CHAPTER I

SUMMARY

FINITE arithmetical differences have proved remarkably successful in dealing with differential equations; for instance, approximate particular solutions of the equation for the diffusion of heat $\partial^2\theta/\partial x^2 = \partial\theta/\partial t$ can be obtained quite simply and without any need to bring in Fourier analysis. An example is worked out in a paper published in *Phil. Trans.* A, Vol. 210*. In this book it is shown that similar methods can be extended to the very complicated system of differential equations, which expresses the changes in the weather. The fundamental idea is that atmospheric pressures, velocities, etc. should be expressed as numbers, and should be tabulated at certain latitudes, longitudes and heights, so as to give a general account of the state of the atmosphere at any instant, over an extended region, up to a height of say 20 kilometres. The numbers in this table are supposed to be given, at a certain initial instant, by means of observations.

It is shown that there is an arithmetical method of operating upon these tabulated numbers, so as to obtain a new table representing approximately the subsequent state of the atmosphere after a brief interval of time, δt say. The process can be repeated so as to yield the state of the atmosphere after $2\delta t$, $3\delta t$ and so on. There is a limit however to the possible number of repetitions, because each table is found to be smaller than its predecessor, in longitude and latitude, having lost a strip round its edge. Only if the table included the whole globe could the repetitions be endless. Also the errors increase with the number of steps.

In Ch. 2 the working of the method is shown by its application to a specially simplified case. In Ch. 3 the coordinate differences are considered in relation to the average size of European cyclones, and the following differences are provisionally selected: in time 6 hours, in longitude the distances between 128 equally spaced meridians, in latitude 200 kilometres of the earth's circumference, and in height the intervals between fixed heights nearly corresponding to the normal pressures of 8, 6, 4, 2 decibars. Thus small-scale phenomena, such as local thunderstorms, have to be smoothed out.

In Ch. 4 the fundamental equations are collected from various sources, set in order and completed where necessary. Those for the atmosphere are then integrated with respect to height so as to make them apply to the mean values of the pressure, density, velocity, etc., in the several conventional strata. Incidentally certain constants relating to friction and to radiation are collected from observational data. It is found to be necessary to eliminate the vertical velocity from all the equations, and in Ch. 5 it is shown how this can be done. Special difficulties arise in connection with the uppermost stratum on account of its great thickness and the enormous ratio of density

* pp. 312, 313.

between its upper and lower surfaces. These difficulties are removed in Ch. 6, as far as high latitudes are concerned. In particular it is shown how the total mass transport above any level may be deduced from a pilot balloon observation which extends well into the stratosphere.

In Ch. 7 the arrangement of the tabular numbers in space and time is discussed with a view to securing the best representation of differential coefficients by difference ratios.

In Ch. 8 the whole system of arithmetical operations is reviewed in order. With regard to the horizontal differential coefficients the general method may be briefly described in the following four sentences: Take the differential equations and replace everywhere the infinitesimal operator ∂ by the finite difference operator δ. Use arithmetic instead of symbols. Attend carefully to the centering of the differences. Leave the errors due to the finiteness of the differences over for consideration at the end of the process. With regard to the vertical differential coefficients, on the contrary, it is often possible to effect an exact transformation to differences, by means of a vertical integration. In arranging the computing, it has constantly to be borne in mind that the rate of change with time of every one of the discrete values of the dependent variables must be calculable from their instantaneous distribution in time and space, excepting only those values near the edge of the horizontal area represented in the table. We may refer to this necessary property by saying, for brevity, that the system must be "lattice-reproducing."

In Ch. 9 will be found an arithmetical table showing the state of the atmosphere observed over middle Europe at 1910 May 20 d. 7 h. G.M.T. This region and instant were chosen because the observations form the most complete set known to me at the time of writing, and also because V. Bjerknes has published large scale charts of the isobaric surfaces, together with collated data for wind, cloud and precipitation. Starting from the table of the initially observed state of the atmosphere at this instant, the method described in the preceding paragraphs is applied, and so the rates of change of the pressures, winds, temperatures, etc. are obtained. Unfortunately this "forecast" is spoilt by errors in the initial data for winds. These errors appear to arise mainly from the irregular distribution of pilot balloon stations, and from their too small number.

In Ch. 10 the smoothing of initial observations is discussed.

In Ch. 11 is a collection of problems still waiting to be solved, with some suggestions for their treatment.

