1
Introduction

1.1 Capsule history of quantum mechanics

Starting in the seventeenth century, and continuing to the present day, physicists developed a body of ideas that describe much about the world around us: the motion of a cannonball, the orbit of a planet, the working of an engine, the crack of a baseball bat. This body of ideas is called *classical mechanics.*

In 1905, Albert Einstein realized that these ideas didn't apply to objects moving at high speeds (that is, at speeds near the speed of light) and he developed an alternative body of ideas called *relativistic mechanics.* Classical mechanics is wrong in principle, but it is a good approximation to relativistic mechanics when applied to objects moving at low speeds.

At about the same time, several experiments led physicists to realize that the classical ideas also didn't apply to very small objects, such as atoms. Over the period 1900–1927 a number of physicists (Planck, Bohr, Einstein, Heisenberg, de Broglie, Schrödinger, and others) developed an alternative *quantum mechanics.* Classical mechanics is wrong in principle, but it is a good approximation to quantum mechanics when applied to large objects.

1.2 What is the nature of quantum mechanics?

I'm not going to spend any time on the history of quantum mechanics, which is convoluted and fascinating. Instead, I will focus on the ideas developed at the end. What sort of ideas required twenty-eight years of development from this stellar group of scientists?

Einstein's theory of relativity is often (and correctly) described as strange and counterintuitive. Yet, according to a widely used graduate level text,

> [the theory of relativity] is a modification of the structure of mechanics which must not be confused with the far more violent recasting required by quantum theory.

Murray Gell-Mann, probably the most prominent living practitioner of the field, said of quantum mechanics that

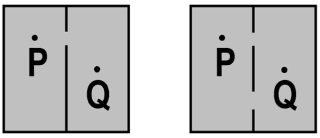
> Nobody feels perfectly comfortable with it.

And the inimitable Richard Feynman, who developed many of the ideas that will be expounded in this book, remarked that

> I can safely say that nobody understands quantum mechanics.

One strange aspect of quantum mechanics concerns predictability. Classical mechanics is *deterministic* — that is, if you know exactly the situation as it is now, then you can predict exactly what it will be at any moment in the future. Chance plays no role in classical mechanics. Of course, it might happen that the prediction is very difficult to perform, or it might happen that it is very difficult to find exactly the current situation, so such a prediction might not be a practical possibility. (This is the case when you flip a coin.) But in principle any such barriers can be surmounted by sufficient work and care. Relativistic mechanics is also deterministic. In contrast, quantum mechanics is *probabilistic* — that is, even in the presence of exact knowledge of the current situation, it is impossible to predict its future exactly, regardless of how much work and care one invests in such a prediction.

Even stranger, however, is quantum mechanical *interference*. I cannot describe this phenomenon in a single paragraph — that is a major job of this entire book — but I can give an example. Suppose a box is divided in half by a barrier with a hole drilled through it, and suppose an atom moves from point P in one half of the box to point Q in the other half. Now suppose a second hole is drilled through the barrier and then the experiment is repeated. The second hole increases the number of possible ways to move from P to Q, so it is natural to guess that its presence will increase the probability of making this move. But in fact — and in accord with the predictions of quantum mechanics — a second hole drilled at certain locations will *decrease* that probability.



The fact that quantum mechanics is strange does not mean that quantum mechanics is unsuccessful. On the contrary, quantum mechanics is the most successful theory that humanity has ever developed; the brightest jewel in our intellectual crown. Quantum mechanics underlies our understanding of atoms, molecules, solids, and nuclei. It is vital for explaining aspects of stellar evolution, chemical reactions, and the interaction of light with matter. It underlies the operation of lasers, transistors, magnets, and superconductors. I could cite reams of evidence backing up these assertions, but I will content myself by describing a single measurement. One electron will be stripped away from a helium atom that is exposed to ultraviolet light below a certain wavelength. This threshold wavelength can be determined experimentally to very high accuracy: it is $50.425\,929\,9 \pm 0.000\,000\,4$ nanometers. The threshold wavelength can also be calculated from quantum mechanics: this prediction is $50.425\,931\,0 \pm 0.000\,002\,0$ nanometers. The agreement between observation and quantum mechanics is extraordinary. If you were to predict the distance from New York to Los Angeles with this accuracy, your prediction would be correct to within the width of your hand. In contrast, classical mechanics predicts that *any* wavelength of light will strip away an electron, that is, that there will be no threshold at all.

