This introduction to general equilibrium modeling takes an integrated approach to the analysis of macroeconomics and finance. It provides students, practitioners, and policymakers with an easily accessible set of tools that can be used to analyze a wide range of economic phenomena.

Key features:

• Provides a consistent framework for understanding dynamic economic models.
• Introduces key concepts in finance in a discrete time setting.
• Develops a simple recursive approach for analyzing a variety of problems in a dynamic, stochastic environment.
• Sequentially builds up the analysis of consumption, production, and investment models to study their implications for allocations and asset prices.
• Reviews business cycle analysis and the business cycle implications of monetary and international models.
• Covers latest research on asset pricing in overlapping generations models and on models with borrowing constraints and transaction costs.
• Includes end-of-chapter exercises allowing readers to monitor their understanding of each topic.

Online resources available at www.cambridge.org/altug_labadie

SUMRU ALTUG is Professor of Economics at Koç University, Istanbul, and Research Fellow on the International Macroeconomics Programme at the Centre for Economic Policy Research (CEPR), London.

PAMELA LABADIE is Professor of Economics at George Washington University, Washington DC.
ASSET PRICING FOR DYNAMIC ECONOMIES

SUMRU ALTUG AND PAMELA LABADIE
Contents

List of figures \hspace{1cm} xi
List of tables \hspace{1cm} xii
Preface \hspace{1cm} xiii

I BASIC CONCEPTS 1

1 Complete contingent claims 3
  1.1. A one-period model 3
    1.1.1. Contingent claims equilibrium 5
    1.1.2. Computing the equilibrium 6
    1.1.3. Pareto optimal allocations 11
  1.2. Security market equilibrium 12
    1.2.1. Definition 12
    1.2.2. Attaining a CCE by an SME 16
    1.2.3. The Pareto optimum and the representative consumer 20
  1.3. Conclusions 22
  1.4. Exercises 22

2 Arbitrage and asset valuation 25
  2.1. Absence of arbitrage: some definitions 25
    2.1.1. The law of one price 27
    2.1.2. Arbitrage opportunities 30
  2.2. Existence of a state-price vector 32
    2.2.1. Risk-free asset 34
    2.2.2. Risk-neutral pricing 35
    2.2.3. The stochastic discount factor 37
  2.3. Binomial security markets 38
    2.3.1. An economy with two dates 39
    2.3.2. A multi-period economy 41
  2.4. Conclusions 47
  2.5. Exercises 47

3 Expected utility 51
  3.1. Expected utility preferences 51
    3.1.1. Some definitions 51
  3.2. Risk aversion 54
  3.3. One-period expected utility analysis 56
    3.3.1. The risk premium 58
## Contents

3.3.2. Measures of risk aversion 58  
3.3.3. Risk aversion in a portfolio choice problem 61  
3.4. Measures of increasing risk 63  
3.5. Conclusions 67  
3.6. Exercises 67  

4 CAPM and APT 72  
4.1. The capital asset-pricing model 72  
4.1.1. The discount factor 72  
4.1.2. Expected utility maximization 74  
4.1.3. Alternative derivations 77  
4.2. Arbitrage pricing theory 80  
4.3. Conclusions 83  
4.4. Exercises 83  

5 Consumption and saving 86  
5.1. A deterministic economy 86  
5.1.1. Properties of the saving function 88  
5.1.2. Optimal consumption over time 90  
5.2. Portfolio choice under uncertainty 94  
5.3. A more general problem 95  
5.3.1. Precautionary saving 100  
5.4. Conclusions 103  
5.5. Exercises 104  

6 Dynamic programming 109  
6.1. A deterministic growth problem 109  
6.1.1. Guess-and-verify 111  
6.1.2. Finite horizon economies 113  
6.2. Mathematical preliminaries 115  
6.2.1. Markov processes 116  
6.2.2. Vector space methods 118  
6.2.3. Contraction mapping theorem 122  
6.3. A consumption-saving problem under uncertainty 126  
6.4. Exercises 129  

7 Intertemporal risk sharing 133  
7.1. Multi-period contingent claims 133  
7.1.1. Aggregate uncertainty 134  
7.1.2. Central planning problem 139  
7.1.3. Sequential trading 140  
7.1.4. Implications for pricing assets 145  
7.2. Idiosyncratic endowment risk 146  
7.2.1. Notation 147  
7.2.2. The economy 148  
7.2.3. Complete contingent claims 149  
7.2.4. Dynamic programming 151
Contents

