In *The Quantum Theory of Fields*, Nobel Laureate Steven Weinberg combines his exceptional physical insight with his gift for clear exposition to provide a self-contained, comprehensive, and up-to-date introduction to quantum field theory. Volume II gives a current and self-contained account of the methods of quantum field theory, and how they have led to an understanding of the weak, strong, and electromagnetic interactions of the elementary particles. The presentation of modern mathematical methods is throughout interwoven with accounts of the problems of elementary particle physics and condensed matter physics to which they have been applied. Many topics are included that are not usually found in books on quantum field theory. The book contains much original material, and is peppered with examples and insights from the author's experience as a leader of elementary particle physics. Problems are included at the end of each chapter.

**From reviews of Volume I**

‘...an impressively lucid and thorough presentation of the subject...Weinberg manages to present difficult topics with richness of meaning and marvellous clarity. Full of valuable insights, his treatise is sure to become a classic, doing for quantum field theory what Dirac's *Quantum Mechanics* did for quantum mechanics. I eagerly await the publication of the second volume.’

S. S. Schweber, *Nature*

‘For over twenty years there has been no good modern textbook on the subject. For all that time, Steven Weinberg has been promising to write one. That he has finally done it is cause for celebration among those who try to teach and try to learn the subject. Weinberg’s book is for serious students of field theory...it is the first textbook to treat quantum field theory the way it is used by physicists today.’

Howard Georgi, *Science*

‘Steven Weinberg, who contributed to the development of quantum chromodynamics and shared the Nobel Prize in Physics for his contributions to the electroweak theory, has written a definitive text on the physical foundations of quantum field theory. His book differs significantly from the long line of previous books on quantum field theory...To summarize, *Foundations* builds the structure of quantum field theory on the sure footing of physical insight. It is beautifully produced and meticulously edited...and it is a real bargain in price. If you want to learn quantum field theory, or have already learned it and want to have a definitive reference at hand, purchase this book.’

O. W. Greenberg, *Physics Today*

‘In addition to a superb treatment of all the conventional topics there are numerous sections covering areas that are not normally emphasized, such as the subject of field redefinitions, higher-rank tensor fields and an unusually clear and thorough treatment of infrared effects...this latest book reinforces his high scholarly standards. It provides a unique exposition that will prove invaluable both to new research students as well as to experienced research workers. Together
with Volume II, this will become a classic text on a subject of central importance to a wide area of theoretical physics.'

M. B. Green, *CERN Courier*

'I believe that what readers will find particularly helpful in this volume is the consistency of the whole approach, and the emphasis on quantities and properties that are directly useful to particle physicists. This is particularly true for those who are interested in the more phenomenological aspects. The reader only needs limited background knowledge, and a clear line is followed throughout the book, making it easy to follow. The author presents extremely thorough but elementary discussions of important physical questions, some of which seem to be an original way of addressing the subject.'

J. Zinn-Justin, *Physics World*

'This is a well-written book by one of the masters of the subject...it is certainly destined to become a standard text book and should find its way to the shelves of every physics library.'

J. Madore, *Classical and Quantum Gravity*

'The book starts out with an excellent historical introduction, not found anywhere else, giving citations to many by now classic papers...a valuable reference work as well as a textbook for graduate students.'

G. Roepstorff, *Zentralblatt für Mathematik*

**From reviews of Volume II**

'It is a majestic exposition. The two volumes are structured in a logical way. Everything is explained with incisive clarity. Weinberg always goes to the heart of any argument, and includes many things that cannot be found elsewhere in the literature. Often I find myself thinking: "Ah! Now I understand that properly." ...I find it hard to imagine a better treatment of quantum field theory than Weinberg’s. All serious students and researchers will want to have these volumes on their shelves.'

John C. Taylor, *Nature*

'Weinberg’s *Modern Applications* goes to the boundaries of our present understanding of field theory. It is unmatched by any other book on quantum field theory for its depth, generality and definitive character, and it will be an essential reference for serious students and researchers in elementary particle physics.’

O. W. Greenberg, *Physics Today*

‘...Steven Weinberg is one of our most gifted makers of theoretical tools as well as a virtuoso in their use. His new book conveys both the satisfaction of understanding nature and the feel of the atelier, for the “modern applications” of its subtitle include both the derivation of physical consequences and the development of new tools for understanding and applying field theory itself...*Modern Applications* is a splendid book, with abundant useful references to the original literature. It is a
very interesting read from cover to cover, for the wholeness Weinberg's personal perspective gives to quantum field theory and particle physics.'

