Principles of Plasma Diagnostics

Plasma physics is currently one of the most active subdisciplines of physics. Measurements of the parameters of laboratory plasmas, termed plasma diagnostics, are based on a wide variety of characteristic plasma phenomena. Understanding these phenomena allows standard techniques to be applied and interpreted correctly and also forms the basis of innovation. This book provides a detailed derivation and discussion of the principles of plasma physics upon which diagnostics are based. These include magnetic measurements, electric probes, refractive index, radiation emission and scattering, and ionic processes.

The second edition of Principles of Plasma Diagnostics brings this classic text completely up to date. It gathers together a body of knowledge previously scattered throughout the scientific literature. The text maintains its first-principles development of the required concepts, so it is accessible to students and researchers with little plasma physics background. Nevertheless, even seasoned plasma physicists should appreciate the work as a valuable reference and find insight in the lucid development of the fundamentals as they apply to diagnostics.

Most of the examples of diagnostics in action are taken from fusion research but the focus on principles will make it useful to all experimental and theoretical plasma physicists, including those interested in space and astrophysical applications as well as laboratory plasmas.

Ian H. Hutchinson has over 25 years of experience in experimental plasma physics, in Australia, Great Britain, and America. Since 1987 he has led the Alcator Tokamak research project, the largest university-based fusion plasma physics group in the U.S. He has published numerous journal articles on a wide variety of plasma experiments, and is Honorary Editor of the journal Plasma Physics and Controlled Fusion. Dr. Hutchinson is a Fellow of the American Physical Society and the Institute of Physics, and as Professor in the Department of Nuclear Engineering at M.I.T., he teaches graduate level plasma physics.
Principles of Plasma Diagnostics

Second edition

I. H. HUTCHINSON

Massachusetts Institute of Technology
S. D. G.
Contents

Preface to the first edition xiii
Preface to the second edition xvi

1 Plasma diagnostics 1
1.1 Introduction 1
1.2 Plasma properties 2
  1.2.1 Moments of the distribution function 3
  1.2.2 Multiple species 5
1.3 Categories of diagnostics 6
Further reading 9

2 Magnetic diagnostics 11
2.1 Magnetic field measurements 11
  2.1.1 The magnetic coil 11
  2.1.2 Hall effect and Faraday effect measurements 13
  2.1.3 Rogowski coils 15
  2.1.4 Ohmic power and conductivity 17
2.2 Magnetohydrodynamic equilibrium 20
  2.2.1 Diamagnetism \( (m = 0 \text{ term}) \) 22
  2.2.2 Position and asymmetry measurements \( (m = 1) \) 24
  2.2.3 Strongly shaped plasmas 30
2.3 Internal magnetic probe measurements 37
  2.3.1 Field measurements 37
  2.3.2 Current density 40
  2.3.3 Electric field 41
  2.3.4 Pressure 43
  2.3.5 Two- and three-dimensional measurements 44
2.4 Fluctuations 45
  2.4.1 External measurements 46
  2.4.2 Internal fluctuation measurements 51
Further reading 52
Exercises 52

3 Plasma particle flux 55
3.1 Preliminaries 55
  3.1.1 Particle flux 56
  3.1.2 Debye shielding 57
  3.1.3 Collisional effects 58
Contents

3.2 Probes in collisionless plasmas without magnetic fields 60
  3.2.1 Sheath analysis 60
  3.2.2 Sheath thickness 65
  3.2.3 Exact solutions 67
  3.2.4 Orbit-limited collection 68
  3.2.5 Interpretation of the characteristic 70

3.3 The effects of a magnetic field 72
  3.3.1 General effects 72
  3.3.2 Quasicollisionless ion collection in a strong magnetic field 74
  3.3.3 Collisions in a magnetic field 78
  3.3.4 Mach probes for plasma velocity measurements 82
  3.3.5 Oblique collection and perpendicular velocity measurements 86

3.4 Applications 89
  3.4.1 Some practical considerations 89
  3.4.2 More sophisticated analyzers 94

Further reading 99
Exercises 100

4 Refractive-index measurements 104

4.1 Electromagnetic waves in plasma 104
  4.1.1 Waves in uniform media 104
  4.1.2 Plasma conductivity 107
  4.1.3 Nonuniform media: The WKBJ approximation and full-wave treatments 110