7.3. Risk sharing with idiosyncratic and aggregate risk 153
  7.3.1. First-best solution 154
7.4. Conclusions 159
7.5. Exercises 160

8 Consumption and asset pricing 162
  8.1. The consumption-based CAPM 162
    8.1.1. Recursive competitive equilibrium 164
    8.1.2. Asset-pricing functions 167
    8.1.3. Risk premia 171
    8.1.4. Volatility bounds for intertemporal MRSs 175
    8.1.5. The “equity premium puzzle” 178
  8.2. Pricing alternative assets 180
    8.2.1. Discount bonds and the yield curve 180
    8.2.2. Pricing derivative instruments 186
    8.2.3. The Black-Scholes options pricing formula 188
  8.3. A growing economy 191
    8.3.1. Cointegration in asset-pricing relations 195
  8.4. Conclusions 198
  8.5. Exercises 199

9 Non-separable preferences 202
  9.1. Non-time-additive preferences 202
    9.1.1. Habit persistence and consumption durability 203
    9.1.2. A more general specification 204
    9.1.3. A recursive framework 206
    9.1.4. Pricing durable consumption goods 209
    9.1.5. Asset-pricing relations 210
    9.1.6. Log-linear asset-pricing formulas 213
  9.2. Non-expected utility 215
    9.2.1. Recursive preferences under certainty 215
    9.2.2. The role of temporal lotteries 217
    9.2.3. Properties of non-expected utility preferences 220
    9.2.4. Optimal consumption and portfolio choices 223
  9.3. Tests of asset-pricing relations 228
  9.4. A model with an external habit 231
  9.5. Conclusions 235
  9.6. Exercises 235

10 Economies with production 239
  10.1. Recursive competitive equilibrium with production 240
    10.1.1. Households own the capital stock 241
    10.1.2. Households lease capital to firms 246
  10.2. Extensions 248
    10.2.1. Economies with distortions 248
    10.2.2. The role of expectations 252
  10.3. Solving models with production 253
    10.3.1. A parametric model 256
    10.3.2. The stationary distribution 260
  10.4. Financial structure of a firm 262
Contents

10.4.1. The irrelevance of debt versus equity financing 266
10.4.2. The equity price and the equity premium 267
10.4.3. Empirical implications 269
10.4.4. Taxes and the debt-equity ratio 273
10.5. Conclusions 277
Appendix: The invariant distribution 278
10.6. Exercises 282

11 Investment 285
11.1. The neoclassical model of investment 286
11.2. The Q theory adjustment-cost model of investment 288
  11.2.1. The Q theory of investment 288
  11.2.2. Adjustment costs 288
  11.2.3. The social planner’s problem 289
  11.2.4. The market economy 291
  11.2.5. Asset-pricing relations 295
11.3. Irreversible investment 296
  11.3.1. A model with partial irreversibility and expandability 297
  11.3.2. A model of irreversible investment 305
11.4. An asset-pricing model with irreversible investment 307
  11.4.1. The model 307
  11.4.2. The social planner’s problem 308
  11.4.3. The competitive equilibrium 314
  11.4.4. The value of the firm and Q 319
  11.4.5. The relation among stock returns, investment, and Q 321
11.5. Conclusions 323
11.6. Exercises 323

12 Business cycles 326
12.1. Business cycle facts 327
12.2. Shocks and propagation mechanisms 331
12.3. Real business cycle models 333
  12.3.1. An RBC model 335
  12.3.2. A model with indivisible labor supply 338
  12.3.3. Other “puzzles” 342
12.4. Solving business cycle models 346
  12.4.1. Quadratic approximation 346
12.5. Business cycle empirics 352
  12.5.1. Dynamic factor analysis 353
  12.5.2. ML and GMM estimation approaches 357
  12.5.3. A New Keynesian critique 360
12.6. Conclusions 366
12.7. Exercises 367