Chris Quigg, Science

'Experienced researchers and beginning graduate students will delight in the gems of wisdom to be found in these pages. This book combines exposition of technical detail with physical insight in a unique manner that confirms the promise of Volume I and I have no doubt that these two volumes will rapidly constitute the classic treatment of this important subject.'

M. B. Green, CERN Courier

'...a valued reference and a mine of useful information for professional field theorists.'

Tom Kibble, New Scientist

'...a clear presentation of the subject, explaining the underlying concepts in much depth and in an accessible style. I expect that these volumes will become the first source we turn to when trying to answer the challenging questions asked by bright postgraduates when they first encounter quantum field theory...I have no doubt that The Quantum Theory of Fields will soon be found on the bookshelves of most particle theorists, and that it will be one of the main sources used in the preparation of lectures on the subject for postgraduate students.'

C. T. C. Sachrajda, The Times Higher Education Supplement

'...Weinberg has produced a masterpiece that will be a standard reference on the field for a long time to come.'

B. E. Y. Svensson, Elementa

From reviews of Volume III

'...has produced a treatise that many of us had long awaited, perhaps without fully realizing it...with the publication of The Quantum Theory of Fields, Vol. III, has performed an analogous service for supersymmetry...Although this volume is the third in a trilogy, it is quite different from its two predecessors, and it stands on its own...May a new generation of students imbibe its content and spirit.'

Physics Today

'The third volume of The Quantum Theory of Fields is a self-contained introduction to the world of supersymmetry and supergravity. It will be useful both for experienced researchers in the field and for students who want to take the first steps towards learning about supersymmetry. Unlike other books in this field, it covers the wide spectrum of possible applications of supersymmetry in physics.'

Hans Peter Nilles, Nature

'Weinberg is of course one of the creators of modern quantum field theory, as well as of its physical culmination, the standard model of all (nongravitational) interactions. It is...very timely that this latest part of his monograph, devoted to supersymmetry and supergravity, has just appeared. As a text, it has been
pretested by Weinberg for a freestanding one-year graduate course; as a clear organizing reference to this extremely vast field, it will help the experts as well... Weinberg's style of presentation is as clear and meticulous as in his previous works.'

Stanley Deser, *Journal of General Relativity and Gravitation*

'Veinberg tries to be as elementary and clear as possible and steers clear of more sophisticated mathematical tools. Together with the previous volumes, this volume will serve as an invaluable reference to researchers and a textbook for graduate students.'

G. Roepstorff, *Zentralblatt MATH*
The Quantum Theory of Fields

Volume II
Modern Applications

Steven Weinberg
University of Texas at Austin
Contents

Sections marked with an asterisk are somewhat out of the book’s main line of development and may be omitted in a first reading.

PREFACE TO VOLUME II xvii

NOTATION xx

15 NON-ABELIAN GAUGE THEORIES 1

15.1 Gauge Invariance 2

Gauge transformations □ Structure constants □ Jacobi identity □ Adjoint representation □ Yang-Mills theory □ Covariant derivatives □ Field strength tensor □ Finite gauge transformations □ Analogy with general relativity

15.2 Gauge Theory Lagrangians and Simple Lie Groups 7

Gauge field Lagrangian □ Metric □ Antisymmetric structure constants □ Simple, semisimple, and U(1) Lie algebras □ Structure of gauge algebra □ Compact algebras □ Coupling constants

15.3 Field Equations and Conservation Laws 12

Conserved currents □ Covariantly conserved currents □ Inhomogeneous field equations □ Homogeneous field equations □ Analogy with energy-momentum tensor □ Symmetry generators

15.4 Quantization 14

Primary and secondary first-class constraints □ Axial gauge □ Gribov ambiguity □ Canonical variables □ Hamiltonian □ Reintroduction of \( A_y \) □ Covariant action □ Gauge invariance of the measure

15.5 The De Witt–Faddeev–Popov Method 19

Generalization of axial gauge results □ Independence of gauge fixing functionals □ Generalized Feynman gauge □ Form of vertices