1.3 How small is small?

I said above that the results predicted by quantum mechanics differed from the results predicted by classical mechanics only when these ideas were applied to "very small objects, such as atoms". How small is an atom? Cells are small: a typical adult contains about 60 trillion cells. But atoms are far smaller: a typical cell contains about 120 trillion atoms. An atom is twice as small, relative to a cell, as a cell is small, relative to a person. In this book, when I say "small" I mean "*very* small". You've never handled objects this small; I've never handled objects this small; none of your friends has ever handled objects this small. They are completely outside the domain of our common experience. As you read this book, you will find that quantum mechanics is contrary to common sense. There is nothing wrong with this. Common sense applies to commonly encountered situations, and we do not commonly encounter the atomic world.

1.4 The role of mathematics in quantum mechanics

One frequently hears statements to the effect that the ideas of quantum mechanics are highly mathematical and can only be understood through the use of complex mathematics (partial differential equations, Fourier transforms, eigenfunction expansions, etc.).

> One can popularize the quantum theory [only] at the price of gross oversimplification and distortion, ending up with an uneasy compromise between what the facts dictate and what it is possible to convey in ordinary language.

It is certainly true that the professional physicist needs a vast mathematical apparatus in order to solve efficiently the problems of quantum mechanics. (For example, the calculation of the helium stripping threshold wavelength described above was a mathematical *tour de force*.) But this is not, I believe,* because quantum mechanics itself is fundamentally difficult or mathematical. I believe instead that the root rules of quantum mechanics are in fact quite simple. (They are unfamiliar and unexpected, but nevertheless simple.) When these rules are applied to particular situations, they are used over and over again and therefore the *applications* are complicated. An analogy helps explain this distinction. The rules of chess are very simple: they can be written on a single page of paper. But when these rules are applied to particular situations they are used over and over again and result in a complicated game: the applications of the chess rules fill a library.

Indeed, can any fundamental theory be highly mathematical? Electrons know how to obey quantum mechanics, and electrons can neither add nor subtract, much less solve partial differential equations! If something as simple-minded as an electron can understand quantum mechanics, then certainly something as wonderfully complex as the reader of this book can understand it too.

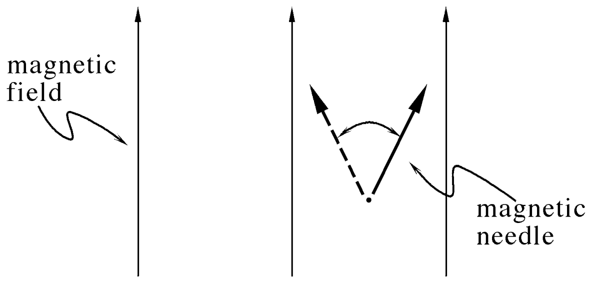
* Not everyone agrees with me.

2
Classical Magnetic Needles

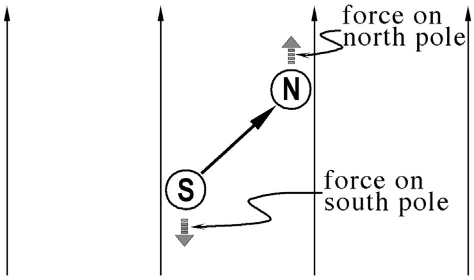
How shall we approach the principles of quantum mechanics? One way is simply to write them down. In fact I have already done that (in the first paragraph of the Preface), but to do so I had to use words and concepts that you don't yet understand. To develop the necessary understanding I will use a particular physical system as a vehicle to propel our exploration of quantum mechanics. Which system? An obvious choice is the motion of a tossed ball. Unfortunately this system, while simple and familiar in classical mechanics, is a complicated one in quantum mechanics. We will eventually get to the quantum mechanics of a tossed ball (in chapter 14, "Quantum mechanics of a bouncing ball", page 103), but as the vehicle for developing quantum mechanics I will instead use a system that is simple in quantum mechanics but that is, unfortunately, less familiar in daily life. That system is the magnetic needle in a magnetic field. This chapter describes the classical motion of a magnetic needle so that we will be able to see how its classical and quantal behaviors differ.

2.1 Magnetic needle in a magnetic field

A magnetic needle — like the one found in any woodsman's compass — has a "north pole" and a "south pole". I will symbolize the magnetic needle by an arrow pointing from its south pole to its north pole. When a magnetic needle is placed in a magnetic field — such as the magnetic field of the earth, or that produced by a horseshoe magnet — then the magnetic field acts to push the north pole in the direction of the field, and to push the south pole in the direction opposite the field. (It is not important for you to understand in detail how this effect works or even what the phrases "north pole" and "magnetic field" mean. Remember that this chapter merely builds a classical scaffolding that will be discarded once the correct quantal structure is built.) These two pushes together twist the needle towards an orientation in which the associated arrow points in the same direction as the field. If the needle starts out pointing parallel to the magnetic field, then it keeps on pointing in that direction. If the needle starts out not pointing parallel to the magnetic field, then it oscillates back and forth about this preferred direction. (If friction is present, then these oscillations will eventually die out and the needle will point precisely parallel to the field. If there is no friction then the oscillations will continue forever. In atomic systems there is no friction.)