Ch. 12 is a list of Notation.

Pressures fictitiously "reduced to sea-level" are not used in the present method. Instead, the varying height of the land is dealt with by the variation of the lower limit of an integral with respect to height. See Ch. 4/2, Ch. 4/4.

The problem of weather prediction is of the "marching" variety. To explain this statement it should be pointed out that the ease or difficulty with which physical problems involving differential equations can be solved, depends on very different things according to whether symbolic methods or arithmetical differences are to be employed. In the former case the main facts in the situation are the "order" and "degree" of the equations and whether they are ordinary or partial. In the latter case what usually matters most is the relation of the "body equation" to the boundary conditions. By "body equation" is here meant the differential equation which holds throughout the region of space and the interval of time with which we have to deal. The "boundary" must be understood to be the limits of either this time or this space. According to the relation between the body equation and the boundary conditions, problems are divided into:

(i) "Jury" problems in which the integral must be determined with reference to the boundary as a whole: for instance the problem of a stone thrown from a given point to hit another given point; or that of the stresses inside a loaded dam. Cases like these frequently require troublesome successive approximations, before a statement is obtained with which the "jurymen," seated round the boundary, will all agree.

(ii) "Marching" problems in which the integral can be stepped out from a part of the boundary: for instance the problem of a stone thrown with given initial vector-velocity, or that of the cooling of a body with given initial and superficial temperatures. Other things being equal, these problems are much more easily solved than those in division (i) above. Weather prediction falls into the "marching" category.

Whilst dealing with the general subject of finite differences it may be well to mention two important properties brought to notice by Mr W. F. Sheppard.

(*a*) The great gain in accuracy, in the representation of a differential coefficient, when the differences are centered instead of progressive; a gain secured by a slight increase of work.

(*b*) That the errors due to centered differences, when small enough, are proportional to the square of the coordinate difference. This fact provides a universal means of checking and correcting the errors.

For further information about centered differences the reader is referred to "Central-Difference Formulae," by W. F. Sheppard, *Proc. Lond. Math. Soc.* Vol. XXXI. (1899) and to "The Approximate Arithmetical Solution by Finite Differences of Physical Problems," by L. F. Richardson, *Phil. Trans.* A, Vol. 210, p. 307 (1910).

1—2

CHAPTER II

INTRODUCTORY EXAMPLE

BEFORE attending to the complexities of the actual atmosphere and their treatment by this numerical method, it may be well to exhibit the working of a much simplified case. Lest the reader, catching sight of numbers of 7 digits, should suppose that these are necessary, let me at once point out that they have been introduced in order to measure the errors due to finite differences, which in this example are very small. An intelligible picture of the sequence of phenomena would remain after the last 4 places of digits had been cut off everywhere.

Suppose now that there is no precipitation, clouds or water vapour, neither solar nor terrestrial radiation, no eddies, and no mountains or land, but an atmosphere in which we can ignore or summarize variations with height moving upon a globe covered by sea. Further to simplify the problem, let us neglect all the quadratic terms in the dynamical equations. Then, in order to summarize the vertical velocity and the density, let us perform an integration with respect to height upon the horizontal dynamical equations and upon the equation of continuity of mass. If the limits of integration are sea-level, and a height so great that the density there is negligible, we thus arrive at a set of equations similar to those used by Laplace in his discussion of Tides on a Rotating Globe (*vide* Lamb, *Hydrodynamics*, 4th ed. § 214):

$$\frac{\partial M_E}{\partial t} = -H' \frac{\partial p_G}{\partial e} + 2\omega \sin\phi \,.\, M_N, \quad \text{.......(1)}$$

$$\frac{\partial M_N}{\partial t} = -H' \frac{\partial p_G}{\partial n} - 2\omega \sin\phi \,.\, M_E, \quad \text{.......(2)}$$

$$\frac{\partial p_G}{\partial t} = -g\left\{\frac{\partial M_E}{\partial e} + \frac{\partial M_N}{\partial n} - \frac{M_N \tan\phi}{a}\right\} \quad \text{.......(3)}$$

$$= -g\left\{\frac{\partial M_E}{\partial e} + \frac{1}{\cos\phi}\frac{\partial}{\partial n}(M_N \cos\phi)\right\}. \quad \text{.......(3a)}$$