15.6 Ghosts 24

Determinant as path integral □ Ghost and antighost fields □ Feynman rules for ghosts □ Modified action □ Power counting and renormalizability
15.7 BRST Symmetry 27
Auxiliary field $h_a$ □ BRST transformation □ Nilpotence □ Invariance of new action □ BRST-cohomology □ Independence of gauge fixing □ Application to electrodynamics □ BRST-quantization □ Geometric interpretation

15.8 Generalizations of BRST Symmetry∗ 36
De Witt notation □ General Faddeev–Popov–De Witt theorem □ BRST transformations □ New action □ Slavnov operator □ Field-dependent structure constants □ Generalized Jacobi identity □ Invariance of new action □ Independence of gauge fixing □ Beyond quadratic ghost actions □ BRST quantization □ BRST cohomology □ Anti-BRST symmetry

15.9 The Batalin–Vilkovisky Formalism∗ 42
Open gauge algebras □ Antifields □ Master equation □ Minimal fields and trivial pairs □ BRST-transformations with antifields □ Antibrackets □ Anticanonical transformations □ Gauge fixing □ Quantum master equation

Appendix A A Theorem Regarding Lie Algebras 50
Appendix B The Cartan Catalog 54
Problems 58
References 59

16 EXTERNAL FIELD METHODS 63

16.1 The Quantum Effective Action 63
Currents □ Generating functional for all graphs □ Generating functional for connected graphs □ Legendre transformation □ Generating functional for one-particle-irreducible graphs □ Quantum-corrected field equations □ Summing tree graphs

16.2 Calculation of the Effective Potential 68
Effective potential for constant fields □ One loop calculation □ Divergences □ Renormalization □ Fermion loops

16.3 Energy Interpretation 72
Adiabatic perturbation □ Effective potential as minimum energy □ Convexity □ Instability between local minima □ Linear interpolation

16.4 Symmetries of the Effective Action 75
Symmetry and renormalization □ Slavnov–Taylor identities □ Linearly realized symmetries □ Fermionic fields and currents

Problems 78
References 78
Contents

17 RENORMALIZATION OF GAUGE THEORIES 80

17.1 The Zinn-Justin Equation 80
Slavnov-Taylor identities for BRST symmetry □ External fields $K_a(x)$ □ Antibrackets

17.2 Renormalization: Direct Analysis 82
Recursive argument □ BRST-symmetry condition on infinities □ Linearity in $K_a(x)$ □ New BRST symmetry □Cancellation of infinities □ Renormalization constants □ Nonlinear gauge conditions

17.3 Renormalization: General Gauge Theories* 91
Are ‘non-renormalizable’ gauge theories renormalizable? □ Structural constraints □ Anticanonical change of variables □ Recursive argument □ Cohomology theorems

17.4 Background Field Gauge 95
New gauge fixing functions □ True and formal gauge invariance □ Renormalization constants

17.5 A One-Loop Calculation in Background Field Gauge 100
One-loop effective action □ Determinants □ Algebraic calculation for constant background fields □ Renormalization of gauge fields and couplings □ Interpretation of infinities

Problems 109

References 110

18 RENORMALIZATION GROUP METHODS 111

18.1 Where do the Large Logarithms Come From? 112
Singularity at zero mass □ ‘Infrared safe’ amplitudes and rates □ Jets □ Zero mass singularities from renormalization □ Renormalized operators

18.2 The Sliding Scale 119
Gell-Mann–Low renormalization □ Renormalization group equation □ One-loop calculations □ Application to $\phi^4$ theory □ Field renormalization factors □ Application to quantum electrodynamics □ Effective fine structure constant □ Field-dependent renormalized couplings □ Vacuum instability

18.3 Varieties of Asymptotic Behavior 130
Singularity at finite energy □ Continued growth □ Fixed point at finite coupling □ Asymptotic freedom □ Lattice quantization □ Triviality □ Universal coefficients in the beta function
18.4 \textbf{Multiple Couplings and Mass Effects} \hspace{1cm} 139
Behavior near a fixed point \hspace{1cm} Invariant eigenvalues \hspace{1cm} Nonrenormalizable theories
\hspace{1cm} Finite dimensional critical surfaces \hspace{1cm} Mass renormalization at zero mass \hspace{1cm} Renormalization group equations for masses

18.5 \textbf{Critical Phenomena}\textsuperscript{*} \hspace{1cm} 145
Low wave numbers \hspace{1cm} Relevant, irrelevant, and marginal couplings \hspace{1cm} Phase transitions and critical surfaces \hspace{1cm} Critical temperature \hspace{1cm} Behavior of correlation length \hspace{1cm} Critical exponent \hspace{1cm} $4 - \epsilon$ dimensions \hspace{1cm} Wilson–Fisher fixed point \hspace{1cm} Comparison with experiment \hspace{1cm} Universality classes

