Now into its eighth edition and with additional material on primality testing, written by J. H. Davenport, *The Higher Arithmetic* introduces concepts and theorems in a way that does not require the reader to have an in-depth knowledge of the theory of numbers but also touches upon matters of deep mathematical significance. A companion website (www.cambridge.org/davenport) provides more details of the latest advances and sample code for important algorithms.

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THE HIGHER ARITHMETIC

AN INTRODUCTION TO THE THEORY OF NUMBERS

Eighth edition

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CONTENTS

INTRODUCTION

The higher arithmetic, or the theory of numbers, is concerned with the properties of the natural numbers 1, 2, 3, ... These numbers must have exercised human curiosity from a very early period; and in all the records of ancient civilizations there is evidence of some preoccupation with arithmetic over and above the needs of everyday life. But as a systematic and independent science, the higher arithmetic is entirely a creation of modern times, and can be said to date from the discoveries of Fermat (1601–1665).

A peculiarity of the higher arithmetic is the great difficulty which has often been experienced in proving simple general theorems which had been suggested quite naturally by numerical evidence. 'It is just this,' said Gauss, 'which gives the higher arithmetic that magical charm which has made it the favourite science of the greatest mathematicians, not to mention its inexhaustible wealth, wherein it so greatly surpasses other parts of mathematics.'

The theory of numbers is generally considered to be the 'purest' branch of pure mathematics. It certainly has very few direct applications to other sciences, but it has one feature in common with them, namely the inspiration which it derives from *experiment*, which takes the form of testing possible general theorems by numerical examples. Such experiment, though necessary in some form to progress in every part of mathematics, has played a greater part in the development of the theory of numbers than elsewhere; for in other branches of mathematics the evidence found in this way is too often fragmentary and misleading.

As regards the present book, the author is well aware that it will not be read without effort by those who are not, in some sense at least, mathematicians. But the difficulty is partly that of the subject itself. It cannot be evaded by using imperfect analogies, or by presenting the proofs in a way which may convey the main idea of the argument, but is inaccurate in detail. The theory of numbers is by its nature the most exact of all the sciences, and demands exactness of thought and exposition from its devotees.

The theorems and their proofs are often illustrated by numerical examples. These are generally of a very simple kind, and may be despised by those who enjoy numerical calculation. But the function of these examples is solely to illustrate the general theory, and the question of how arithmetical calculations can most effectively be carried out is beyond the scope of this book.

The author is indebted to many friends, and most of all to Professor Erdős, Professor Mordell and Professor Rogers, for suggestions and corrections. He is also indebted to Captain Draim for permission to include an account of his algorithm.

The material for the fifth edition was prepared by Professor D. J. Lewis and Dr J. H. Davenport. The problems and answers are based on the suggestions of Professor R. K. Guy.

Chapter VIII and the associated exercises were written for the sixth edition by Professor J. H. Davenport. For the seventh edition, he updated Chapter VII to mention Wiles' proof of Fermat's Last Theorem, and is grateful to Professor J. H. Silverman for his comments.

For the eighth edition, many people contributed suggestions, notably Dr J. F. McKee and Dr G. K. Sankaran. Cambridge University Press kindly re-typeset the book for the eighth edition, which has allowed a few corrections and the preparation of an electronic complement: `www.cambridge.org/davenport`. References to further material in the electronic complement, when known at the time this book went to print, are marked thus: ♠:0.