4.2 Measurement of electron density 112
  4.2.1 Interferometry 112
  4.2.2 Determining the phase shift 116
  4.2.3 Modulation and detection methods 120
  4.2.4 Coherence, diffraction and refraction 122
  4.2.5 Choice of frequency, vibration 126
  4.2.6 Interferometric imaging 127
  4.2.7 Schlieren and shadowgraph imaging 129
  4.2.8 Phase contrast interferometry 132
  4.2.9 Scattering from refractive-index perturbations 133

4.3 Magnetic field measurement 133
  4.3.1 Effect of a magnetic field 133
  4.3.2 Faraday rotation 134
  4.3.3 Propagation close to perpendicular 137
  4.3.4 Measurement of the polarization 140

4.4 Abel inversion 141

4.5 Reflectometry 144
Contents

4.5.1 Calculation of the phase delay 146
4.5.2 Implementation of reflectometry 148
4.5.3 Relative merits of reflectometry and interferometry 151
Further reading 151
Exercises 152

5 Electromagnetic emission by free electrons 155

5.1 Radiation from an accelerated charge 155
5.1.1 The radiation fields 155
5.1.2 Frequency spectrum in the far field 157

5.2 Cyclotron radiation 158
5.2.1 Radiation by a single electron 158
5.2.2 Plasma emissivity 162
5.2.3 Nonrelativistic plasma 163
5.2.4 Radiation transport, absorption, and emission 166
5.2.5 Wave polarization and finite density effects 169
5.2.6 Spatially varying magnetic field 174
5.2.7 Diagnostic applications in thermal plasmas 175
5.2.8 Nonthermal plasmas 179
5.2.9 Čerenkov emission 183

5.3 Radiation from electron–ion encounters 186
5.3.1 Classical bremsstrahlung 186
5.3.2 Quantum-mechanical bremsstrahlung 192
5.3.3 Integration over velocities 195
5.3.4 Recombination radiation contribution 197
5.3.5 Temperature measurement 201
5.3.6 Multiple species: $Z_{\text{eff}}$ measurement 203
5.3.7 Absorption: blackbody level bremsstrahlung 204
5.3.8 x-ray imaging 206
5.3.9 Nonthermal emission 210
Further reading 214
Exercises 214

6 Electromagnetic radiation from bound electrons 217

6.1 Radiative transitions: the Einstein coefficients 218
6.2 Types of equilibria 221
6.2.1 Thermal equilibrium 221
6.2.2 Saha–Boltzmann population distribution 222
6.2.3 Nonthermal populations 223
6.2.4 Coronal equilibrium 225
6.2.5 Time-dependent situations 227

6.3 Rate coefficients for collisional processes 229
6.3.1 Radiative recombination 230
6.3.2 The classical impact approximation 232
x

Contents

6.3.3 The dipole approximation 234
6.3.4 Ionization and excitation rates 237
6.3.5 Dielectronic recombination 241
6.3.6 Example: carbon ν 243
6.3.7 Charge-exchange recombination 245

6.4 Line broadening 245
6.4.1 Natural line broadening 246
6.4.2 Doppler broadening 247
6.4.3 Pressure broadening 247
6.4.4 Combinations of broadening effects 251
6.4.5 Reabsorption: optically thick lines 252

6.5 Applications 253
6.5.1 Line intensities 253
6.5.2 Doppler broadening 257
6.5.3 Ion flow velocity 259
6.5.4 Stark widths 261
6.5.5 Bolometry 262

6.6 Active diagnostics 263
6.6.1 Resonant fluorescence 263
6.6.2 Zeeman splitting: magnetic field measurements 266
Further reading 267
Exercises 268

7 Scattering of electromagnetic radiation 273
7.1 Relativistic electron motion in electromagnetic fields 274
7.2 Incoherent Thomson scattering 276
7.2.1 Nonrelativistic scattering: the dipole approximation 276
7.2.2 Conditions for incoherent scattering 277
7.2.3 Incoherent Thomson scattering (B = 0) 280
7.2.4 Experimental considerations 285
7.3 Coherent scattering 293
7.3.1 The scattered field and power 293
7.3.2 Scattering form factor for a uniform unmagnetized plasma 296
7.3.3 Problems of diagnostics using the ion feature 303
7.3.4 Scattering from macroscopic density fluctuations 304
7.4 Scattering when a magnetic field is present 308
7.4.1 Incoherent scattering from magnetized electrons 309
7.4.2 Presence of the harmonic structure 311
7.4.3 Magnetic field measurement 314
7.4.4 Coherent scattering in a magnetic field 315
Further reading 320
Exercises 320
Contents