18.6 \textbf{Minimal Subtraction} \hspace{1cm} 148
Definition of renormalized coupling \hspace{1cm} Calculation of beta function \hspace{1cm} Application to electrodynamics \hspace{1cm} Modified minimal subtraction \hspace{1cm} Non-renormalizable interactions

18.7 \textbf{Quantum Chromodynamics} \hspace{1cm} 152
Quark colors and flavors \hspace{1cm} Calculation of beta function \hspace{1cm} Asymptotic freedom \hspace{1cm} Quark and gluon trapping \hspace{1cm} Jets \hspace{1cm} $e^+e^-$ annihilation into hadrons \hspace{1cm} Accidental symmetries \hspace{1cm} Non-renormalizable corrections \hspace{1cm} Behavior of gauge coupling \hspace{1cm} Experimental results for $g$, and $\Lambda$

18.8 \textbf{Improved Perturbation Theory}\textsuperscript{*} \hspace{1cm} 157
Leading logarithms \hspace{1cm} Coefficients of logarithms

Problems \hspace{1cm} 158

References \hspace{1cm} 159

19 \textbf{SPONTANEOUSLY BROKEN GLOBAL SYMMETRIES} \hspace{1cm} 163

19.1 \textbf{Degenerate Vacua} \hspace{1cm} 163
Degenerate minima of effective potential \hspace{1cm} Broken symmetry or symmetric superpositions? \hspace{1cm} Large systems \hspace{1cm} Factorization at large distances \hspace{1cm} Diagonalization of vacuum expectation values \hspace{1cm} Cluster decomposition

19.2 \textbf{Goldstone Bosons} \hspace{1cm} 167
Broken global symmetries imply massless bosons \hspace{1cm} Proof using effective potential \hspace{1cm} Proof using current algebra \hspace{1cm} $F$ factors and vacuum expectation values \hspace{1cm} Interactions of soft Goldstone bosons

19.3 \textbf{Spontaneously Broken Approximate Symmetries} \hspace{1cm} 177
Pseudo-Goldstone bosons \hspace{1cm} Tadpoles \hspace{1cm} Vacuum alignment \hspace{1cm} Mass matrix \hspace{1cm} Positivity
Contents

19.4 Pions as Goldstone Bosons 182

SU(2) × SU(2) chiral symmetry of quantum chromodynamics □ Breakdown to isospin □ Vector and axial-vector weak currents □ Pion decay amplitude □ Axial form factors of nucleon □ Goldberger-Treiman relation □ Vacuum alignment □ Quark and pion masses □ Soft pion interactions □ Historical note

19.5 Effective Field Theories: Pions and Nucleons 192

Current algebra for two soft pions □ Current algebra justification for effective Lagrangian □ σ-model □ Transformation to derivative coupling □ Nonlinear realization of SU(2) × SU(2) □ Effective Lagrangian for soft pions □ Direct justification of effective Lagrangian □ General effective Lagrangian for pions □ Power counting □ Pion–pion scattering for massless pions □ Identification of F-factor □ Pion mass terms in effective Lagrangian □ Pion–pion scattering for real pions □ Pion–pion scattering lengths □ Pion–nucleon effective Lagrangian □ Covariant derivatives □ g4 ≠ 1 □ Power counting with nucleons □ Pion–nucleon scattering lengths □ σ-terms □ Isospin violation □ Adler–Weisberger sum rule

19.6 Effective Field Theories: General Broken Symmetries 211

Transformation to derivative coupling □ Goldstone bosons and right cosets □ Symmetric spaces □ Cartan decomposition □ Nonlinear transformation rules □ Uniqueness □ Covariant derivatives □ Symmetry breaking terms □ Application to quark mass terms □ Power counting □ Order parameters

19.7 Effective Field Theories: SU(3) × SU(3) 225

SU(3) multiplets and matrices □ Goldstone bosons of broken SU(3) × SU(3) □ Quark mass terms □ Pseudoscalar meson masses □ Electromagnetic corrections □ Quark mass ratios □ Higher terms in Lagrangian □ Nucleon mass shifts

