. Appendix F shows that the influential Fehr–Schmidt model for welfare evaluations is a special case of rank dependent utility, and Appendix J contains the elaborations of the exercises in the book.

Our five-step presentation of decision models

We usually present decision theories in five steps that serve to make the empirical meaning of the theories tangible for the readers. The first step is, simply, to define the decision model. We specify the subjective parameters of the model and give a formula describing how these parameters imply preferences. In expected utility with given probabilities (risk), the subjective parameter is utility. In prospect theory for risk, the subjective parameters are utility, probability weighting, and loss aversion. In the second step, it is demonstrated how decisions can be derived from the model using simple numerical exercises.

Although we do not endorse Lord Kelvin's maxim "science is measurement" as a universal principle, measurement is central to this book. Thus, the third step in our analysis presents what is called the elicitation method. It demonstrates how the subjective parameters of a decision theory can be measured from observed preferences in as simple and direct a way as possible. The third step reverses the second step. Now, preferences are not derived from the subjective parameters but the subjective parameters are derived from preferences.

To illustrate the third step for readers who know the expected utility model, assume expected utility with utility U for given probabilities. Assume a scaling U(\$0) = 0 and U(\$100) = 1 (such scaling is always possible as we will see later). Then, for any \$α between \$0 and \$100, an indifference between receiving \$α for sure or receiving

\$100 with probability p and receiving \$0 with probability 1−p immediately implies $U(\$\alpha) = p$, as we will see later. In this manner we can relate utility to observed choice in a direct and simple manner. Using such observations, we always obtain the right utility function whatever this function is. For instance, we need not assume that utility belongs to a particular family such as the family of power functions ("CRRA"; $U(\alpha) = \alpha^{\theta}$ for some power θ). In descriptive applications we can use such measurements of utility to make predictions about future preferences. In prescriptive applications, simple indifferences as just described can be used to determine preferences in real and complex situations.

A necessary condition for a model to hold is that different ways to measure its subjective parameters should not run into contradictions. This leads to the fourth step in our analysis of decision models: we define preference conditions to preclude such contradictions. These preference conditions then turn out to be not only necessary, but also sufficient, for the decision model considered to hold. That is, they provide behavioral foundations of the models, presented in the fifth step. This means that the behavioral foundations in this book will all be closely related to direct empirical measurements of the subjective parameters of models. The preference conditions are thus easily observable and testable.[1] They all involve a limited number of tractable choices. Comment 2.6.4 will describe the five steps for the expected utility model in Chapter 2. I first developed these five steps when I taught prospect theory to a selected group of high school students (in 30 hours), where this approach worked well. I have maintained it ever since.

After having thus introduced decision models, we discuss their empirical properties, and give suggestions for applications to various fields. I hope that this book will enhance further applications, to be developed by readers specialized in such fields.

[1] Gilboa (2009 end of Ch. 8) pointed out that, conversely, simple preference axioms (= conditions) can be used to obtain simple elicitation methods: "Yet, the language of the axioms, which is concrete and observable, and the quest for simple axioms often facilitate the elicitation/calibration problem significantly."

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Part I

Expected utility

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1 The general model of decision under uncertainty and no-arbitrage (expected utility with known utilities and unknown probabilities)

1.1 Basic definitions

This section introduces the general notation and terminology for decision under uncertainty.

Examples of decision making under uncertainty

Example 1.1.1 [Vendor]. Imagine that you are a street vendor. You have to decide which merchandise to take along with you tomorrow. Merchandise can be ice cream, hot dogs, newspapers, magazines, and so on. The net profits resulting from the merchandise depend on the weather tomorrow, and can be negative because goods not sold tomorrow are lost. Although weather conditions constitute a continuum of possibilities, we assume for simplicity that we need to distinguish only between three eventualities: either there will be no rain (s_1), or there will be some rain (s_2), or it will rain all day (s_3). You do not know for sure whether s_1, s_2, or s_3 will occur. Table 1.1.1 presents, in dollar units, the profits that can result from your decision depending on the weather tomorrow.

Table 1.1.1. *Net profits obtained from merchandise, depending on the weather*

	no rain (s_1)	*some rain* (s_2)	*all rain* (s_3)
x ("ice cream")	400	100	−400
y ("hot dogs")	−400	100	400
0 ("neither")	0	0	0
x + y ("both")	0	200	0

We assume, for simplicity, that the supply of ice cream does not affect the profits obtained from hot dogs, and vice versa, in the lowest row "both." □

Example 1.1.2 [Finance]. Imagine that you want to speculate on the copper price next month. You can buy a financial portfolio, denoted x and costing 30K (K = $1000),