# Index

 $a^{x}$ , definition and continuity, 161 Abel's limit theorem, 333 absolute convergence test, 104 absolute convergence theorem, 332, 337 absolute value, 12 absolutely convergent series, 104 addition formula for  $tan^{-1}$ , 166 alternating test, 107 antipodal points theorem, 145 approximation by Taylor polynomials, 317 Archimedean property of  $\mathbb{R}$ , 7 arithmetic in  $\mathbb{R}$ , 8, 30 arithmetic mean - geometric mean inequality, 18, 21 asymptotic behaviour of functions, 176 basic continuous functions, 142 differentiable functions, 224 null sequences, 48 power series, 325 series, 97 Bernoulli's inequality, 20, 237 Bessel function, 329 Bijection, 355 binomial theorem, 358, 342 blancmange function, 244 Bolzano-Weierstrass theorem, 70 bound, greatest lower, 26 bound, least upper, 25 boundedness theorem, 62, 92, 149 Cauchy condensation test, 97 Cauchy's mean value theorem, 239 Cauchy-Schwarz inequality, 20, 311 Chain Rule, 218 combination rules continuous functions, 136 convergent sequences, 55 convergent series, 89 differentiable functions, 216 functions which tend to  $\infty$ , 177

inequalities, 14 infimum, 275 integrable functions, 278 limits of functions, 172 null sequences, 46 power series, 338

primitives, 284 sequences which tend to  $\infty$ , 65 series, 89 supremum, 275 tilda. 305 common limit criterion, 268 common refinement, 262 comparison test, 94 composition rule asymptotic behaviour of functions, 180 continuous functions, 136 differentiable functions, 218 limits of functions, 173 conditionally convergent series, 104 continuity continuity, 132, 194 limits of functions, 171 one-sided, 134, 194 uniform, 201 continuous functions basic, 142 boundedness theorem, 149 combination rules, 136 composition rule, 136 extreme values theorem, 169 integrability, 277 intermediate value theorem, 143 inverse function rule, 153 squeeze rule, 137 convergence, interval of, 330 radius of, 330 convergent sequence, 53 combination rules, 55 quotient rule, 55 squeeze rule, 58 convergent series, 85 combination rules, 89  $cos^{-1}$ , 156 cosh, power series, 339  $cosh^{-1}$ , 158 cosine function derived function, 214

Darboux's theorem, 254 decimal representation of numbers, 3, 87 density property of  $\mathbb{R}$ , 7 derivatives, 207 higher order, 215

power series, 326

one-sided, 210 standard, 359 difference quotient, 206 differentiability of  $f(x) = e^{x}$ , 214  $f(x) = a^x$ , 223  $f(x) = x^{\alpha}, 223$  $f(x) = x^x$ , 223 trigonometrical functions, 214 differentiable functions combination rules, 216 composition rules, 218 inequalities involving, 237 inverse function rule, 221 differentiation, 210 rule for power series, 341 Dirichlet's function, 197, 310 discontinuity, removable, 172 divergent sequence, 61 series, 85 domination hierarchy, 56 e, definition, 74 irrationality, 175  $\varepsilon - \delta$  game, 43, 187 Euclidean algorithm, 78 even subsequence, 66  $e^x$ , definition, 75, 122 fundamental property, 127 inverse property, 75 exponent laws, 163 exponential function continuity, 141 derived function, 214 differentiability, 223 inequalities, 141 power series, 326 extreme values theorem, 149 extremum, 228

field, 9 first subsequence rule, 67 function, nowhere differentiable, 244 fundamental inequality for integrals, 288 fundamental theorem of Algebra, 146 of Calculus, 283, 292

general binomial theorem, 342 geometric series, 87

## CAMBRIDGE

Cambridge University Press 978-0-521-68424-8 - A First Course in Mathematical Analysis David Alexander Brannan Index More information

### 458

Goethe, ix greatest lower bound, 26, 272 property of  $\mathbb{R}$ , 29 Gregory's series for estimating  $\pi$ , 346

harmonic series, 93, 109 higher-order derivatives, 215 hyperbolic functions, differentiability, 224 hypergeometric series, 352

increasing-decreasing theorem, 234 inequalities, power rule, 10 rules for integrals, 289 infimum, 26, 272 infinite series, 85 inheritance property of subsequences, 66 Int I, 234 Integrability, 264 combination rules, 278 continuous functions, 277 integral test, 297 modulus rule, 278 monotonic functions 276 Riemann's criterion, 267 integral, 264 additivity, 280 inequality rules, 289 lower, 264 upper, 264 integration by parts, 285 by substitution, 286 integration rule for power series, 341 integration, reduction of order method, 293 intermediate value theorem, 143 interval image theorem, 149 interval of convergence, 230 inverse function rule for continuous functions, 153 differentiable functions, 221 inverse hyperbolic functions, 158 inverse trigonometric functions, 156 irrational numbers, 5