19.8 Anomalous Terms in Effective Field Theories* 234

Wess–Zumino–Witten term □ Five-dimensional form □ Integer coupling □ Uniqueness and de Rham cohomology

19.9 Unbroken Symmetries 238

Persistent mass conjecture □ Vafa–Witten proof □ Small non-degenerate quark masses

19.10 The U(1) Problem 243

Chiral U(1) symmetry □ Implications for pseudoscalar masses

Problems 246

References 247
xii

Contents

20 OPERATOR PRODUCT EXPANSIONS 252

20.1 The Expansion: Description and Derivation 253
Statement of expansion □ Dominance of simple operators □ Path-integral derivation

20.2 Momentum Flow* 255
$\phi^2$ contribution for two large momenta □ Renormalized operators □ Integral equation for coefficient function □ $\phi^3$ contribution for many large momenta

20.3 Renormalization Group Equations for Coefficient Functions 263
Derivation and solution □ Behavior for fixed points □ Behavior for asymptotic freedom

20.4 Symmetry Properties of Coefficient Functions 265
Invariance under spontaneously broken symmetries

20.5 Spectral Function Sum Rules 266
Spectral functions defined □ First, second, and third sum rules □ Application to chiral $SU(N) \times SU(N)$ □ Comparison with experiment

20.6 Deep Inelastic Scattering 272
Form factors $W_1$ and $W_2$ □ Deep inelastic differential cross section □ Bjorken scaling □ Parton model □ Callan–Gross relation □ Sum rules □ Form factors $T_1$ and $T_2$ □ Relation between $T_1$ and $W_1$ □ Symmetric tensor operators □ Twist □ Operators of minimum twist □ Calculation of coefficient functions □ Sum rules for parton distribution functions □ Altarelli–Parisi differential equations □ Logarithmic corrections to Bjorken scaling

20.7 Renormalons* 283
Borel summation of perturbation theory □ Instanton and renormalon obstructions □ Instantons in massless $\phi^4$ theory □ Renormalons in quantum chromodynamics

Appendix Momentum Flow: The General Case 288
Problems 292
References 293

21 SPONTANEOUSLY BROKEN GAUGE SYMMETRIES 295

21.1 Unitarity Gauge 295
Elimination of Goldstone bosons □ Vector boson masses □ Unbroken symmetries and massless vector bosons □ Complex representations □ Vector field propagator □ Continuity for vanishing gauge couplings
Contents

21.2 Renormalizable $\xi$-Gauges 300
Gauge fixing function □ Gauge-fixed Lagrangian □ Propagators

21.3 The Electroweak Theory 305
Lepton-number preserving symmetries □ $SU(2) \times U(1)$ □ $W^\pm$, $Z^0$, and photons □ Mixing angle □ Lepton-vector boson couplings □ $W^\pm$ and $Z^0$ masses □ Muon decay □ Effective fine structure constant □ Discovery of neutral currents □ Quark currents □ Cabibbo angle □ $c$ quark □ Third generation □ Kobayashi–Maskawa matrix □ Discovery of $W^\pm$ and $Z^0$ □ Precise experimental tests □ Accidental symmetries □ Nonrenormalizable corrections □ Lepton nonconservation and neutrino masses □ Baryon nonconservation and proton decay

21.4 Dynamically Broken Local Symmetries* 318
Fictitious gauge fields □ Construction of Lagrangian □ Power counting □ General mass formula □ Example: $SU(2) \times SU(2)$ □ Custodial $SU(2) \times SU(2)$ □ Technicolor

21.5 Electroweak–Strong Unification 327
Simple gauge groups □ Relations among gauge couplings □ Renormalization group flow □ Mixing angle and unification mass □ Baryon and lepton nonconservation

21.6 Superconductivity* 332
$U(1)$ broken to $Z_2$ □ Goldstone mode □ Effective Lagrangian □ Conservation of charge □ Meissner effect □ Penetration depth □ Critical field □ Flux quantization □ Zero resistance □ ac Josephson effect □ Landau–Ginzburg theory □ Correlation length □ Vortex lines □ $U(1)$ restoration □ Stability □ Type I and II superconductors □ Critical fields for vortices □ Behavior near vortex center □ Effective theory for electrons near Fermi surface □ Power counting □ Introduction of pair field □ Effective action □ Gap equation □ Renormalization group equations □ Conditions for superconductivity

