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978-0-521-74701-1 - Sphere Packing, Lewis Carroll, and Reversi: Martin Gardner's
New Mathematical Diversions

Martin Gardner

Frontmatter

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SPHERE PACKING, LEWIS CARROLL, AND REVERSI

For 25 of his 94 years, Martin Gardner wrote “Mathematical Games and Recreations,” a monthly column for *Scientific American* magazine. These columns have inspired hundreds of thousands of readers to delve more deeply into the large world of mathematics. He has also made significant contributions to magic, philosophy, children’s literature, and the debunking of pseudoscience. He has produced more than 60 books, including many best sellers, most of which are still in print. His *Annotated Alice* has sold more than one million copies. He continues to write a regular column for the *Skeptical Inquirer* magazine.



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From 1957 through 1986, Martin Gardner wrote the “Mathematical Games” columns for *Scientific American* that are the basis for these books. *Scientific American* editor Dennis Flanagan noted that this column contributed substantially to the success of the magazine. The exchanges between Martin Gardner and his readers gave life to these columns and books. These exchanges have continued, and the impact of the columns and books has grown. These new editions give Martin Gardner the chance to bring readers up to date on newer twists on old puzzles and games, on new explanations and proofs, and on links to recent developments and discoveries. Illustrations have been added and existing ones improved, and the bibliographies have been greatly expanded throughout.

1. Hexaflexagons, Probability Paradoxes, and the Tower of Hanoi: Martin Gardner's First Book of Mathematical Puzzles and Games
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The Mathematical Association of America



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CAMBRIDGE UNIVERSITY PRESS
 Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi

Cambridge University Press
 32 Avenue of the Americas, New York, NY 10013-2473, USA
www.cambridge.org
 Information on this title: www.cambridge.org/9780521747011

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First published 2009

First edition published as *New Mathematical Diversions from Scientific American*,
 Simon and Schuster, 1966

Printed in the United States of America

A catalog record for this publication is available from the British Library.

Library of Congress Cataloging in Publication data

Gardner, Martin, 1914–
 Sphere packing, Lewis Carroll, and reversi : Martin Gardner's new mathematical
 diversions / Martin Gardner.

p. cm. – (The new Martin Gardner mathematical library ; 3)
 Includes bibliographical references and index.

ISBN 978-0-521-75607-5 (hardback) – ISBN 978-0-521-74701-1 (pbk.)
 1. Mathematical recreations. 2. Carroll, Lewis, 1832–1898. I. Title. II. Series.
 QA95.G323 2009
 793.74–dc22 2009010976

ISBN 978-0-521-75607-5 hardback
 ISBN 978-0-521-74701-1 paperback

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Acknowledgments

The chapters of this book first appeared as columns in various issues of *Scientific American*, published under the copyright of Scientific American, Inc., in 1959 (Chapters 2, 14), 1960 (Chapters 1, 3–10, 12, 13), and 1961 (Chapters 11, 15–20). The author gratefully acknowledges the support and encouragement of Scientific American, Inc., for the publication of this anthology. The author is solely responsible for all revisions and corrections to the original columns. He also thanks the artists who contributed to the success of these columns and books for allowing reuse of their work: James D. Egleson (via heirs Jan and Nicholas Egleson), Harold Jacobs, Amy Kasai, Patra McElwee, Peter Renz, Alex Semenoick, and Bunji Tagawa (via Donald Garber for the Tagawa Estate). Artists' names are cited where these were known. All rights other than use in connection with these materials lie with the original artists.

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Introduction

“A good mathematical joke,” wrote the British mathematician John Edensor Littlewood (in the introduction to his *Mathematician's Miscellany*), “is better, and better mathematics, than a dozen mediocre papers.”

This is a book of mathematical jokes, if “joke” is taken in a sense broad enough to include any kind of mathematics that is mixed with a strong element of fun. Most mathematicians relish such play, though of course they keep it within reasonable bounds. There is a fascination about recreational mathematics that can, for some persons, become a kind of drug. Vladimir Nabokov's great chess novel, *The Defense*, is about such a man. He permitted chess (one form of mathematical play) to dominate his mind so completely that he finally lost contact with the real world and ended his miserable life-game with what chess problemists call a *sui mate* or self-mate. He jumped out of a window. It is consistent with the steady disintegration of Nabokov's chess master that as a boy he had been a poor student, even in mathematics, at the same time that he had been “extraordinarily engrossed in a collection of problems entitled *Merry Mathematics*, in the fantastical misbehavior of numbers and the wayward frolic of geometric lines, in everything that the schoolbook lacked.”

The moral is this: Enjoy mathematical play, if you have the mind and taste for it, but don't enjoy it too much. Let it provide occasional holidays. Let it stimulate your interest in serious science and mathematics. But keep it under firm control.

And if you can't keep it under control, you can take some comfort from the point of Lord Dunsany's story “The Chess-Player, the

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Financier, and Another.” A financier recalls a friend named Smoggs who was on the road to becoming a brilliant financier until he got sidetracked by chess. “It came gradually at first: he used to play chess with a man during the lunch hour, when he and I both worked for the same firm. And after a while he began to beat the fellow. . . . And then he joined a chess club, and some kind of fascination seemed to come over him; something, like drink, or more like poetry or music . . . he could have been a financier. They say it’s no harder than chess, though chess leads to nothing. I never saw such brains wasted.”

“There are men like that,” agrees the prison warden. “It’s a pity . . .” And he locks the financier back in his cell for the night.

My thanks again to *Scientific American* for permission to reprint these 20 columns. As in the two previous book collections, the columns have been expanded, errors corrected, and much new material added that was sent to me by readers. I am grateful, also, to my wife for help in proofing; to my editor, Nina Bourne; and above all, to that still-growing band of readers, scattered throughout the nation and the world, whose welcome letters have so greatly enriched the material reprinted here.

MARTIN GARDNER