8    Neutral atom diagnostics  322
8.1  Neutral particle analysis  322
     8.1.1 Collision processes  322
     8.1.2 Neutral transport  325
     8.1.3 The fast neutral spectrum  328
     8.1.4 Dense plasma cases  332
     8.1.5 Nonthermal plasmas  333
     8.1.6 Neutral density measurement  334
8.2  Active probing with neutral particles  336
     8.2.1 Neutral-beam attenuation  337
     8.2.2 Active charge exchange  340
     8.2.3 Doping species, lithium beams  341
8.3  Charge-exchange spectroscopy  344
     8.3.1 Charge-exchange cross sections  345
     8.3.2 Diagnostic applications of charge-exchange spectroscopy  351
8.4  Emission from beam atoms  354
     8.4.1 Beam emission spectroscopy  355
     8.4.2 Motional Stark effect  358
8.5  Other neutral particle diagnostics  363
     8.5.1 Rutherford scattering  363
     8.5.2 Pellet injection  364
Further reading  367
Exercises  367

9    Fast ions and fusion products  369
9.1  Neutron diagnostics  369
     9.1.1 Reactions and cross sections  369
     9.1.2 Complicating factors  372
     9.1.3 Neutron spectrum  374
     9.1.4 Collimated neutron measurements  379
9.2  Charged particle diagnostics  380
     9.2.1 Charged reaction products  380
     9.2.2 Orbits of energetic charged particles  383
     9.2.3 Lost charged particle orbit diagnosis  387
     9.2.4 Ion probing beams  388
Further reading  391
Exercises  392

Appendix 1  Fourier analysis  395
Appendix 2  Errors, fluctuations, and statistics  402
Appendix 3  Survey of radiation technology  407
Contents

Appendix 4 Definitions and identities of fundamental parameters 414

Appendix 5 Atomic rates for beam diagnostics 415

Glossary 420
References 426
Index 434
Preface to the first edition

The practice of plasma diagnostics is a vast and diverse subject, far beyond the span of a single volume, such as this, to cover in all its detail. Therefore, some limitations on the objectives adopted here have to be accepted. The title *Principles of Plasma Diagnostics* refers to the fact that the physical principles used for plasma measurements are to be our main concern. In brief, this book seeks to give a treatment of the fundamental physics of plasma diagnostics, and thus to provide a sound conceptual foundation upon which to base any more detailed study of applications. I hope, therefore, to bring the reader to the point where he or she may, with confidence and understanding, study the details of any diagnostic discussed in the literature.

Most journal articles and reviews on plasma diagnostics tend, of necessity, to begin from a mere citing of the required equations governing the principles employed. For all but the experienced specialist, this means that the reader must accept the equations without much justification or else pursue a deeper understanding through references to original papers. One of my main objectives here is to overcome this difficulty by a systematic presentation from first principles. Therefore, if in some cases it may seem that the development stops just as we approach the point of practicality, I can only plead that, in bringing the reader to the point of being able comfortably to understand the basis of any application, I have fulfilled a major part of my task.

Some justification of the fact that I provide very little detailed discussion of instruments and techniques may be appropriate, since they are by no means uninteresting or irrelevant.

First, to describe the various experimental technologies in a way accessible to the uninitiated, at anything other than a pure “cookbook” level, would require so much space as to be overwhelming for a single volume. Second, instruments and technology are not really unique to the plasma field, in most cases, even though the needs of plasma diagnostics are sometimes the driving force behind their development. Third, the technology is developing so rapidly that any extensive treatment tends to become outdated almost immediately. Fourth, there are several recent journal article reviews and specialist book series that emphasize instrumentation.

My philosophy, then, is to include only sufficient description of the technology to provide a fundamental understanding of the applications,
Preface to the first edition

rather than a detailed analysis of the instruments themselves. Only when the plasma is virtually part of the instrumental configuration, such as in an interferometer, is more detailed discussion given. As compensation, brief summaries of some of the present technological capabilities are given in an appendix.