Appendix General Unitarity Gauge 352
Problems 353
References 354

22 ANOMALIES 359

22.1 The $\pi^0$ Decay Problem 359
Rate for $\pi^0 \rightarrow 2\gamma$ □ Naive estimate □ Suppression by chiral symmetry □ Comparison with experiment

22.2 Transformation of the Measure: The Abelian Anomaly 362
Chiral and non-chiral transformations □ Anomaly function □ Chern–Pontryagin density □ Nonconservation of current □ Conservation of gauge-non-invariant
Contents

22.3 Direct Calculation of Anomalies: The General Case 370

22.4 Anomaly-Free Gauge Theories 383

22.5 Massless Bound States 389

22.6 Consistency Conditions 396

22.7 Anomalies and Goldstone Bosons 408

Problems 416

References 417

23 EXTENDED FIELD CONFIGURATIONS 421

23.1 The Uses of Topology 422

23.2 Homotopy Groups 430

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Contents

23.3 Monopoles 436
SU(2)/U(1) model □ Winding number □ Electromagnetic field □ Magnetic monopole moment □ Kronecker index □ ’t Hooft–Polyakov monopole □ Another Bogomol’nyi inequality □ BPS monopole □ Dirac gauge □ Charge quantization □ G/(H' × U(1)) monopoles □ Cosmological problems □ Monopole–particle interactions □ G/H monopoles with G not simply connected □ Irrelevance of field content

23.4 The Cartan–Maurer Integral Invariant 445
Definition of the invariant □ Independence of coordinate system □ Topological invariance □ Additivity □ Integral invariant for S1 → U(1) □ Bott’s theorem □ Integral invariant for S3 → SU(2)

23.5 Instantons 450
Evaluation of Cartan–Maurer invariant □ Chern–Pontryagin density □ One more Bogomol’nyi inequality □ v = 1 solution □ General winding number □ Solution of U(1) problem □ Baryon and lepton non-conservation by electroweak instantons □ Minkowskian approach □ Barrier penetration □ Thermal fluctuations

23.6 The Theta Angle 455
Cluster decomposition □ Superposition of winding numbers □ P and CP non-conservation □ Complex fermion masses □ Suppression of P and CP non-conservation by small quark masses □ Neutron electric dipole moment □ Peccei–Quinn symmetry □ Axions □ Axion mass □ Axion interactions

23.7 Quantum Fluctuations around Extended Field Configurations 462
Fluctuations in general □ Collective parameters □ Determinental factor □ Coupling constant dependence □ Counting collective parameters

23.8 Vacuum Decay 464
False and true vacua □ Bounce solutions □ Four dimensional rotational invariance □ Sign of action □ Decay rate per volume □ Thin wall approximation

Appendix A Euclidean Path Integrals 468
Appendix B A List of Homotopy Groups 472
Problems 473
References 474

AUTHOR INDEX 478
SUBJECT INDEX 484
OUTLINE OF VOLUME I

1 HISTORICAL INTRODUCTION
2 RELATIVISTIC QUANTUM MECHANICS
3 SCATTERING THEORY
4 THE CLUSTER DECOMPOSITION PRINCIPLE
5 QUANTUM FIELDS AND ANTIPARTICLES
6 THE FEYNMAN RULES
7 THE CANONICAL FORMALISM
8 ELECTRODYNAMICS
9 PATH-INTEGRAL METHODS
10 NON-PERTURBATIVE METHODS
11 ONE-LOOP RADIATIVE CORRECTIONS IN QUANTUM ELECTRO-DYNAMICS
12 GENERAL RENORMALIZATION THEORY
13 INFRARED EFFECTS
14 BOUND STATES IN EXTERNAL FIELDS
Preface To Volume II

This volume describes the advances in the quantum theory of fields that have led to an understanding of the electroweak and strong interactions of the elementary particles. These interactions have all turned out to be governed by principles of gauge invariance, so we start here in Chapters 15–17 with gauge theories, generalizing the familiar gauge invariance of electrodynamics to non-Abelian Lie groups.

Some of the most dramatic aspects of gauge theories appear at high energy, and are best studied by the methods of the renormalization group. These methods are introduced in Chapter 18, and applied to quantum chromodynamics, the modern non-Abelian gauge theory of strong interactions, and also to critical phenomena in condensed matter physics. Chapter 19 deals with general spontaneously broken global symmetries, and their application to the broken approximate $SU(2) \times SU(2)$ and $SU(3) \times SU(3)$ symmetries of quantum chromodynamics. Both the renormalization group method and broken symmetries find some of their most interesting applications in the context of operator product expansions, discussed in Chapter 20.