#### $K\varepsilon$ lemma, 49, 193

least upper bound, 25, 262 property of  $\mathbb{R}$ , 29 Leibniz notation, 207 Leibniz test, 107 Leibniz's series for estimating  $\pi$ , 346 l'Hôpital's rule, 241 limit comparison test, 95 limit inequality rule for limits, 175 sequences, 60 limit of a function, 169, 177, 185 as  $x \rightarrow \infty$ , 177 one-sided, 175 limit of a sequence, 53, 182 limits of functions and continuity, 171 limits of functions, combination rules, 172 composition rule, 173 one-sided, 175 squeeze rule, 174 local extremum theorem, 229  $log_e(1 + x)$ , power series for, 325 lower integral, 264 lower Riemann sum, 259

Maclaurin integral test, 297 mathematical induction, 357 maximum, 23 mean value theorem, 233 minimum, 24 modulus function, continuity, 135 modulus rule for integrable functions, 278 modulus, 12 monotone convergence theorem, 68 monotonic function, 153 integrability, 276 monotonic sequence theorem, 69 multiplication of series, 114

n!, Stirling's formula for, 306
neighbourhood, 169
non-null test, 91
nth partial sum of series, 85
nth root function, continuity, 155
nth root of a positive real number, 32, 155
null partition criterion, 269
null sequence, 43

odd subsequence, 66 one-one function, 152, 355 one-sided derivative, 210 one-sided limit, 175 onto function, 355 order properties of  $\mathbb{R}$ , 7

π, 76 irrationality of, 348 numerical estimates, 346 partial sum of series, 85 partition of [a, b], 258 standard, 258 power series absolute convergence, 332 basic, 325 combination rules, 338 differentiation rule, 341 integration rule, 341 uniqueness theorem, 342 primitives, 282 combination rules, 278 scaling rule, 278 standard, 360 uniqueness theorem, 284

ratio test, 96 for radius of convergence, 330 rational function, continuity, 136 differentiability, 217 rational power, 34 rearrangement of series, 109 reciprocal rule for functions which tend to  $\infty$ , 177 sequences which tend to  $\infty$ , 64 tilda, 305 recursion formula, 71 reduction of order method, 293 refinement, 262 remainder estimate, 322 removable discontinuity, 172 Riemann's rearrangement theorem, 112 Riemann's function, 198 Riemann's-criterion for integrability, 267

Rolle's theorem, 231

radius of convergence theorem, 329

scaling rule for primitives, 284 second derivative test, 236 second subsequence rule, 67 sequences, 38 basic null, 48 combination rules, 55 convergent, 53 divergent, 61 limit inequality rule, 60 monotonic, 40 null, 43 unbounded, 62 which tend to  $\infty$ , 63 which tend to  $-\infty$ , 65 squeeze rule, 58 series, 85 absolutely convergent, 104 basic, 97 convergent, 85 divergent, 85 geometric, 87 harmonic, 93, 109 hypergeometric, 352 integral test, 297 multiplication, 114 product rule, 114 Taylor, 325 telescoping, 88 sine function derived function, 214 power series, 326 sine inequality, 140  $sin^{-1}$ , 156 sinh, 158 power series, 339 square root function, continuity, 134 squeeze rule as  $x \to \infty$ , 178 continuous functions, 137

### Index

# CAMBRIDGE

Cambridge University Press 978-0-521-68424-8 - A First Course in Mathematical Analysis David Alexander Brannan Index More information

### Index

convergent sequences, 58 limits of functions, 174 null sequences, 47 sequences which tend to  $\infty$ , 179 standard derivatives, 359 standard partition, 258 standard primitives, 360 Stirling's formula, 306 strategy for testing for convergence, 116 sub-interval theorem, 280 subsequence, 66 sum function, 325 supremum, 35

*tan*<sup>-1</sup>, 157 power series, 341

 $tanh^{-1}$ , 159 power series, 344 tangent approximation, 314 Taylor polynomial, 316 Taylor series, 325 Taylor's theorem, 320 tilda notation, 300, 304 combination rules, 305 transitive property of  $\mathbb{R}$ , 7 transitive rule for inequalities, 291 triangle inequality, 15, 16 backwards form, 15 infinite form, 105 integrals, 291 trichotomy property of  $\mathbb{R}$ , 7 trigonometric functions

continuity, 139 differentiability, 213

unbounded sequence, 62 uniform continuity, 201 uniqueness theorem, power series, 342 primitives, 284 upper integral, 264 upper Riemann sum, 259

Wallis's formula, 293 Weierstrass, K., x

zero derivative theorem, 235 zero of polynomial, 295 zeros localisation theorem, 147