1

What this book is about and how to use it

1.1 The proper treatment of quantification in ordinary Human

In *The proper treatment of quantification in ordinary English* Montague sets forth his goal as follows:[1]

> "The aim of this paper is to present in a rigorous way the syntax and semantics of a certain fragment of a certain dialect of English. For expository purposes the fragment has been made as simple and restricted as it can be while accommodating all the more puzzling cases of quantification and reference with which I am acquainted." (Montague 1974a: 247)

The goal of this book is to survey a good chunk of the research that has been directed at Montague's puzzles and their natural extensions in the past 35 years. The survey has a dual focus. One is on how the understanding of "quantification" and "quantifier" has been changing over time. The way I see it, we have witnessed three main stages of research:

Grand uniformity (the 1970s and 1980s)
Foundational work that affords a uniform treatment of initially disparate-looking phenomena: generalized quantifiers for all noun phrases, a kind-based treatment of existential and generic readings of bare plurals, etc.

Diversity (the 1980s and 1990s)
Dynamic semantics for definites and indefinites, choice-functional indefinites vs. others, the differential behavior of quantifiers

Internal composition (from 2000 on)
Quantifier-phrase-internal and, most recently, quantifier-word-internal compositionality

The other focus is on the core notion of scope and its implementation in several varieties of generative syntax and categorial grammar. We may disagree about what the best syntax is, but any serious attempt at compositionality must be built on a credible syntax. It is important to see that at least the core ideas can be implemented in various different ways.

Montague's puzzles include the interaction of quantifier phrases among themselves and with intensional predicates, and the binding of pronouns by quantifiers. We will not attempt to cover the research on intensionality, save for a brief discussion in **§5.7**, although Chapter 3 takes up quantification over individuals vs. worlds and times. Another major self-imposed limitation has been to set aside quantificational binding (see **§2.3.3**).

The structure of the discussion is as follows.

Chapters 2 through 4 offer an introduction to generalized quantifiers, with an eye on the implications for scope and the syntax/semantics interface, non-nominal domains of quantification, and on semantic properties that turn out to be significant for empirical work. These chapters do not attempt to rehash what existing excellent introductions do (see some recommended readings in **§2.1**); they attempt to give a picture that cannot be found elsewhere.

Chapters 5 and 6 pull together some of the questions and data that led to the major transformation in how we approach "quantifiers" and "scope". (The transformation explains why this introduction does not start with a substantial definition of "quantification" – there is no need to set up a strawman and fight with it throughout the book.)

Chapters 7 through 10 discuss some of the issues that have been in the focus of much research: existential scope, distributivity, numeral indefinites, and modified numeral expressions. Here a major limitation is that the discussion of plural noun phrases (especially of collective readings) is kept to the minimum.

Chapter 11 surveys recent approaches to the syntax of clause-internal scope, with special attention to how they account for the diversity of scopal behavior. Chapter 12 pulls together the even more recent work on the internal structure of universal quantifiers – quantifier phrases as well as quantifier words.

The last four chapters survey more controversial and more preliminary ideas than the ones preceding them. Seeing that this is a research survey, not a textbook, it hopes to stimulate further work by giving a sense of where we actually are.

Throughout the book I attempt to link up the results of serious semantics and serious syntax. Occasionally I am mainly talking to the semanticist or to the syntactician, but my hope is that many readers will put themselves in the shoes of both.

Although a great many formal semanticists are native speakers of languages other than English, the bulk of our efforts has been directed at analyzing English or, sometimes, at disguising research on another language as work on English. This survey makes an attempt to bring multiple languages to bear on the questions under discussion, or at least to point out the existence of some high-quality literature on various languages. I am definitely not doing as good a job as I would like to, simply because I have not processed all this literature in sufficient depth.

1.2 How to use this book

This is not a textbook. Many things follow from this. It does not single out one theory and endow the reader with a working knowledge of it. It selects a story-line and shows what a relatively wide range of literature has to say about it. Although some formalization is offered, the discussion is kept as informal as possible, to maintain readability and to remain neutral as to technicalities. Sometimes it does not make sense to avoid the formalism; if the reader feels that a part is too difficult, they should breeze through it and rest assured that they will be able to pick up the thread afterwards.

The endnotes typically supply further important empirical or formal detail. Their contents are an integral part of the text, at least for some readers. They are relegated to note status to avoid disrupting the train of thought in the main text. The best thing is to keep a bookmark at the notes and consult them systematically.

