Geostatistics Explained

An Introductory Guide for Earth Scientists

This reader-friendly introduction to geostatistics provides a lifeline for students and researchers across the Earth and environmental sciences who until now have struggled with statistics. Using simple and clear explanations for both introductory and advanced material, it demystifies complex concepts and makes formulas and statistical tests easy to understand and apply.

The book begins with a discussion and critical evaluation of experimental and sampling design before moving on to explain essential concepts of probability, statistical significance and Type 1 and Type 2 error. Tests for one and two samples are presented, followed by an accessible graphical explanation of analysis of variance (ANOVA). More advanced ANOVA designs, correlation and regression, and non-parametric tests including chi-square, are then considered. Finally, it introduces the essentials of multivariate techniques such as principal components analysis, multidimensional scaling and cluster analysis, analysis of sequences (especially autocorrelation and simple regression models) and concepts of spatial analysis, including the semivariogram and its application in Kriging.

Illustrated with wide-ranging and interesting examples from topics across the Earth and environmental sciences, *Geostatistics Explained* provides a solid grounding in the basic methods, as well as serving as a bridge to more specialized and advanced analytical techniques. It can be used for an undergraduate course or for self-study and reference. Worked examples at the end of each chapter help reinforce a clear understanding of the statistical tests and their applications.

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Preface

This book presents an introduction to statistical methods that is specifically written for "earth science" students who do not have a strong background in mathematics.

The earth sciences are increasingly (and appropriately) recognized as environmental sciences that overlap and integrate with other disciplines, especially geography, hydrology, soil science, oceanography, environmental management, environmental impact assessment, bioremediation, remote sensing and conservation. As a result, the skills required of earth scientists have become far more diverse, as have the interests and backgrounds of students who enroll in these programs. Today's earth scientists need to be able to critically evaluate sampling designs, to understand the concept of statistical analysis, and be able to evaluate and interpret the results of statistical tests applied in a wide range of fields.

A sound grounding in statistical concepts and methods is especially important, but an increasing proportion of earth science students do not have this. Some have told us that math avoidance is the reason why they have pursued earth sciences instead of chemistry, biology and physics. Many such students are afraid of mathematics (often because they did badly in such subjects at high school) and dread doing an introductory statistics course.

This book has been developed for university and college courses in introductory geostatistics and as a guide for new users to learn statistics on their own. We assume very little prior knowledge of mathematics and start from first principles to develop an understanding of significance testing that can be applied to all statistical tests and related to experimental design. We use a carefully structured conceptual approach to introduce and explain what statistical tests actually do, using a minimum of terminology. Concepts that other introductory texts present as a daunting series of

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formulae are explained in a way that even the "math-phobic" student will find refreshing. The examples we have given are deliberately simple to help the reader understand the statistical concepts being explained. In cases where we have not given a reference for an example, the data have been deliberately contrived (or simplified from actual data) for clarity. Perhaps most importantly, this text develops a strong conceptual understanding that can be applied to the range of statistical methods used in the geosciences.

If you only take an introductory course, then this book will provide the background and understanding you need to interpret and critically evaluate results and summary reports produced by statisticians. If you go on further in geostatistics, this introduction will serve as a bridge to more advanced courses that use texts such as Borradaile (2003) *Statistics of Earth Science Data*, and Davis (1986, 2002) *Statistics and Data Analysis in Geology*.

We have many people to thank. Erick Bestland introduced us by email. Comments by reviewers improved the text. We thank our editors, Susan Francis and Jon Billam, for their considerable help and their good humor. Both our families provided enormous support and tolerated a great deal of absent-mindedness.

For Steve, Ruth McKillup provided constant encouragement and read, commented on, and reread several drafts. Lynn Stewart's constructive help was particularly appreciated, as were Haylee Weaver's insightful comments.

For Darby, thanks are due to Harold Andrews, who introduced her to statistics as an undergraduate in a course that has proven useful in many ways over the years. Tekla Harms humored many thoughtful geologic discussions at 6 a.m. Peter, Duncan and Lindy Crowley provided necessary distractions from this project and a constant reminder of what is really important. At her feet, dogs waited patiently for walks that were postponed by "one last change" to various chapters; they are glad to know that they will now have their day!