The key to the understanding of the electroweak interactions is the spontaneous breaking of gauge symmetries, which are explored in Chapter 21 and applied to superconductivity as well as to the electroweak interactions. Quite apart from spontaneous symmetry breaking is the possibility of symmetry breaking by quantum-mechanical effects known as anomalies. Anomalies and various of their physical implications are presented in Chapter 22. This volume concludes with a discussion in Chapter 23 of extended field configurations, which can arise either as new ingredients in physical states, such as skyrmions, monopoles, or vortex lines, or as non-perturbative quantum corrections to path integrals, where anomalies play a crucial role.

It would not be possible to provide a coherent account of these developments if they were presented in a historical order. I have chosen instead to describe the material of this book in an order that seems to me to work best pedagogically — I introduce each topic at a point where
the motivation as well as the mathematics can be understood with the least possible reference to material in subsequent chapters, even where logic might suggest a somewhat different order. For instance, instead of having one long chapter to introduce non-Abelian gauge theories, this material is split between Chapters 15 and 17, because Chapter 15 provides a motivation for the external field formalism introduced in Chapter 16, and this formalism is necessary for the work of Chapter 17.

In the course of this presentation, the reader will be introduced to various formal devices, including BRST invariance, the quantum effective action, and homotopy theory. The Batalin-Vilkovisky formalism is presented as an optional side track. It is introduced in Chapter 15 as a compact way of formulating gauge theories, whether based on open or closed symmetry algebras, and then used in Chapter 17 to study the cancellation of infinities in ‘non-renormalizable’ gauge theories, including general relativity, and in Chapter 22 to show that certain gauge theories are anomaly-free to all orders of perturbation theory. The effective field theory approach is extensively used in this volume, especially in applications to theories with broken symmetry, including the theory of superconductivity. I have struggled throughout for the greatest possible clarity of presentation, taking time to show detailed calculations where I thought it might help the reader, and dropping topics that could not be clearly explained in the space available.

The guiding aim of both Volumes I and II of this book is to explain to the reader why quantum field theory takes the form it does, and why in this form it does such a good job of describing the real world. Volume I outlined the foundations of the quantum theory of fields, emphasizing the reasons why nature is described at accessible energies by effective quantum field theories, and in particular by gauge theories. (A list of chapters of Volume I is given at the end of the table of contents of this volume.) The present volume takes quantum field theory and gauge invariance as its starting points, and concentrates on their implications.

This volume should be accessible to readers who have some familiarity with the fundamentals of quantum field theory. It is not assumed that the reader is familiar with Volume I (though it wouldn’t hurt). Aspects of group theory and topology are explained where they are introduced.

Some of the formal methods described in this volume (such as BRST invariance and the renormalization group) have important applications in speculative theories that involve supersymmetry or superstrings. I am enthusiastic about the future prospects of these theories, but I have not included them in this book, because it seems to me that they require a whole book to themselves. (Perhaps supersymmetry and supergravity will be the subjects of a Volume III.) I have excluded some other interesting topics here, such as finite temperature field theory, lattice gauge calcula-
Preface

In recent years, quantum field theory has been a primary focus of theoretical physics. The large $N_c$ approximation, because they were not needed to provide either motivation or mathematical techniques for the rest of the book, and the book was long enough.

The great volume of the literature on quantum field theory and its applications makes it impossible for me to read or quote all relevant articles. I have tried to supply citations to the classic papers on each topic, as well as to papers that describe further developments of material covered here, and to references that present detailed calculations, data, or proofs referred to in the text. As before, the mere absence of a citation should not be interpreted as a claim that the material presented is original, but some of it is.

In my experience this volume provides enough material for a one-year course for graduate students on advanced topics in quantum field theory, or on elementary particle physics. Selected parts of Volumes I and II would be suitable as the basis of a compressed one-year course on both the foundations and the modern applications of quantum field theory. I have supplied problems for each chapter. Some of these problems aim simply at providing exercise in the use of techniques described in the chapter; others are intended to suggest extensions of the results of the chapter to a wider class of theories.