The chapters and sections address theoretical issues, rather than descriptive topics, whenever possible. For this reason the discussion is somewhat fragmented and repetitive: a particular descriptive topic and a particular piece of work may be relevant for various different questions. So one descriptive topic may be discussed in many places in the book, and different claims made in one and the same piece of work may be brought to bear on various different issues. Usually there are pointers to the other relevant sections and occasionally brief summaries are given of what has already been said; the reader is encouraged to also make good use of the index. A certain amount of repetition is necessary in any case, because not every reader will want to go through the whole book. No issue or piece of work is discussed completely. It is assumed that the reader will go on to consult some of the literature surveyed herein.

The publisher and the author were unanimous in wanting a slender volume, so a certain amount of background is presupposed. For the basics I recommend the syntax and semantics chapters of the twelve-author textbook Fromkin (2000). A good thorough introduction to syntax is Koopman, Sportiche and Stabler (to appear). For formal foundations, the ideal background is a combination of Gamut (1991) and Chapters 2, 6, and 7 of Landman (1991). For lighter fare, use Allwood et al. (1977) and Szabolcsi (1997d). It will be extremely helpful if the reader is comfortable with

λ's. For a boost I recommend Chris Barker's famed Lambda Tutorial, http://homepages.nyu.edu/~cb125/Lambda/.

Where appropriate the text will point to handbook articles or textbooks, or to original works that have acquired comparable status, for background on the topic under discussion. To draw the reader's attention to these items the authors' names appear in small capitals.

1.3 Notation and terminology

As Montague (1974a,b) points out, the syntax of the object language may be directly interpreted in models, or translation into a suitably rich logical language may induce a model-theoretic interpretation for the object-language syntax. Montague uses the translation strategy; HEIM AND KRATZER (1998) use direct interpretation. The present book follows the translation strategy, because it makes it much easier to calculate with somewhat complex expressions. The reader should be aware of the following: (i) Expressions are translated into a logical language; the λ-operator for example is not used as part of the English meta-language; (ii) Square brackets indicating scope are not abandoned in favor of right-unbounded dots; (iii) The domain of quantification is either not indicated or its type appears as an index on the prefix. For example:

$$\text{Heim and Kratzer: } \lambda x \in D \;.\; P(x) = 1$$
$$\text{this book: } \lambda x_e[P(x)]$$

Following current syntactic practice we refer to syntactic units like *every dragon* as "quantifier phrases", "noun phrases", "DPs", or "QPs". The label "NP" is reserved for the complement of the determiner, as in the schematic form *every NP*. Notice that "NP" is not short for "noun phrase": *every dragon* is a noun phrase but *dragon* is a NP.

Plain italics, as in *every dragon*, indicate a mention of a natural-language expression. Adding a prime (in the text or in numbered examples), as in *every dragon*$'$, signifies both the counterpart of a natural-language expression in the syntax of some logical language, and the interpretation (denotation, meaning) of the expression. This convention allows us to avoid clumsy things like $[\![\textit{every dragon}]\!]^{M,g}$. Although the convention is obviously sloppy and can be seen as complicit in promoting the confusion of logical syntax with model theoretic semantics, if the reader bears the distinction in mind it will always be clear which of the two things we are talking about in a given context.

Sometimes the interpretation of a linguistic example is prefixed with OK or #. Such annotation indicates that the example is acceptable or unacceptable on the given interpretation, and that no claim is being made as to whether the example has other interpretations.

2

Generalized quantifiers and their elements: operators and their scopes

2.1 Generalized quantifiers – heroes or old fogeys?

Starting with Montague (1974a) but at least with the almost simultaneous appearance of BARWISE AND COOPER (1981), Higginbotham and May (1981), and Keenan and Stavi (1986) generalized quantifiers became the staple of formal semantics. For decades it has been taken for granted that they serve as the interpretations of the most widely researched grammatical category in the field, i.e. noun phrases. Nevertheless, there is mounting evidence that generalized quantifiers are not the panacea magna they were once thought to be, and these days one reads more about what they cannot do than about what they can. So are generalized quantifiers a thing of the past? If not, what are they good for? What are the main reasons for them to be superseded, and by what?