III MONETARY AND INTERNATIONAL MODELS 371
13 Models with cash-in-advance constraints 373
13.1. “Evil is the root of all money” 374
13.2. The basic cash-in-advance model 376
### Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.2.1.</td>
<td>Solution for velocity 383</td>
</tr>
<tr>
<td>13.2.2.</td>
<td>Empirical results 385</td>
</tr>
<tr>
<td>13.2.3.</td>
<td>Inflation risk and the inflation premium 386</td>
</tr>
<tr>
<td>13.2.4.</td>
<td>Velocity shock 389</td>
</tr>
<tr>
<td>13.3.</td>
<td>Inflation and interest rates 390</td>
</tr>
<tr>
<td>13.4.</td>
<td>Transactions services model 394</td>
</tr>
<tr>
<td>13.5.</td>
<td>Growing economies 399</td>
</tr>
<tr>
<td>13.6.1.</td>
<td>Money and real activity 401</td>
</tr>
<tr>
<td>13.6.2.</td>
<td>Consumption-leisure choices 402</td>
</tr>
<tr>
<td>13.6.3.</td>
<td>Business cycle implications 410</td>
</tr>
<tr>
<td>13.7.</td>
<td>Conclusions 416</td>
</tr>
<tr>
<td>13.8.</td>
<td>Exercises 417</td>
</tr>
<tr>
<td>14.</td>
<td>International asset markets 422</td>
</tr>
<tr>
<td>14.1.</td>
<td>A two-country model 423</td>
</tr>
<tr>
<td>14.2.</td>
<td>International monetary model 430</td>
</tr>
<tr>
<td>14.2.1.</td>
<td>The terms of trade and the exchange rate 435</td>
</tr>
<tr>
<td>14.2.2.</td>
<td>Pricing alternative assets 439</td>
</tr>
<tr>
<td>14.3.</td>
<td>Variants of the basic model 444</td>
</tr>
<tr>
<td>14.3.1.</td>
<td>Non-traded goods 444</td>
</tr>
<tr>
<td>14.3.2.</td>
<td>Exchange rates and international capital flows 448</td>
</tr>
<tr>
<td>14.4.</td>
<td>Conclusions 456</td>
</tr>
<tr>
<td>14.5.</td>
<td>Exercises 456</td>
</tr>
<tr>
<td>15.</td>
<td>Asset pricing with frictions 461</td>
</tr>
<tr>
<td>15.1.</td>
<td>The role of idiosyncratic risk for asset pricing 462</td>
</tr>
<tr>
<td>15.2.</td>
<td>Transactions costs 467</td>
</tr>
<tr>
<td>15.2.1.</td>
<td>A model with bid-ask spreads 469</td>
</tr>
<tr>
<td>15.3.</td>
<td>Volatility bounds with frictions 472</td>
</tr>
<tr>
<td>15.4.</td>
<td>Conclusions 475</td>
</tr>
<tr>
<td>15.5.</td>
<td>Exercises 476</td>
</tr>
<tr>
<td>16.</td>
<td>Borrowing constraints 478</td>
</tr>
<tr>
<td>16.1.</td>
<td>Idiosyncratic risk and borrowing constraints 479</td>
</tr>
<tr>
<td>16.1.1.</td>
<td>The basic model 480</td>
</tr>
<tr>
<td>16.1.2.</td>
<td>Restrictions on markets 480</td>
</tr>
<tr>
<td>16.1.3.</td>
<td>Pure insurance economy 481</td>
</tr>
<tr>
<td>16.1.4.</td>
<td>Pure credit model 484</td>
</tr>
<tr>
<td>16.1.5.</td>
<td>Asset span 491</td>
</tr>
<tr>
<td>16.2.</td>
<td>Townsend turnpike model 492</td>
</tr>
<tr>
<td>16.2.1.</td>
<td>Description of the model 493</td>
</tr>
<tr>
<td>16.2.2.</td>
<td>Borrowing-constrained households 496</td>
</tr>
<tr>
<td>16.2.3.</td>
<td>Borrowing constraints as netting schemes 498</td>
</tr>
<tr>
<td>16.2.4.</td>
<td>Liquidity-constrained households 500</td>
</tr>
<tr>
<td>16.2.5.</td>
<td>Debt-constrained economies 501</td>
</tr>
<tr>
<td>16.3.</td>
<td>Conclusions 502</td>
</tr>
<tr>
<td>16.4.</td>
<td>Exercises 502</td>
</tr>
</tbody>
</table>
Contents

17 Overlapping generations models 504
  17.1. The environment 505
    17.1.1. Primitives 505
    17.1.2. Autarky in the absence of an outside asset 506
  17.2. The stochastic overlapping generations model 508
    17.2.1. Central planning problem 510
    17.2.2. Equal-treatment Pareto-optimal solution 514
  17.3. Competitive equilibrium 515
    17.3.1. Deterministic economy 515
    17.3.2. Fiat money 517
    17.3.3. The stochastic economy 518
  17.4. Equity pricing in a growing economy 526
    17.4.1. Risk premia 529
  17.5. Capital accumulation and social security 533
    17.5.1. Social security 540
  17.6. Conclusions 542
  17.7. Exercises 542

