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0521662222 - A Unified Theory of Voting: Directional and Proximity Spatial Models

Samuel Merrill III and Bernard Grofman

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## A Unified Theory of Voting

### Directional and Proximity Spatial Models

This book addresses the questions: How do voters use their own issue positions and those of candidates to decide how to vote? How do candidates choose policy positions in response to the behavior of voters? Does a voter tend to choose the candidate who most nearly shares the views of the voter or rather a candidate who holds more extreme or intense views but in the same direction as the voter, perhaps because voters discount candidates' abilities to implement the policies they advocate? The authors develop a unified model that incorporates these and other voter motivations and, using conditional logit and other statistical methods, assess its empirical predictions – for both voter choice and candidate strategy – in the United States, Norway, and France. The analyses show that a combination of motivations involving proximity, direction, discounting, and party identification is compatible with the choices made by voters and with the mildly but not extremely divergent policies that are characteristic responses to these choices in both two-party and multiparty electorates. All of these motivations are necessary to understand the linkage between candidate issue positions and voter preferences.

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No work stands on its own. We draw heavily from the basic Downsian proximity voting framework and the huge literature extending Downs's work, particularly that of Enelow, Hinich, and Munger.<sup>1</sup> We were directly inspired by the directional voting ideas of Matthews (1979), which, in turn, can be linked to ideas in social choice in Schofield (1983, 1985); and by the work of Rabinowitz, Macdonald, and their co-authors (e.g., Rabinowitz and Macdonald, 1989) and the literature their work has inspired.<sup>2</sup>

<sup>1</sup> Recent important works focusing on proximity spatial models of electoral competition include Enelow and Hinich's *The Spatial Theory of Voting* (1984); Ordeshook's *Game Theory and Political Theory* (1986); Mueller's *Public Choice II* (1989); and Hinich and Munger's *Ideology and the Theory of Political Choice* (1994). Enelow and Hinich's second book (*Advances in the Spatial Theory of Voting*, 1990) treats multi-candidate electoral competition and the effect of policy preferences on the part of candidates, among other topics. While the principal focus of Coughlin's *Probabilistic Voting Theory* (1992) is the proximity model, he also derives equilibrium results for candidate strategy under assumptions based on directionality and for a fixed-sum constraint on the available resources to be redistributed to voters. Pierce's *Choosing the Chief: Presidential Elections in France and the United States* (1995a), van der Eijk and Franklin's *Choosing Europe?* (1996), and Hinich and Munger's recent book, *Analytical Politics* (1997) include comparison of the Rabinowitz–Macdonald directional model and the proximity model as one among many topics. However, no previous book has systematically compared directional spatial models with the traditional proximity models or investigated the implications of a unified approach for candidate strategy and equilibrium as we do here.

<sup>2</sup> Our bibliography lists 35 papers since 1989 dealing with the Rabinowitz–Macdonald directional model, of which at least 29 have already appeared in print at the time of this writing. There is also a direct link between the 1970s work of the mathematical psychologist Douglas Carroll and the Rabinowitz–Macdonald directional model. The Matthews directional model, on the other hand, has received much less attention in the recent literature on candidate and political party competition and to our knowledge has not been the subject of any empirical analysis prior to the publication of four of our own papers on directional models in the 1990s.

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