Like many other books, this one starts out with generalized quantifiers, but it does so bearing the controversy around them in mind. This will also make it easier to highlight some of the underlying assumptions and some of the firm advantages of generalized quantifiers. Building on these foundations the book will survey two areas of research. One has to do with alternative approaches to scope assignment. The other has to do with the diversity in the behavior of quantifier phrases and with recent attempts to explain it in a compositional fashion. In this way the book will place an emphasis on ongoing work. Apart from the hope of stimulating research in these newer areas, making the unquestioningly generalized-quantifier-theoretic part relatively brief is justified by the fact that there are so many superb texts available on the topic. From the 1990s one would recommend KEENAN (1996), KEENAN AND WESTERSTÅHL (1997), and LANDMAN (1991). In recent years the most comprehensive and authoritative text is PETERS AND WESTERSTÅHL (2006); GLANZBERG (2006) and RUYS AND WINTER (2008) are excellent handbook chapters.

2.2 Generalized quantifiers and their elements: operators and their scopes

In many logics, operators are introduced syncategorematically. They are not expressions of the logical language; the syntax only specifies how they combine with expressions to yield new expressions, and the semantics specifies what their effect is:

(1) If ϕ is a formula, $\forall x[\phi]$ is a formula.
$\forall x[\phi]$ is true if and only if every assignment of values to the variable x makes ϕ true.

The quantifier prefix $\forall x$ functions like a diacritic in the phonetic alphabet: $'$ is not a character of the IPA but attaching it to a consonant symbol indicates that the sound is palatal (e.g. [t′]). In line with most of the linguistic literature we are going to assume that operators embodied by morphemes or phrases are never syncategorematic.[2] But if *every* and *every dragon* are ordinary expressions that belong to some syntactic category, then, by the principle of compositionality, they must have their own self-contained interpretations. This contrasts with the situation in predicate logic. In (2) the contributions of *every* and *every dragon* are scattered all over the formula without being subexpressions of it. Everything in (2) other than *guard treasure′* comes from *every dragon*, and everything other than *guard treasure′* and *dragon′* comes from *every*.

(2) Every dragon guards treasure.
$\forall x[dragon'(x) \rightarrow guard\ treasure'(x)]$

Not only would we like to assign a self-contained interpretation to *every dragon*, we would also like to assign it one that resembles, in significant respects, the kind of interpretations we assign to *Smaug* and *more than three dragons*. The reason why these are all categorized as DPs in syntax is that they exhibit very similar syntactic behavior. It is then natural to expect them to have in some respects similar semantics. If they did not, then the syntactic operations involving DPs (e.g. merging DP with a head, in current terminology) could not be given uniform interpretations. To a certain point it is easy to see how that interpretation would go. Assume that the DP *Smaug* refers to the individual s and the predicate (TP, a projection of Tense) *guards treasure* to the set of individuals that guard treasure. Interpreting the DP–TP relation as the set theoretical element-of relation, *Smaug guards treasure* will be interpreted as $s \in guard\ treasure'$. Now consider *Every dragon guards treasure*. The DP *every dragon* does not denote an individual, but we can associate with it a unique set of individuals, the set of dragons. Reinterpreting DP–TP using the subset relation, *Every dragon guards treasure* is compositionally interpreted as $dragon' \subseteq guard\ treasure'$. To achieve uniformity, we can go back and recast $s \in guard\ treasure'$ as $\{s\} \subseteq guard\ treasure'$, with $\{s\}$ the singleton set that contains just Smaug. But indefinite DPs like *more than three dragons* still cannot be accommodated, because there is no unique set of individuals they could be associated with. In a universe of just 5 dragons, sets of more than three dragons can be picked in various different ways.

One of Montague's (1974a) most important innovations was to provide a self-contained and uniform kind of denotation for all DPs in the form of generalized quantifiers, introduced mathematically in Mostowski (1957) based on Frege's fundamental idea. The name is due to the fact we generalize from the first order logical ∀ and ∃ and their direct descendants *every dragon* and *some dragon* to the whole gamut, *less than five dragons*, *at least one dragon*, *more dragons than serpents*, *the dragon*, etc., even including proper names like *Smaug*.

A generalized quantifier is a set of properties. In the examples below the generalized quantifiers are defined using English and, equivalently, in the language of set theory and in a simplified Montagovian notation, to highlight the fact that they do not have an inherent connection to any particular logical notation. The main simplification is that we present denotations extensionally. Thus each property is traded for the set of individuals that have the property (rather than the intensional analogue, a function from worlds to such sets of individuals), but the term "property" is retained, as customary, to evoke the relevant intuition. This approach fits all three of our examples equally well:

(3) a. *Smaug* denotes the set of properties that Smaug has. If Smaug is hungry, then the property of being hungry is an element of this set.
   b. *Smaug* denotes $\{P : s \in P\}$. If Smaug is hungry, then $\{a : a \in hungry'\} \in \{P : s \in P\}$.
   c. *Smaug* denotes $\lambda P[P(s)]$. If Smaug is hungry, then $\lambda P[P(s)]\ (hungry')$ yields the value True.

