The Student's Introduction to *Mathematica*®

Second edition

The unique feature of this compact student's introduction is that it presents concepts in an order that closely follows a standard mathematics curriculum, rather than structured along features of the software. As a result, the book provides a brief introduction to those aspects of the Mathematica® software program most useful to students. The second edition of this well-loved book is completely rewritten for Mathematica[®] 6, including coverage of the new dynamic interface elements, several hundred exercises, and a new chapter on programming. This book can be used in a variety of courses, from precalculus to linear algebra. Used as a supplementary text it will aid in bridging the gap between the mathematics in the course and Mathematica®. In addition to its course use, this book will serve as an excellent tutorial for those wishing to learn Mathematica[®] and brush up on their mathematics at the same time.

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The Student's Introduction to *Mathematica*®

A Handbook for Precalculus, Calculus, and Linear Algebra

Second edition

Bruce F. Torrence Eve A. Torrence



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For

Alexandra and Robert

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Preface

The mathematician and juggler Ronald L. Graham has likened the mastery of computer programming to the mastery of juggling. The problem with juggling is that the balls go exactly where you throw them. And the problem with computers is that they do exactly what you tell them.

This is a book about *Mathematica*, a software system described as "the world's most powerful global computing environment." As software programs go, *Mathematica* is big—really big. We said that back in 1999 in the preface to the first edition of this book. And it's gotten a good deal bigger since then. There are more than 900 new documented symbols in version 6 of *Mathematica*. It's been said that there are more *new* commands in version 6 than there were commands in version 1. It's gotten so big that the documentation is no longer produced in printed form. Our trees and our backs are grateful. Yes, *Mathematica* will do exactly what you ask it to do, and it has the potential to amaze and delight—but you have to know how to ask, and that can be a formidable task.

That's where this book comes in. It is intended as a supplementary text for high school and college students. As such, it introduces commands and procedures in an order that roughly coincides with the usual mathematics curriculum. The idea is to provide a coherent introduction to *Mathematica* that does not get ahead of itself mathematically. Most of the available reference materials make the assumption that the reader is thoroughly familiar with the mathematical concepts underlying each *Mathematica* command and procedure. This book does not. It presents *Mathematica* as a means not only of solving mathematical problems, but of exploring and clarifying the concepts themselves. It also provides examples of procedures that students will need to master, showing not just individual commands, but sequences of commands that together accomplish a larger goal.

While written primarily for students, the first edition was well-received by many non-students who just wanted to learn *Mathematica*. By following the standard mathematics curriculum, we were told, the presentation exudes a certain familiarity and coherence. What better way to learn a computer program than to rediscover the beautiful ideas from your foundational mathematics courses?

What's New in this Edition?

The impetus for a second edition was driven by the software itself. The first edition coincided with the release of *Mathematica* 4. While version 5 introduced a few notable new commands, much of the innovations in that release were kept under the hood, so to speak. The algorithms associated with many well-used commands were improved, but the user interface underwent minimal changes. *Mathematica* 6 on the other hand is a different beast entirely. Perhaps the most fundamental innovation is the introduction of dynamic user interface elements with commands such as **Manipulate**. It is now possible to take essentially any *Mathematica* expression and add sliders or buttons that permit a user to adjust parameters in real time. The second edition was re-written from the ground up to take these and other changes into account. Virtually every section of every chapter has undergone extensive revision and expansion. This edition reflects the software as it exists today.

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The organization of the book has not changed, but there are two notable new additions:

The second edition has exercises, several hundred in fact. These provide a means for experimenting with and extending the ideas outlined in each section. They also provide a concrete and structured framework for interacting with the software. It is through such interactions that familiarity and (ultimately) competence and even mastery will be attained. Complete solutions are freely available online, as discussed in the next section.

In addition, a new chapter has been added (Chapter 8) to address the fundamental aspects of programming with *Mathematica*. While this topic is far too expansive to cover thoroughly in a single chapter, many of the fundamentals of programming are conveyed here. It is a fact that many of the new features of version 6 require a working knowledge of pure functions and other ideas that fit naturally into this context. You are likely to find yourself reading a section of this chapter here and there as you explore certain topics in the earlier chapters. Think of it as a handy reference.

How to Use this Book

Of course, this is a printed book and as such is perfectly suitable for bedtime reading. But in most cases you will want to have the book laid open next to you as you work directly with *Mathematica*. You can mimic the inputs and then try variations. After you get used to the syntax conventions it will be fun.

The first chapter provides a brief tutorial for those unfamiliar with the software. The second delves a bit deeper into the fundamental design principles and can be used as a reference for the rest of the book. Chapters 3 and 4 provide information on those *Mathematica* commands and procedures relevant to the material in a precalculus course. Chapter 5 adds material relevant to single-variable calculus, and Chapter 6 deals with multivariable calculus. Chapter 7 introduces commands and procedures pertinent to the material in a linear algebra course.

 \triangle Some sections of the text carry this warning sign. These sections provide slightly more comprehensive information for the advanced user. They can be skipped by less hardy souls.

Beginning in Chapter 3, each section has exercises. Solutions to *every* exercise can be freely down-loaded from the website at www.studentsmathematica.com.

Mathematica runs on every major operating system, from Macs and PCs to Linux workstations. For the most part it works exactly the same on every platform. There are, however, a few procedures (such as certain keyboard shortcuts) that are platform specific. In such cases we have provided specific information for both the Mac OS and Microsoft Windows platforms. If you find yourself running *Mathematica* on some other platform you can be assured that the procedure you need is virtually identical to one of these.

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Acknowledgments

Time flies. When we wrote the first edition of this book Robert and Alexandra were toddlers who would do anything to get our attention and wanted to sit on our laps while we worked. Now they are teenagers who just want our laptops. Like *Mathematica* our kids have grown up. They have become our best friends and terrific travel buddies. This project has again disrupted their lives and we thank them for their attempts at patience. To quote Robert, "You guys aren't going to write any more books, are you?" Don't worry kids, at this rate you'll both be in college.

Special thanks go out to Paul Wellin at Wolfram Research, who handled the page design and who dealt tirelessly with countless other issues, both editorial and technical. We would like to thank Randolph-Macon College and the Walter Williams Craigie Endowment for the support we received throughout this project. And we thank Peter Thompson, our editor at Cambridge, for his professional acumen and ongoing encouragement and support.