V Supplementary Material 547
A Mathematical appendix 549
  A.1 Stochastic processes 549
  A.2 Some useful theorems 553

Bibliography 558
Index 581
# List of figures

1.1 The consumer’s optimum in an economy with two states  page 8  
2.1 Three-period binomial tree  41  
3.1 Expected utility indifference curves  53  
3.2 Attitudes towards risk  56  
3.3 First-order stochastic dominance  64  
3.4 Mean-preserving spread  66  
4.1 Security market line  77  
5.1 Consumer’s first-order conditions  87  
5.2 Alternating deterministic endowment  93  
6.1 Plots of $\varphi_n(t)$ for $n = 2, 3$  121  
6.2 A fixed point on the real line  123  
8.1 Mean-variance frontier for MRSs  176  
9.1 Early versus late resolution of uncertainty  218  
10.1 Configurations of capital stocks  242  
10.2 Optimal consumption and capital stocks  245  
10.3 Stationary distribution for the capital stock  261  
12.1 Amplitude and duration of a business cycle  328  
12.2 Impulse responses to a shock in technology  353  
13.1 Timing of trades in the Lucas model  379  
13.2 Timing of trades in the consumption/leisure model  404  
A.1 An upper hemi-continuous correspondence  555  
A.2 A lower hemi-continuous correspondence  556
# List of tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>An economy with four dates and two states</td>
<td>45</td>
</tr>
<tr>
<td>10.1</td>
<td>Optimal savings levels as a function of ((k, z))</td>
<td>259</td>
</tr>
<tr>
<td>10.2</td>
<td>Unconditional moments of consumption, capital stock and output</td>
<td>262</td>
</tr>
<tr>
<td>12.1</td>
<td>Cyclical properties of key variables</td>
<td>352</td>
</tr>
</tbody>
</table>
Preface

The starting point for any analysis in finance involves assigning a current price to a future stream of uncertain payoffs. This is the basic notion behind any asset-pricing model. Take, for example, the price of a share to a competitive firm. Since the share entitles the owner to claims for the future profits of the firm, a central problem is to assign a value to these future profits. Take another asset – a house. This provides housing services in all states of nature and at all dates. Consequently, the value of the house today must reflect the value of these future services. Other examples include the pricing of durable goods or investment projects based on their future expected marginal products. One approach to monetary economics also follows this basic principle – if money as an asset has value in equilibrium (in the absence of any legal restrictions), then this value must reflect the stream of services provided by this asset.

Our approach is to derive pricing relationships for different assets by specifying the economic environment at the outset. One of the earliest examples of this approach is Merton [342]. However, Merton does not relate the technological sources of uncertainty to the equilibrium prices of the risky assets. Alternatively, he assumes a given stochastic process for the returns of different types of assets and then prices them given assumptions about consumer preferences. Consequently, the supply side is not explicitly considered by Merton. The asset-pricing model of Lucas [317] is fully general equilibrium but it is an endowment economy, so that consumption and investment decisions are trivial. Brock [76] develops an asset-pricing model with both the demand and supply side fully specified and links it up to Ross’s [369] arbitrage pricing model.

In this book, we will start from an explicit economic environment and deduce the implications for asset prices, and the form of the asset-pricing function from the equilibrium in these environments. To study the problem of asset pricing, we could also follow another approach: we could take a very general and abstract approach, viewing asset pricing as the valuation of a future stream of uncertain payoffs from the asset according to a general pricing function. Given a minimal set of assumptions about the set of payoffs, we could try to characterize the properties of this abstract pricing function. This is the approach taken by Ross [371],
Harrison and Kreps [240], Chamberlain and Rothschild [100], amongst others. One general point to note about the relationship between the two approaches to asset pricing is that the former abstract approach acquires economic content when interpreted in terms of the equilibrium approach. In fact, the benchmark payoff in the pricing function used to price streams of uncertain payoffs turns out to be the intertemporal marginal rate of substitution function for consumption. Depending on the nature of heterogeneity among consumers, the existence of complete contingent claims markets, and the role of money for acquiring consumption goods, the form of this intertemporal marginal rate of substitution function changes.