Here M_E, M_N are the components of the whole momentum of the column of atmosphere standing upon a horizontal square centimetre at sea-level, p_G is the pressure at sea-level, g is gravity, t is time, ϕ is latitude, a is the radius of the earth, ω its angular velocity, ∂e and ∂n are distances to east and to north, and H' is an empirical height, used to convert $\int_{h=0}^{h=\infty} \frac{\partial p}{\partial e}\, dh$ into $H' \frac{\partial p_G}{\partial e}$. The data given by Mr W. H. Dines for the difference of pressure between cyclones and anticyclones up to 14 km, and at 20 km, when combined with the extrapolation into the stratosphere according to the method to be described in Ch. 6 below, indicate that $H' = 9{\cdot}2$ km $= 0{\cdot}92 \times 10^6$ cm, on the average.

Proof of (1), (2), (3) is not here given, because at this stage we are more concerned

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with the procedure of solving these equations, than with their naturalness. Let it suffice to point out that this is a set of three simultaneous equations, involving three dependent variables M_E, M_N and p_G and three independent variables, time, latitude and longitude.

The atmospheric pressure at sea-level, p_G, plays the same rôle in these equations as does the elevation of the sea above its equilibrium level in the tidal equations. Much of tidal theory* is directly applicable, but its interest has centered mainly in forced and free oscillations, whereas now we are concerned with unsteady circulations.

The free periods of oscillation of the hypothetical atmosphere governed by equations (1), (2), (3) are the same as those of an ocean of uniform depth H' in which the particles of water do not attract one another to an appreciable extent.

To proceed to the numerical solution of (1), (2) and (3) we take a piece of paper ruled in large squares, like a chessboard, and let it represent a map. The lines forming the squares are taken as meridians and parallels of latitude—an unusual thing in map projection. Next, we lay down the convention that all numbers written inside a square relate to the latitude and longitude of its centre. The longitude of the centres of the squares is written down at the top of the table, and the latitude along the left-hand margin. The difference δn is constant and equal to 400 km between the points where like quantities such as p and p, or M and M, are tabulated, although the squares are only 200 km in the side. The distance δe, between the points where like quantities are tabulated, is that between meridians equally spaced at the rate of 64 to the equator; it is nearly 400 km in latitude 50°.

The symbols p_G, M_E, M_N are written in the squares in the places shown in the table. It is seen that pressure and momentum alternate in a pattern, which is such that, if a chessboard had been used, the pressures would all appear on the red squares, and the momenta all on the white ones, or vice versa. The reason for this pattern will appear as we go on.

Now to represent the initial observations of pressure we are at liberty to write down any arbitrary set of numbers, at the points of the map where p_G is required, only with this qualification: that if the assumed pressure gradients be unnaturally steep, the consequent changes will be perplexingly violent. Since p_G enters the equations only by way of its differential, we may dispense with superfluous digits by tabulating differences of pressure from any standard value such as the general mean. These differences are here denoted by Δp_G.

When the pressure distribution has been chosen, we have next to represent the initial observations of momentum-per-area by writing down numbers in the alternate chequers. These numbers might have been chosen independently of the pressure, and in fact quite arbitrarily, with a qualification similar to that mentioned above. But it has been thought to be more interesting to sacrifice the arbitrariness in order to test our familiar idea, the geostrophic wind, by assuming it initially and watching the ensuing changes.

* E.g. Lamb, *Hydrodynamics*, 4th ed. §§ 213 to 223 and 314 to 316. *Vide* also Gold in M. O. publication 203 on diurnal variation of the trade winds of the Atlantic Ocean.

In order to convince the reader of the reliability of the numerical method, a problem has been selected which has been solved analytically. This has imposed a further great restriction on the arbitrariness. One of the most powerful of analytic methods is that of expansion in a series, by Maclaurin's Theorem. (*Vide* Forsyth's *Differential Equations.*) But in order that this method may be applicable, the initial distribution must be free from discontinuities even in remote regions. To get isobars running partly north and south, we may try $p_G = \sin \lambda$, where λ is longitude eastward. To avoid a discontinuity of pressure at the pole we can multiply by $\cos \phi$. To avoid an infinite geostrophic wind at the equator we can multiply by $(\sin \phi)^2$.

So the selected form of the initial pressure distribution has been

$$\Delta p_G = \sin \lambda \cos \phi \, (\sin \phi)^2 \times 10^5 \text{ dynes cm}^{-2}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

where Δp_G signifies the deviation from the general mean.