By concentrating on the physical principles, my intention has been to produce a book of interest to plasma physicists as a whole, whatever the area of their major specialization. However, it is necessary in a work such as this to have a fairly clear perspective. Otherwise, one is forever qualifying statements in a way that ought to be implicit. My perspective is that of laboratory plasma diagnostics. What is more, most of the examples are taken from controlled fusion research applications, partly because fusion is the area in which by far the most study of plasma diagnostics has been done. I trust, nevertheless, that the material may be useful also to experimentalists and theoreticians in other plasma fields, such as space or astrophysical research, since it is a discussion mostly of general principles, applicable to these very different plasmas as well as those in the laboratory.

The level of the treatment may perhaps best be described as intermediate graduate. This means that a good basic undergraduate physics background should be sufficient to enable the reader to follow the material, even though the approach may be more demanding than in an undergraduate text. Very little detailed prior knowledge of plasma physics is assumed; therefore, researchers entering the plasma field should find most of the material accessible. There is, however, no pretense at a systematic introduction to plasma physics, and the presumption is that basic plasma physics concepts, at least, are familiar. The more senior researcher I hope will also find useful material here for reference and to gain a broader perspective, although length restrictions prevent discussion of many important practical details.

The literature and references cited are intended to serve two limited purposes: to provide representative examples of the principles in action and to provide starting points for more detailed study of the scientific literature in any specialized area. There is no attempt to provide exhaustive references and I apologize to anyone who feels their own work to have been unjustly omitted.

I also thank all my teachers, friends, students, and colleagues who have provided information, figures, criticism, suggestions, corrections, and so on. Like all of science, plasma research is a cooperative enterprise, and so the material here represents an overview of the work of a large number of people over many years. Special thanks also go to my colleagues at MIT who have taken time to read sections of the manuscript and make suggestions for improvements, especially Bruce Lipschultz, Earl
Preface to the first edition

Marmar, Steve McCool, Jim Terry, Reich Watterson, and Steve Wolfe. The shortcomings of the book are mine, though! Thanks to Cathy Lydon for managing so much difficult word processing.

Thank you, Fran, for making it all possible by your constant support and love.
Preface to the second edition

Plasma diagnostics has grown in accomplishment and importance in the sixteen years since the first edition of this book was written. The fusion research field has reached the threshold of energy breakeven, and of committing to a burning plasma experiment. But more important perhaps, the accuracy and comprehensiveness of measurements on major magnetic plasma confinement devices now give us unprecedented information on plasma behaviour. Plasmas have gained in importance in industrial processes and of course in electronic manufacturing; so the economic necessity of monitoring them accurately has become increasingly evident. Astrophysical and space plasma diagnosis has continued to be the basis of investigations of a host of phenomena from black hole accretion to planetary magnetospheres.

In preparing a second edition, my objective was to retain the original emphasis on the physical principles upon which plasma measurements are based, and to maintain an accessible teaching style. Both of these aspects have proven attractive to students and researchers. Also, the examples are still predominantly drawn from my own field of fusion research, but some discussion of the broader applications is included. It became increasingly pressing in recent years that the book should be updated to include the latest techniques and applications. It has thus been impossible to avoid some expansion of the length, because of the substantial additional material. A few obsolete sections have been removed, but I have endeavored to keep as much of the first edition as possible, bringing the topics up to date by discussions of the recent developments and modern references.

Although the expansion makes the book more useful as a comprehensive reference, it undoubtedly makes its use as a teaching text somewhat more difficult. Selection of material has to be more ruthless in skipping sections, particularly those that explore the details of specialized applications. I find that it is now inadvisable to attempt to teach more than about half the material in a term. I have nevertheless been encouraged by the experience of a number of teachers and students who have found the previous edition to be a valuable breadth reference for courses introducing plasma physics, as well as for more specialized contexts.

I have changed my convention for expressing spectral densities to use cyclic frequency, and changed my Fourier transform convention accordingly. This is explained at the end of Appendix 1. It means that
Preface to the second edition

there are a number of factor $2\pi$ changes in equations relative to the first edition.

I am grateful for the hospitality of the Australian National University Plasma Research Laboratory, where much of the writing of the second edition was done, to my students over the years who helped me see the places that were unclear, as well as finding many typographical errors, to my colleagues who provided figures, advice and critiques, and always to my wife, Fran, for her endless support.

Cambridge, Massachusetts, August 2001.