If the magnetic field has the same strength at all points in the vicinity of the needle, that is, if the field is uniform, then the upward force acting on the north pole of the needle is exactly cancelled by the downward force acting on the south pole and there is no net force on the needle. So in a uniform field there is an impetus for the needle to oscillate, but no impetus for it to move up or down, or left or right.

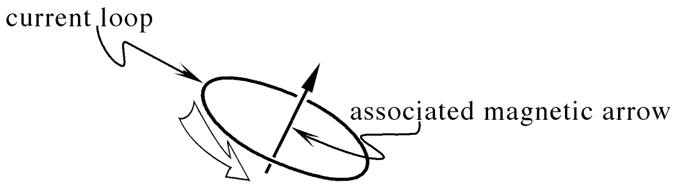


2.2 Magnetic effects on electric current

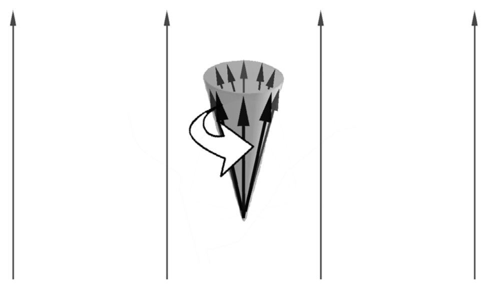
A loop of wire carrying electric current behaves in many ways like a compass needle. The associated magnetic arrow* points perpendicular to the current loop, so if the current loop is placed in a magnetic field, the

* This associated arrow is purely abstract — there's nothing actually located there.

arrow "wants" to point parallel to the field. But the current loop's arrow



isn't *exactly* like a compass needle's arrow, because the current loop arrow *precesses* rather than *oscillates* in a magnetic field. "Precession" means that the tip of the symbolic arrow moves around a circle while its base is fixed. Thus a precessing arrow traces out the figure of a cone. You



can make your index finger precess by holding it up in the air and then twisting its tip around in a circle while keeping your hand fixed.

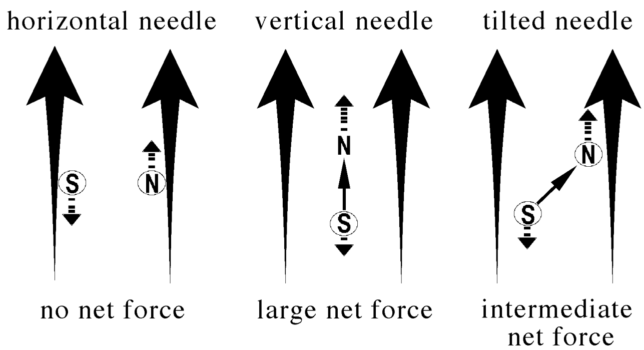
I wish I could describe for you an experiment that you could do to prove this fact to yourself. Unfortunately, this cannot be done with the equipment available in the typical home. It is, however, quite easy to do a parallel home experiment with an analogous system. A top rotating in a gravitational field happens to behave very much like a current loop in a magnetic field. (The rotating body of the top is analogous to the moving electric charge — the current — in the loop. The axle of the top is analogous to the magnetic arrow.) I urge you to spin a top, put it on the floor, tip the rotation axis away from the vertical, and then watch the top precess.

2.3 Magnetic needle in a non-uniform magnetic field

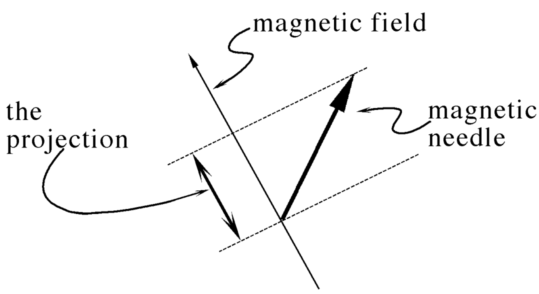
We have seen that a magnetic needle in a uniform magnetic field feels zero net magnetic force, because the upward force on the north pole is cancelled by the downward force on the south pole. But if a magnetic needle is placed in a *non-uniform* magnetic field, then there *can* be a net force on the needle.