The purpose of this book is to provide an integrated treatment of a variety of dynamic equilibrium frameworks and to examine their empirical implications. The book is organized in four main parts. In Part I, we present material that constitutes the basis for much thinking in dynamic macroeconomics and finance. We begin by describing a simple version of the Arrow-Debreu contingent claims model, which is one of the building blocks of asset pricing. We also present the basics of arbitrage and asset valuation, expected utility analysis, CAPM and APT, and consumption/savings decisions under uncertainty. In Part II, we present a more fully developed set of results for dynamic economies under uncertainty using a recursive approach. In this part, we describe a pure exchange, representative consumer economy as well as economies with production. This framework allows us to derive the form of the asset-valuation function and to examine such issues as the effects of taxation on asset returns, the optimal financial structure of a firm, and the role of uncertainty in determining asset pricing and equilibrium allocations. Part III is devoted to cash-in-advance models, which allow us to examine the effects of inflation and exchange rate risk. Part IV presents material at a slightly more advanced level. In this part, we examine questions related to market incompleteness and the effects of frictions such as transactions costs. We consider the effects of borrowing constraints on equilibrium allocations and prices in a model with consumer heterogeneity and idiosyncratic risk. The stochastic overlapping generations model has been suggested as a convenient framework for analyzing issues related to “bubbles” in asset prices and the determinants of savings decisions with intergenerational heterogeneity among consumers. We examine a variety of issues using the stochastic overlapping generations framework. In many recent empirical applications of dynamic models, numerical solution methods have been combined with simulation or estimation procedures to assess quantitatively the importance of alternative model features. In this book, we also describe how numerical dynamic programming methods and other numerical methods can be used for solving and simulating a variety of dynamic economic problems.

There are many excellent texts in macroeconomics and finance that also cover material that is presented in this text. Cochrane's [109] text is an
excellent reference that covers all of the standard issues in finance, updated using the modern approach to asset pricing. The texts by Darell Duffie [159, 161] also present the modern general equilibrium approach to finance but they are more technical in nature and help to serve as useful references, especially for advanced graduate students. The texts by Ingersoll [261], Huang and Litzenberger [256], Jarrow [263], Copeland and Weston [127], Hull [259], amongst others, present many of the standard issues of finance at differing levels of abstraction. They are recommended for students who (i) either lack a more traditional knowledge of finance and financial markets, or (ii) wish to obtain more detailed knowledge of some of the issues that we cover in this book. There is also some overlap between the topics we consider in this book and other texts dealing with dynamic general equilibrium modeling or macroeconomics such as Sargent [384] or Ljungqvist and Sargent [325].

A unifying feature of our discussion is that many of the dynamic equilibrium models that we consider can be formulated as dynamic programming problems and solved using a contraction mapping approach. Rather than introduce explicit measure-theoretic considerations for analyzing dynamic stochastic models, we describe uncertainty in terms of Markov uncertainty in a discrete-time setting. In Chapter 6, we provide a review of some results from functional analysis that we use in later chapters. For a review of basic results from functional analysis, we refer the reader to Kreyszig [290] and Naylor and Sell [351], and to Papoulis [356] for a review of probability theory and stochastic processes.

We have provided a set of detailed exercises at the end of each chapter and their solutions as a separate file. These exercises are intended to introduce some new topics at the same time that they allow the student to apply the methods described earlier. We developed this book from our teaching of finance, graduate financial economics and macroeconomics at the University of York, Duke University, the University of Minnesota, the University of Wisconsin, and Columbia University. It reflects our desire to provide a unified treatment of material that we could not find in one place. For teaching purposes, this text can be used as the basis for a graduate macroeconomics or financial economics course. We hope that this text will also prove useful to students and practitioners in the fields of macroeconomics, finance, applied general equilibrium modeling, and structural econometrics.

Paul Soderlind gave many useful comments that helped to improve the current version of this text. We have also received helpful comments from various colleagues for the first edition of the text, including Erdem Başçi, Thomas Cooley, Scott Freeman, Christian Gilles, Jeremy Greenwood, Steve LeRoy, Bruce Smith, Allan Stockman, the participants of the International Workshop at the University of Rochester, and of a series of seminars at Bilkent University in Ankara, Turkey. We thank Zhenyu Wang
for many helpful comments and the numerical calculations. Finally, we are grateful to İrem Demirci and Muharrem Yeşilirmak from Koç University for reading through the chapters and providing editorial comments, for writing exercises and solutions to various chapters, and assisting with other aspects of the production of this manuscript.