(4) a. *Every dragon* denotes the set of properties that every dragon has. If every dragon is hungry, then the property of being hungry is an element of this set.
   b. *Every dragon* denotes $\{P : dragon' \subseteq P\}$. If every dragon is hungry, then $\{a : a \in hungry'\} \in \{P : dragon' \subseteq P\}$.
   c. *Every dragon* denotes $\lambda P \forall x[dragon'(x) \rightarrow P(x)]$. If every dragon is hungry, then $\lambda P \forall x[dragon'(x) \rightarrow P(x)](hungry')$ yields the value True.

(5) a. *More than one dragon* denotes the set of properties that more than one dragon has. If more than one dragon is hungry, then the property of being hungry is an element of this set.

b. *More than one dragon* denotes $\{P : |dragon' \cap P| > 1\}$. If more than one dragon is hungry, then $\{a : a \in hungry'\} \in \{P : |dragon' \cap P| > 1\}$.

c. *More than one dragon* denotes $\lambda P \exists x \exists y[x \neq y \wedge dragon'(x) \wedge dragon'(y) \wedge P(x) \wedge P(y)]$. If more than one dragon is hungry, then $\lambda P \exists x \exists y[x \neq y \wedge dragon'(x) \wedge dragon'(y) \wedge P(x) \wedge P(y)](hungry')$ yields the value True.

To make this set of sets of individuals more vivid, it is useful to invoke some simple notions of set theory. The powerset of a set A is the set of all A's subsets. The powerset is so called because a set of n elements has 2^n subsets (2 to the nth power). Imagine a universe of discourse with 4 elements. Its powerset, i.e. the set of all its 16 subsets, is as follows:

(6) Let the universe of discourse be the set $\{a, b, c, d\}$. Then the set of all its subsets, i.e. its powerset is $\{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}\}$.

Extensional semantics can distinguish just these 16 sets of individuals (properties) in a 4-element universe. For example, if the set of dragons is $\{a, b, c\}$ and the set of things that fly is $\{a, b, d\}$, then the properties of being a dragon and being a thing that flies can be distinguished. But if both sets happen to have the same elements, then an extensional semantics cannot distinguish them.

Some sets in the universe have names such as *dragon*, *flies*, etc. whereas others do not. But for our purposes all these are on a par. The most useful label for $\{a, b\}$ is not 'dragon that flies' but, rather, 'entity that is identical to a or b'. When we ask whether a particular sentence, e.g. *Smaug flies* is true, we are interested in sets with particular linguistic labels, but when we study the quantifiers themselves, we are interested in all the sets that are elements of the quantifier and in their relation to all the other subsets of the universe.

To visualize a generalized quantifier we draw the Hasse-diagram of the powerset of the universe. The lines represent the subset relation, thus $\{a\}$ is below $\{a, b\}$ and $\{a, b\}$ below $\{a, b, c\}$, because $\{a\} \subseteq \{a, b\} \subseteq \{a, b, c\}$. Each generalized quantifier is represented as an area (a subset) in this diagram. If Smaug is the individual a, and the set of dragons is $\{a, b, c\}$, the generalized quantifiers denoted by the DPs *Smaug*, *every dragon*, and *more than one dragon* are the shaded areas in Figures 2.1, 2.2, and 2.3, respectively. Such diagrams will be used over and over in Chapter 4.