Cambridge University Press
978-0-521-66104-1 - The Strange World of Quantum Mechanics
Daniel F. Styer
Excerpt
More information

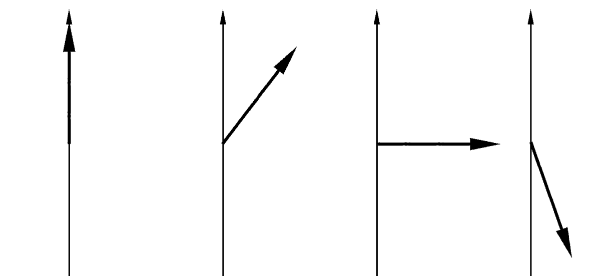
The figure below shows a magnetic field which is stronger at the top of the figure than at the bottom of the figure. For the horizontal needle, both the north and south poles are at the same height and experience the same magnetic field strength, so the two poles experience equal but opposite forces and the net force vanishes. But for the vertical needle, the north pole experiences a stronger magnetic field than does the south pole, so there is a larger upward force on the north pole and a smaller downward force on the south pole. As a result the two forces don't completely cancel — there remains a net upward force. The tilted needle is intermediate between these two situations. It experiences a net upward force, but that force is not as strong as the force on the vertical needle.



You can see that the net force depends upon the angle between the arrow and the field. In fact, the force is proportional to a quantity bearing the awkward name of "the projection of the magnetic arrow onto the direction of the magnetic field". This quantity is defined through a four-stage process: (1) Draw a line to show the direction of the magnetic field (in the illustration below, it tilts to the left). (2) Draw in the magnetic arrow with its base on the field line. (3) Draw a line perpendicular to the field line through the base of the arrow, and another through the tip of the arrow (these are shown dashed). (4) The distance between these two lines is the desired "projection".



Examples of projections:



If an electric current loop is placed in a non-uniform magnetic field, its arrow will precess and at the same time the loop will move. During this precession the projection remains constant,† and hence the force remains constant. For example, suppose the field is stronger at the top than at the bottom (as in the figure on page 8) and suppose a stationary current loop with a small positive projection is placed into the field. Then the current loop will move upward, and as it moves it precesses in such a way that the impetus to move upward stays constant. If the initial projection is negative, then the current loop moves downward.

2.4 Explanation *vs.* description

Have I *explained* the motion of magnetic needles in magnetic fields? Have I *explained* the nature of a magnetic field? Not at all! I have simply *described* these phenomena. Sometimes a description in science can be explained through an appeal to more fundamental principles. For example, I have spoken about the north and south magnetic poles of a compass needle. The poles of a compass needle can in fact be explained in terms of the motion of electrons within the needle's atoms. But in other cases the description is simply the most fundamental thing there is and cannot be "explained" by something else. What is a magnetic field? I have described it, in essence, as "that which makes a compass needle want to oscillate". There are more elaborate and more mathematical descriptions of magnetic field, but none are more fundamental. Science has no explanation for magnetic field, only a description of it.

† Spin your top again and notice that as the top precesses, the tip of the axle remains always the same distance from the floor. The vertical distance from the floor to the tip of the axle is the projection of the axle onto a vertical line. (If you wait long enough that friction slows the rotation of the top, then this projection — the height of the axle tip — will decrease. But if friction can be ignored, then the projection does not change.)

What does "explanation" mean, anyway? Suppose you ask me "Why did it rain yesterday?" I might reply "Because a cold front moved in." Then you could ask "But why did a cold front move in?" I might say "Because the jet stream pushed it." You: "But why did the jet stream push it?" Me: "Because the sun warmed Saskatchewan and so deflected the jet stream."‡ You: "But why does sunlight warm objects?" And at this level I really can't answer your question. I know *that* sunlight carries energy (so do you), and science can describe this energy transport with exquisite accuracy. But science cannot *explain* this energy transport or tell *why* it happens.

This story illustrates that "explanation" means "explanation in terms of something more fundamental". At some point any chain of questioning descends to the most fundamental ideas, and there it must stop. Currently, the most fundamental ideas in physics are called "quantum electrodynamics" and "quantum chromodynamics", two theories which fall squarely within the framework of quantum mechanics that I will describe in this book. Probably there will someday be even more fundamental ideas, so that "why" questions concerning quantum electrodynamics could be answered in terms of these new ideas. However, "why" questions concerning these more fundamental ideas will then be unanswerable! Ultimately, at the bottom of any descending chain of questions, science can only give descriptions (facts) and not explanations (reasons for those facts).

2.5 Problems

> *Above all things we must beware of what I will call "inert ideas" — that is to say, ideas that are merely received into the mind without being utilized, or tested, or thrown into fresh combinations.*
>
> — Alfred North Whitehead

Reading books, listening to lectures, watching movies, running computer simulations, performing experiments, participating in discussions … all these are fine tools for learning quantum mechanics. But you will not *really* become familiar with the subject until you get it under your skin by working problems. The problems in this book do not simply test your comprehension of the material you read in the text. They are instead an important component of the learning process, designed to extend and solidify the concepts presented. Solving problems is a more active, and

‡ Anyone who has raised a child is all too familiar with such chains of questions.