The Econometric Modelling of Financial Time Series

Terence Mills’ best-selling graduate textbook provides detailed coverage of the latest research techniques and findings relating to the empirical analysis of financial markets. In its previous editions it has become required reading for many graduate courses on the econometrics of financial modelling.

This third edition, co-authored with Raphael Markellos, contains a wealth of new material reflecting the developments of the last decade. Particular attention is paid to the wide range of non-linear models that are used to analyse financial data observed at high frequencies and to the long memory characteristics found in financial time series. The central material on unit root processes and the modelling of trends and structural breaks has been substantially expanded into a chapter of its own. There is also an extended discussion of the treatment of volatility, accompanied by a new chapter on non-linearity and its testing.

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The Econometric Modelling of Financial Time Series

Third edition

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Preface to the third edition

In the nine years since the manuscript for the second edition of *The Econometric Modelling of Financial Time Series* was completed there have continued to be many advances in time series econometrics, some of which have been in direct response to features found in the data coming from financial markets, while others have found ready application in financial fields. Incorporating these developments was too much for a single author, particularly one whose interests have diverged from financial econometrics quite significantly in the intervening years! Raphael Markellos has thus become joint author, and his interests and expertise in finance now permeate throughout this new edition, which has had to be lengthened somewhat to accommodate many new developments in the area.

Chapters 1 and 2 remain essentially the same as in the second edition, although examples have been updated. The material on unit roots and associated techniques has continued to expand, so much so that it now has an entire chapter, 3, devoted to it. The remaining material on univariate linear stochastic models now comprises chapter 4, with much more on fractionally differenced processes being included in response to developments in recent years. Evidence of non-linearity in financial time series has continued to accumulate, and stochastic variance models and the many extensions of the ARCH process continue to be very popular, along with the related area of modelling volatility. This material now forms chapter 5, with further non-linear models and tests of non-linearity providing the material for chapter 6. Chapter 7 now contains the material on modelling return distributions and transformations of returns. Much of the material of chapters 8, 9 and 10 (previously chapters 6, 7 and 8) remains as before, but with expanded sections on, for example, non-linear generalisations of cointegration.