The isobars are shown in Fig. 1 on the map of the globe. The equator and the meridian of Greenwich are isobars. There is high pressure over Asia, and over the

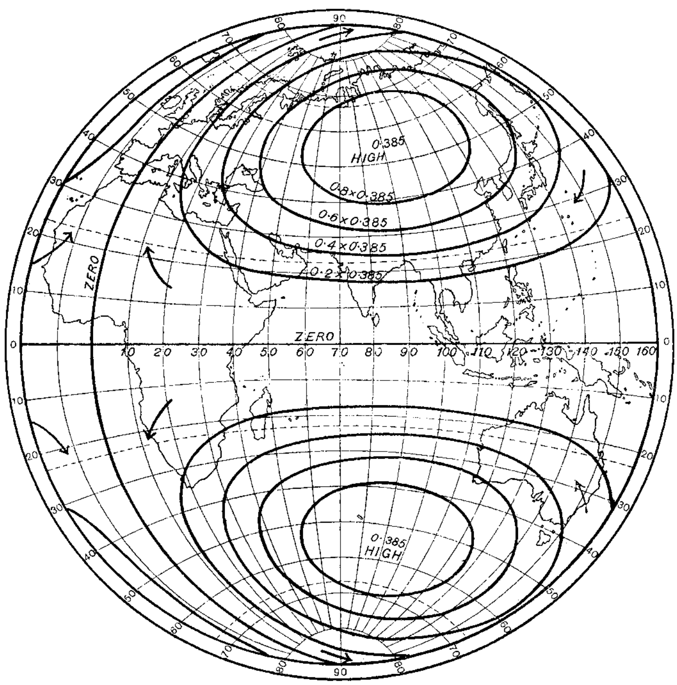

Fig. 1. Isopleths of $\sin l \cos \phi \, (\sin \phi)^2$ corresponding to the isobars of the initial distribution studied in Chapter II.

south Indian Ocean. Low pressure is at the antipodes of each high. The maxima and minima of pressure occur in latitude $54^\circ{\cdot}7$, that is $6{\cdot}08 \times 10^8$ cm from the equator, and amount to $38{\cdot}5$ millibars above or below the mean.

A small portion of this distribution of pressure near England is entered numerically on the table.

To find the geostrophic momenta-per-area, we next insert the above value of Δp_G

in equations (1) and (2), at the same time putting $\partial M_E/\partial t = 0$; $\partial M_N/\partial t = 0$. Then it follows that initially

$$M_E = -\frac{H'}{2\omega a} \sin\lambda \left\{\tfrac{1}{2} + \tfrac{3}{2}\cos 2\phi\right\} \times 10^5 \text{ grm sec}^{-1}\text{ cm}^{-1}, \quad \ldots\ldots\ldots\ldots\ldots(5)$$

$$M_N = +\frac{H'}{2\omega a} \cos\lambda \sin\phi \times 10^5 \text{ grm sec}^{-1}\text{ cm}^{-1}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

These initial momenta-per-area, expressed in numbers, are entered at the appropriate points of the numerical table.

Next, inserting the numbers from the table into equations (1), (2), (3), and treating difference-ratios as if they were differential coefficients, we get the rates of increase of M_E, M_N, p_G. These rates have been multiplied by a δt equal to 2700 seconds, that is by $\frac{3}{4}$ hour, in order to get the increases in that time. An example of the calculation of the increase of M_E will now be given at the point having longitude $-3\delta\lambda$, latitude $6{\cdot}4 \times 10^8$ cm from equator, which will be found in the top left-hand corner of the table of initial data:

$$M_N = 827578{\cdot}6 \text{ grm cm}^{-1}\text{ sec}^{-1},$$

$$2\omega \sin\phi \,.\, \delta t = 0{\cdot}3324746, \text{ a pure number};$$

multiplying these together we get

$$2\omega\sin\phi \,.\, M_N = +275148{\cdot}9 \text{ grm cm}^{-1}\text{ sec}^{-1},$$

