Insights into Game Theory

Few branches of mathematics have been more influential in the social sciences than game theory. In recent years, it has become an essential tool for all social scientists studying the strategic behavior of competing individuals, firms, and countries. However, the mathematical complexity of game theory is often very intimidating for students who have only a basic understanding of mathematics. *Insights into Game Theory* addresses this problem by providing students with an understanding of the key concepts and ideas of game theory without using formal mathematical notation. The authors use four very different topics (college admissions, social justice and majority voting, coalitions and cooperative games, and a bankruptcy problem from the Talmud) to investigate four areas of game theory. The result is a fascinating introduction to the world of game theory and its increasingly important role in the social sciences.

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Insights into Game Theory

An Alternative Mathematical Experience

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AND
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This book is dedicated to the memory of Michael Maschler,
who passed away on July 20, 2008.

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Preface

This book is a *tour de force* of what may be called "verbal mathematics." It demonstrates conclusively that mathematics is not a matter of symbols and equations; rather, it may be characterized as "precise reasoning that has considerable depth, complexity, or sophistication." The book is accessible to everyone who can think.

Also, it is a wonderful introduction to game theory; rather than "explaining" what the theory is about, it simply *does* it. If somebody came from Mars and wanted to know what we mean by "music," you could try to "explain" it; but it would be better to play a Bach fugue, a Verdi aria, some Louis Armstrong jazz, and "Lucy in the Sky with Diamonds." The second alternative is what Gura and Maschler do. Enjoy!

Robert J. Aumann

Introduction

Game theory is a relatively young branch of mathematics that goes back to the publication of *Theory of Games and Economic Behavior* by John von Neumann and Oskar Morgenstern in 1944.[1]

Game theory undertakes to build mathematical models and draw conclusions from these models in connection with interactive decision-making: situations in which a group of people not necessarily sharing the same interests are required to make a decision.

The choice of the topics reflects our purpose: we wanted to present material that does not require mathematical prerequisites and yet involves deep game-theoretic ideas and some mathematical sophistication. Thus, we ruled out topics from non-cooperative game theory, which requires some knowledge of probability, matrices, and point-set topology.

Broadly speaking, the topics chosen are all related to the various meanings that can be given to the concept of "fair division." The four chapters illustrate this.

The first, "Mathematical Matching," concerns, among other things, the problem of assigning applicants to institutions of higher learning. Each applicant ranks the universities according to his scale of preferences. The institutions of higher learning, in turn, rank the applicants for admission according to their own scale of preferences. The question is how to effect the "matching" between the applicants and the universities. The reader will discover that this problem leads to unexpected solutions.

The second chapter, "Social Justice," concerns social decision rules. In a democratic society it is customary to make decisions by

[1] Several "game-theoretic" topics had been discussed before this publication, but not in any systematic way.

a vote. The decision supported by the majority of voters is adopted. But the reader will discover that "majority rule" does not always yield clear-cut solutions. The attempt to find other voting rules raises unexpected difficulties.

The third chapter, "The Shapley Value in Cooperative Games," addresses, among other things, the following problem: a group of people come before an arbitrator and inform him of the expected profits of every subgroup, as well as of the whole group, if the groups operate independently. It seems that these data are sufficient for the arbitrator to decide how to divide the profits if all the litigants operate jointly.

The fourth chapter, "Analysis of a Bankruptcy Problem from the Talmud," addresses the following problem: several creditors have claims to an estate, but the total amount of the claims exceeds the value of the estate. How should the estate be divided among the creditors? In the chapter several solutions are accepted, two of which are discussed in the Talmud.

As explained above, this book is not a textbook in game theory. Rather, it is a collection of a few topics from the theory intended to open a window onto a new and fascinating world of mathematical applications to the social sciences. Our hope is that it will motivate the reader to take a solid course in game theory.

One of the aims of the book is to acquaint the reader and the student with "a different mathematics" – a mathematics that is not buried under complicated formulas, yet contains deep mathematical thinking. Another aim is to show that mathematics can efficiently handle social issues. A third aim is to deepen the mathematical thinking of the person who studies this book.

We believe that by studying the topics of this book, the mathematical thinking of the student will be enriched.

This book selects a small number of topics and studies them in depth. It shows the student of the social sciences how a mathematical model can be constructed for real-life issues.

The chapters are independent. A teacher and a student can choose one chapter or several and cover them in any order.

In high schools, the book can be used by students on any program track or as extracurricular material. The teacher can proceed to the deeper parts of each chapter if she has a mathematically inclined class or skip some of the proofs if the class cannot handle them. The book can also be used by students who want to read independently or under the guidance of a teacher beyond what is required in school.

At universities and colleges, the book can be used in courses whose aim is to introduce general game-theoretic topics and deepen mathematical thinking.

This book owes its origin to the PhD thesis of co-author Ein-Ya Gura. We thank the Science Teaching Center at the Hebrew University of Jerusalem for permission to publish this translation from Hebrew, the Center for the Study of Rationality for funding the translation, and the translator, Michael Borns, not only for the accuracy of his translation, but for the competence of his editing. We thank James Morrow for taking the time to read and comment on the manuscript, Zur Shapira for recommending it to Cambridge University Press, and Chris Harrison and the staff of Cambridge University Press for their encouragement and help in bringing the book to its final form. Last, but not least, we thank Robert Aumann for providing the impetus for both the Hebrew and the English publication of this book.

Ein-Ya Gura and Michael Maschler,
The Hebrew University of Jerusalem,
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