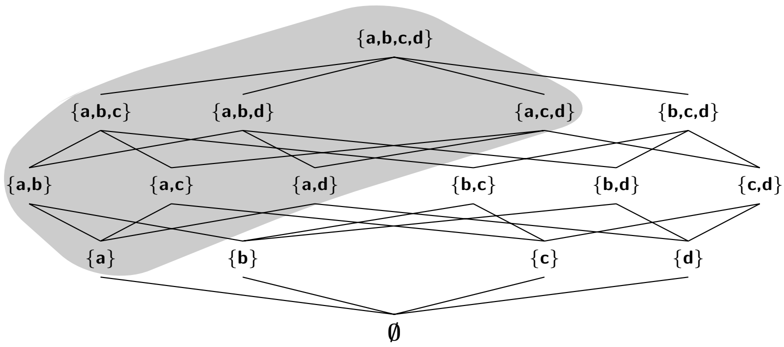

Fig. 2.1 The set of properties Smaug has: all the sets that have a as an element

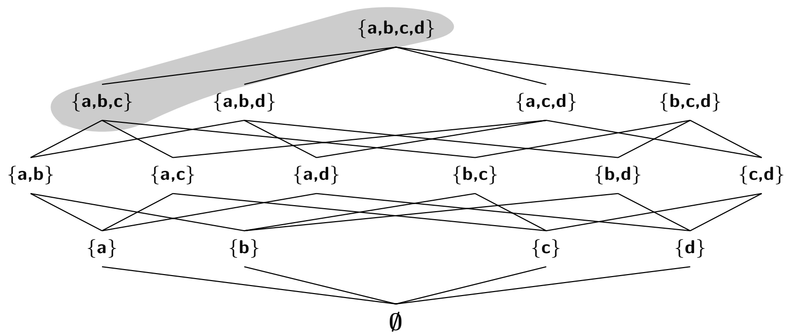

Fig. 2.2 The set of properties every dragon has: all the sets that have $\{a, b, c\}$ as a subset

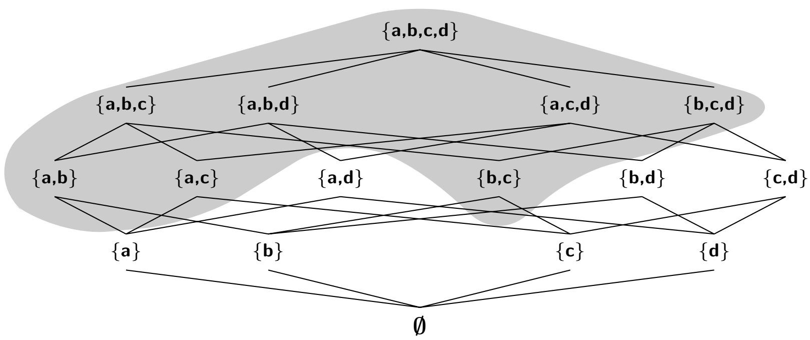

Fig. 2.3 The set of properties more than one dragon has: all the sets whose intersection with $\{a, b, c\}$ has more than one element

Recall that our desire for a uniform interpretation stems from the fact that all DPs play similar roles in syntax.[3] We now have such an interpretation. The specific notion of a generalized quantifier is furthermore useful in two main respects. First, it provides a foundation for the treatment of quantifier scope. Second, it enables one to study the semantic properties of DPs, and to do so in a way that possibly subsumes them under cross-categorial generalizations. We start with scope. The property (*is*) *hungry′* mentioned above has a simple description, but that is an accident. Properties might have arbitrarily complex descriptions:

(7) If every dragon flies or lumbers, then the property of being an individual such that he/she/it flies or he/she/it lumbers is in the set of properties every dragon has.

(8) If there is more than one dragon that spotted every adventurer, then the property of being an individual such that he/she/it spotted every adventurer is an element of the set of properties more than one dragon has.

(9) If every adventurer was spotted by more than one dragon, then the property of being an individual such that there is more than one dragon that spotted him/her/it is an element of the set of properties every adventurer has.

Properties with simple descriptions and ones with complex descriptions are entirely on a par. We are not adding anything to the idea of generalized quantifiers by allowing properties of the latter kind. But once the possibility is recognized, quantifier scope is taken care of. In each case above, some operation is buried in the description of the property that is asserted to be an element of the generalized quantifier. In (7) the buried operation is disjunction; thus (7) describes a configuration in which universal quantification scopes over disjunction. (8) and (9) correspond to the subject wide scope, $S > O$, and the object wide scope, $O > S$, readings of the sentence *More than one dragon spotted every adventurer*. In (8) the main assertion is about the properties shared by more than one dragon, thus the existential quantifier in subject position is taking wide scope. In (9) the main assertion is about the properties shared by every man, thus the universal quantifier in object position is taking wide scope.

This is all there is to it:

(10) Scope
The scope of a quantificational DP, on a given analysis of the sentence, is that part of the sentence which denotes a property that is asserted to be an element of the generalized quantifier denoted by DP on that analysis.