Nearest Δp_G to east $= -3744{\cdot}11$ grm cm^{-1} sec^{-2}

Nearest Δp_G to west $= -7452{\cdot}17$,, ,,

difference $+3708{\cdot}06$,, ,,

but $$\frac{\delta t \,.\, H'}{\delta e} = +74{\cdot}17322 \text{ seconds};$$

multiplying, we get

$$-\frac{\delta t \,.\, H' \,.\, \delta p_G}{\delta e} = -275038{\cdot}8 \text{ grm cm}^{-1}\text{ sec}^{-1};$$

adding, we get from equation (1),

$$\frac{\partial M_E}{\partial t}\delta t = +110{\cdot}1 \text{ grm cm}^{-1}\text{ sec}^{-1}.$$

This change of 110·1 is entered on the initial table in parenthesis under the value of M_E. Its value is, in this case, a measure of the error due to the finite difference, for we took M_N such as to make $\partial M_E/\partial t$ vanish, when the calculation was performed exactly by analysis. It is seen that the error is in this example quite small, being only 1/7000 of the resultant of M_E and M_N. That is why such large numbers of digits have been taken. The changes in M_E at other points are all worked in the same way, but of course the coefficients vary with latitude. The computation of $\delta t \,.\, \partial M_N/\partial t$ from equation (2) is so similar that it need not be illustrated.

Next, as to the pressure changes: it happens conveniently that $\cos\phi \,.\, \delta n$ is a fixed

Auxiliary functions of latitude				Distance north of equator centim $10^8 \times$	Initial distribution, with, in parentheses, increases in $\frac{3}{4}$ hour								
$\frac{2700 H'}{\delta e}$	$\frac{2700 \times}{2\omega \sin \phi}$	$\frac{2700 g}{\delta e}$	$\frac{\delta e}{\delta n}$		$-4\delta\lambda$	$-3\delta\lambda$	$-2\delta\lambda$	$-\delta\lambda$	Longitude 0	$\delta\lambda$	$2\delta\lambda$	$3\delta\lambda$	$4\delta\lambda$
78·07616	0·3389380	$8{\cdot}308322 \times 10^{-2}$	·7953772	6·6		Δp_G −5533·74	M_N 848791·2	Δp_G −1850·52	M_N 852898·2	Δp_G 1850·52	M_N 848791·2	Δp_G 5533·74	
74·17322	·3324746	$7{\cdot}892998 \times 10^{-2}$	·8372294	6·4	Δp_G −7452·17	M_E −20161·5 (+110·1) M_N 827578·6 (−6·7)	Δp_G −3744·11 (+2192·21)	M_E −6742·2 (+111·7) M_N 835626·0 (−2·1)	Δp_G 0·00 (+2202·79)	M_E 6742·2 (+111·7) M_N 835626·0 (+2·1)	Δp_G 3744·11 (+2192·21)	M_E 20161·5 (+110·1) M_N 827578·6 (+6·7)	Δp_G 7452·17
70·70837	·3256830	$7{\cdot}524293 \times 10^{-2}$	·8782553	6·2	M_E −10088·4	Δp_G −5641·79 (2285·89)	M_E −5068·6 (+106·3) M_N 815597·2 (−7·9)	Δp_G −1886·65 (+2308·14)	M_E 0·0 (+107·5) M_N 819543·4 (0·0)	Δp_G 1886·65 (2308·14)	M_E 5068·6 (+106·3) M_N 815597·2 (+7·9)	Δp_G 5641·79 (+2285·89)	M_E 10088·4
67·61653	·3185700	$7{\cdot}195281 \times 10^{-2}$	·9184145	6·0	Δp_G −7505·32	M_E +5303·1 (+101·8) M_N 792967·7 (−17·7)	Δp_G −3770·82 (+2405·14)	M_E +1773·2 (+102·5) M_N 800679·0 (−6·0)	Δp_G 0·00 (+2416·75)	M_E −1773·2 (+102·5) M_N 800679·0 (+6·0)	Δp_G 3770·82 (+2405·14)	M_E −5303·1 (+101·8) M_N 792967·7 (+17·7)	Δp_G 7505·32
64·84507	·3111427	$6{\cdot}900361 \times 10^{-2}$	·9576673	5·8	M_E +24543·9	Δp_G −5614·87 (+2492·95)	M_E 12331·3 (+97·2) M_N 779184·4 (−15·8)	Δp_G −1877·65 (+2517·16)	M_E 0·0 (+97·9) M_N 782954·5 (0·0)	Δp_G 1877·65 (+2517·16)	M_E −12331·3 (+97·2) M_N 779184·4 (+15·8)	Δp_G 5614·87 (+2492·95)	M_E −24543·9
62·35096	·3034082	$6{\cdot}634956 \times 10^{-2}$	·9959750	5·6	Δp_G −7382·85	M_E +31830·7 (+92·4) M_N 755228·0 (−28·5)	Δp_G −3709·29 (+2608·54)	M_E 10644·4 (+92·8) M_N 762572·0 (−9·7)	Δp_G 0·00 (+2621·15)	M_E −10644·4 (+92·8) M_N 762572·0 (+9·7)	Δp_G 3709·29 (+2608·54)	M_E −31830·7 (+92·4) M_N 755228·0 (+28·5)	Δp_G 7382·85
60·09873	·2953744	$6{\cdot}395288 \times 10^{-2}$	1·0332998	5·4	M_E +60313·5	Δp_G −5459·81 (+2690·07)	M_E 30302·7 (+88·0) M_N 739696·5 (−22·5)	Δp_G −1825·80 (+2716·23)	M_E 0·0 (+88·0) M_N 743275·3 (0·0)	Δp_G 1825·80 (+2716·23)	M_E −30302·7 (+88·0) M_N 739696·5 (+22·5)	Δp_G 5459·81 (+2690·07)	M_E −60313·5
58·05882	·2870490	$6{\cdot}178216 \times 10^{-2}$	1·0696048	5·2	Δp_G −7096·70	M_E +59004·1 (+82·5) M_N 714507·5 (−39·0)	Δp_G −3565·52 (+2801·67)	M_E 19731·4 (+83·2) M_N 721455·5 (−12·8)	Δp_G 0·00 (2815·24)	M_E −19731·4 (+83·2) M_N 721455·5 (+12·8)	Δp_G 3565·52 (+2801·67)	M_E −59004·1 (+82·5) M_N 714507·5 (+39·0)	Δp_G 7096·70
56·20651	·2784404	$5{\cdot}981105 \times 10^{-2}$	1·1048544	5·0		Δp_G −5187·70	M_N 697289·2	Δp_G −1734·80	M_N 700663·0	Δp_G 1734·80	M_N 697289·2	Δp_G 5187·70	