* * *

I must acknowledge my special intellectual debt to colleagues at the University of Texas, notably Luis Boya, Phil Candelas, Bryce and Cecile De Witt, Willy Fischler, Joaquim Gomis, and Vadim Kaplunovsky, and especially Jacques Distler. Also, Luis Alvarez-Gaumé, Sidney Coleman, John Dixon, Tony Duncan, Jürg Fröhlich, Arthur Jaffe, Marc Henneaux, Roman Jackiw, Joe Polchinski, Michael Tinkham, Cumrun Vafa, Don Weingarten, Edward Witten and Bruno Zumino gave valuable help with special topics. Jonathan Evans read through the manuscript of this volume, and made many valuable suggestions. For corrections to the first printing of this volume I am indebted to several students and colleagues, including Stephen Adler, Mark Byrd, Vincent Liu, Herbert Neuberger, Chun-yen Wang, and especially Michio Masujima. Thanks are due to Alyce Wilson, who prepared the illustrations and typed the \LaTeX{} input files until I learned how to do it, to Terry Riley for finding countless books and articles, and to Jan Duffy for many helps. I am grateful to Maureen Storey and Alison Woollatt of Cambridge University Press for working to ready this book for publication, and especially to my editor, Rufus Neal, for his continued friendly good advice.

STEVEN WEINBERG

Austin, Texas
December, 1995
Notation

Latin indices $i, j, k$, and so on generally run over the three spatial coordinate labels, usually taken as 1, 2, 3. Where specifically indicated, they run over values 1, 2, 3, 4, with $x^4 = it$.

Greek indices $\mu, \nu, \ldots$ etc. from the middle of the Greek alphabet generally run over the four spacetime coordinate labels 1, 2, 3, 0, with $x^0$ the time coordinate.

Greek indices $\alpha, \beta, \ldots$ etc. from the beginning of the Greek alphabet generally run over the generators of a symmetry algebra.

Repeated indices are generally summed, unless otherwise indicated.

The spacetime metric $\eta_{\mu\nu}$ is diagonal, with elements $\eta_{11} = \eta_{22} = \eta_{33} = 1$, $\eta_{00} = -1$.

The d'Alembertian is defined as $\Box \equiv \eta^{\mu\nu} \partial^2 / \partial x^\mu \partial x^\nu = \nabla^2 - \partial^2 / \partial t^2$, where $\nabla^2$ is the Laplacian $\partial^2 / \partial x^i \partial x^i$.

The ‘Levi–Civita tensor’ $\epsilon^{\mu\nu\rho\sigma}$ is defined as the totally antisymmetric quantity with $\epsilon^{0123} = +1$.

Spatial three-vectors are indicated by letters in boldface.

Three-vectors in isospin space are indicated by arrows.

A hat over any vector indicates the corresponding unit vector: Thus, $\hat{v} \equiv v / |v|$.

A dot over any quantity denotes the time-derivative of that quantity.

Dirac matrices $\gamma_\mu$ are defined so that $\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2\eta_{\mu\nu}$. Also, $\gamma_5 = i\gamma_0 \gamma_1 \gamma_2 \gamma_3$, and $\beta = i\gamma^0 = \gamma_4$.

The step function $\theta(s)$ has the value +1 for $s > 0$ and 0 for $s < 0$. 

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Notation

The complex conjugate, transpose, and Hermitian adjoint of a matrix or vector $A$ are denoted $A^*$, $A^\dagger$, and $A^\dagger = A^*\dagger$, respectively. The Hermitian adjoint of an operator $O$ is denoted $O^\dagger$, except where an asterisk is used to emphasize that a vector or matrix of operators is not transposed. +H.c. or +c.c. at the end of an expression indicates the addition of the Hermitian adjoint or complex conjugate of the foregoing terms. A bar on a Dirac spinor $u$ is defined by $\bar{u} = u^\dagger \beta$. The antifield of a field $\chi$ in the Batalin–Vilkovisky formalism is denoted $\chi^\dagger$ rather than $\chi^*$ to distinguish it from the ordinary complex conjugate or the antiparticle field.

Units are usually used with $\hbar$ and the speed of light taken to be unity. Throughout $-e$ is the rationalized charge of the electron, so that the fine structure constant is $\alpha = e^2/4\pi \simeq 1/137$.

Numbers in parenthesis at the end of quoted numerical data give the uncertainty in the last digits of the quoted figure. Where not otherwise indicated, experimental data are taken from ‘Review of Particle Properties,’ Phys. Rev. D50, 1173 (1994).