Note: $\frac{2700 H'}{\delta n}$ and $\frac{2700 g}{\delta n}$ are constant and equal to 62·1 exactly and 0·06608250 respectively. For H', g and ω see page 13.

multiple of δe, so that by combining the multiplier with $M_N \cos\phi$, the arithmetic is considerably shortened. Equation (3a) may then be written

$$\delta_t p_G = -\frac{g\delta t}{\delta e}\left\{\delta_E M_E + \delta_N\left(M_N \frac{\delta e}{\delta n}\right)\right\}, \quad\quad (7)$$

where the suffixes in δ_t, δ_E, δ_N indicate the variable which alone is varied. As an example of the computation of these pressure-changes we may take the following, which refers to the point at longitude $-3\delta\lambda$, latitude $6{\cdot}2 \times 10^8$ cm north.

Cm from equator	$10^8 \times 6{\cdot}4$	$10^8 \times 6{\cdot}2$	$10^8 \times 6{\cdot}0$
M_N	827578·6		792967·7 grm cm^{-1} sec^{-1}
$M_N \frac{\delta e}{\delta n}$	692873·1		728273·0 ,, ,,
$\delta_N\left(M_N \frac{\delta e}{\delta n}\right)$		$-35399{\cdot}9$ grm cm^{-1} sec^{-1}	
$\delta_E M_E$		$+\ 5019{\cdot}8$,, ,,	
Sum		$-30380{\cdot}1$,, ,,	

Sum $\times -g\,.\,\delta t/\delta e = +2285{\cdot}89 = \delta_t p$ expressed in dynes cm^{-2} per 2700 secs.

This time-change is written in parenthesis under the initial pressure. The pressure-changes at all the points are worked in the same way, except that the coefficients vary with latitude. The advantage of the chessboard pattern is now seen to be that the time-rates are given at the points at which the variables are initially tabulated. By adding the changes in $\frac{3}{4}$ hour we obtain a new table which has the same pattern as the initial one, so that it can in turn be taken as a starting-point. In this way the process can be continued with no limit; except that set by the loss of a strip, round the edge of the map, at every step.

If we had begun with p_G, M_E, M_N all three tabulated in every square, the distribution might have been regarded as two interpenetrating chessboard patterns. In the subsequent steps these interpenetrating systems would have been propagated quite independently of each other.

Let us now compare with observation the result so far obtained. In this deduction the distribution distorts and moves west. Actual cyclones move eastward. It is natural to expect a slip of a sign in the process, but that expectation may be very simply disposed of. For, substitute the geostrophic momenta from (1) and (2) into (3), then there results

$$\frac{\partial p_G}{\partial t} = \frac{gH'}{2\omega\sin\phi}\cdot\frac{\partial p_G}{\partial e}\cdot\frac{\cot\phi}{a}. \quad\quad (8)$$

The deduction of this equation has been abundantly verified. It means that where pressure increases towards the east, there the pressure is rising if the wind be momentarily geostrophic. This statement may be taken as a criticism of the geostrophic wind as an adequate idea when pressure-changes have to be deduced.

The same point was brought to notice by Sir Napier Shaw in his *Principia Atmospherica.* But he was able to escape from the dilemma by supposing that the congestion of air in a northward wind was somehow relieved in an upper level. Here we are dealing with the effect of all levels combined by integration, and the dilemma remains.

I am indebted to Dr E. H. Chapman for a value of the correlation between $\partial p_G/\partial t$ and $\partial p_G/\partial e$. Taking $\delta p_G/\delta t$ for 12 hours at Pembroke and representing $\delta p_G/\delta e$ by the excess of pressure at Bath over that at Roche's Point, an interval of space which has Pembroke nearly at its centre, he finds for observations at 7^h and 18^h on 1915 July 1st to 26th a correlation of $-0{\cdot}5_7$, the negative sign of which is in direct conflict with the equation above.

These facts bring one into sympathy with the view emphasized by Dr Harold Jeffreys (*Phil. Mag.* Jan. 1919) that, to understand cyclones, it is essential that the small terms in the equations be taken into account. But owing to treating latitude as constant in deducing his equation (9) from his (6), Dr Jeffreys concludes that a geostrophic wind implies no change of pressure. The success of Sir Napier Shaw's comparison between theory and the weather map for the case of a rigid portion of air, moving like a wheel with its hub rolling along a parallel of latitude (*Manual of Meteorology*, IV, Ch. 9), again turns on the fact that he has taken exact account of the small terms in the dynamical equations.

$(\delta t)^2 \frac{\partial^2}{\partial t^2}$ *(of initial distribution) with, written below,* $(\delta t)^3 \frac{\partial^3}{\partial t^3}$ *of the same*

	$-3\delta\lambda$	$-2\delta\lambda$	$-\delta\lambda$	0	$\delta\lambda$	$2\delta\lambda$	$3\delta\lambda$
6·4			M_E'' −785·5		M_E'' 785·5		
6·2		M_E'' −1575·9 M_N'' 13188·4	p_G'' −0·43	M_E'' 0 (4255·1) M_N'' 13251·9 (0·0)	p_G'' 0·43	M_E'' 1575·9 M_N'' 13188·4	
6·0	M_N'' 12826·0	p_G'' −0·64	M_E'' −786·9 (4081·4) M_N'' 12947·4 (256·3)	p_G'' 0·00 (−78·12)	M_E'' 786·9 (4081·4) M_N'' 12947·4 (−256·3)	p_G'' 0·64	M_N'' 12826·0
5·8		M_E'' −1574·8 M_N'' 12600·9 (+494·3)	p_G'' −0·34 (−81·53)	M_E'' 0 (3895·8) M_N'' +12662·7 (0·0)	p_G'' +0·34 (−81·53)	M_E'' 1574·8 M_N'' 12600·9 (−494·3)	
5·6	M_N'' 12213·2	p_G'' −0·57	M_E'' −789·1 (3706·7) M_N'' 12334·0 (244·4)	p_G'' 0·00 (−84·75)	M_E'' 789·1 (3706·7) M_N'' 12334·0 (−244·4)	p_G'' 0·57	M_N'' 12213·2
5·4		M_E'' −1579·2 M_N'' 11967·4	p_G'' −0·26	M_E'' 0 (3521·2) M_N'' 12027·0	p_G'' 0·26	M_E'' 1579·2 M_N'' 11967·4	
5·2			M_E'' −791·6		M_E'' 791·6		