THE REASON FOR ANTIPARTICLES

Richard P. Feynman

The title of this lecture is somewhat incomplete because I really want to talk about two subjects: first, why there are antiparticles, and, second, the connection between spin and statistics. When I was a young man, Dirac was my hero. He made a breakthrough, a new method of doing physics. He had the courage to simply guess at the form of an equation, the equation we now call the Dirac equation, and to try to interpret it afterwards. Maxwell in his day got his equations, but only in an enormous mass of 'gear wheels' and so forth.

I feel very honored to be here. I had to accept the invitation, after all he was my hero all the time, and it is kind of wonderful to find myself giving a lecture in his honor.

Dirac with his relativistic equation for the electron was the first to, as he put it, wed quantum mechanics and relativity together. At first he

Paul Dirac Richard Feynman

thought that the spin, or the intrinsic angular momentum that the equation demanded, was the key, and that spin was the fundamental consequence of relativistic quantum mechanics. However, the puzzle of negative energies that the equation presented, when it was solved, eventually showed that the crucial idea necessary to wed quantum mechanics and relativity together was the existence of antiparticles. Once you have that idea, you can do it for any spin, as Pauli and Weisskopf proved, and therefore I want to start the other way about, and try to explain why there must be antiparticles if you try to put quantum mechanics with relativity.

Working along these lines will permit us to explain another of the grand mysteries of the world, namely the Pauli exclusion principle. The Pauli exclusion principle says that if you take the wavefunction for a pair of spin $\frac{1}{2}$ particles and then interchange the two particles, then to get the new wavefunction from the old you must put in a minus sign. It is easy to demonstrate that if Nature was nonrelativistic, if things started out that way then it would be that way for all time, and so the problem would be pushed back to Creation itself, and God only knows how that was done. With the existence of antiparticles, though, pair production of a particle with its antiparticle becomes possible, for example with electrons and positrons. The mystery now is, if we pair produce an electron and a positron, why does the new electron that has just been made have to be antisymmetric with respect to the electrons which were already around? That is, why can't it get into the same state as one of the others that were already there? Hence, the existence of particles and antiparticles permits us to ask a very simple question: if I make two pairs of electrons and positrons and I compare the amplitudes for when they annihilate directly or for when they exchange before they annihilate, why is there a minus sign?

All these things have been solved long ago, in a beautiful way which is simplest in the spirit of Dirac with lots of symbols and operators. I am going to go further back to Maxwell's 'gear wheels' and try to tell you as best I can a way of looking at these things so that they appear not so mysterious. I am adding nothing to what is already known; what follows is simply exposition. So here we go as to how things work–first, why there must be antiparticles.

Relativity and antiparticles

In ordinary nonrelativistic quantum mechanics, if you have a disturbing potential U acting on a particle which is initially in a state ϕ_0, then the state will be different after the disturbance. Up to a phase factor and taking $\hbar = 1$, the amplitude to end up in a state χ is given by the projection of χ onto $U\phi_0$. In fact, we have:

$$\mathrm{Amp}_{\phi_0 \to \chi} = -\mathrm{i}\int \mathrm{d}^3x\, \chi^* U\phi_0 = -\mathrm{i}\langle \chi | U | \phi_0 \rangle. \quad (1)$$

The expression $\langle \chi | U | \phi_0 \rangle$ is Dirac's elegant bra and ket notation for amplitudes, although I will not use it much here. I will suppose though that

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this formula is true when we go to relativistic quantum mechanics.

Now suppose that there are two disturbances, one at a time t_1 and another at a later time t_2, and we would like to know what the amplitude is for the second disturbance to restore the particle to its original state ϕ_0. Call the first disturbance U_1 at time t_1, and the second U_2 at time t_2. We will need to express the successive operations of: the disturbance U_1, evolution from time t_1 to t_2, and the disturbance U_2–this we will do using perturbation theory. Of course, the simplest thing that could happen is that we go straight from ϕ_0 to ϕ_0 direct, with amplitude $\langle\phi_0|\phi_0\rangle = 1$. This is the leading order term of the perturbation expansion. It is the next to leading order term that corresponds to the disturbance U_1 putting the state ϕ_0 into some intermediate state ψ_m of energy E_m, which lasts for time $(t_2 - t_1)$, before the other disturbance U_2 converts back to ϕ_0. All possible intermediate states must be summed over. The total amplitude for the state ϕ_0 to end up in the same state ϕ_0 is then:

$$\mathrm{Amp}_{\phi_0\to\phi_0} = 1 - \sum_m \langle\phi_0|U_2(\mathbf{x}_2)|\psi_m\rangle \times \exp(-\mathrm{i}E_m(t_2 - t_1))\langle\psi_m|U_1(\mathbf{x}_1)|\phi_0\rangle. \quad (2)$$

(I have assumed, for simplicity, that there is no first order amplitude to go from ϕ_0 to ϕ_0; that is, that $\langle\phi_0|U_1|\phi_0\rangle = 0$ and $\langle\phi_0|U_2|\phi_0\rangle = 0$.) If we use plane waves for the intermediate states ψ_m and expand out the amplitudes $\langle\phi_0|U_2|\psi_m\rangle$ and $\langle\psi_m|U_1|\phi_0\rangle$, we see that

$$\text{Amp}_{\phi_0\to\phi_0} = 1 - \int d^3\mathbf{x}_1\, d^3\mathbf{x}_2 \int \frac{d^3\mathbf{p}}{(2\pi)^3 2E_p} b^*(\mathbf{x}_2) \times \exp\{-i[E_p(t_2 - t_1) - \mathbf{p}\cdot(\mathbf{x}_2 - \mathbf{x}_1)]\}\, a(\mathbf{x}_1). \quad (3)$$

Here

$$a(\mathbf{x}_1) = U_1(\mathbf{x}_1)\phi_0(\mathbf{x}_1)\surd(2E_p),$$

$$b(\mathbf{x}_2) = U_2(\mathbf{x}_2)\phi_0(\mathbf{x}_2)\surd(2E_p),$$

and $E_p = \surd(p^2 + m^2)$ for a particle of mass m. These E_p factors are arranged just to make the relativistic properties more apparent, as $d^3\mathbf{p}/(2\pi)^3 2E_p$ is an invariant momentum density. The process can be written pictorially as in Fig. 1.

We are going to study some special cases of the above formula. The way I am going to do it is first

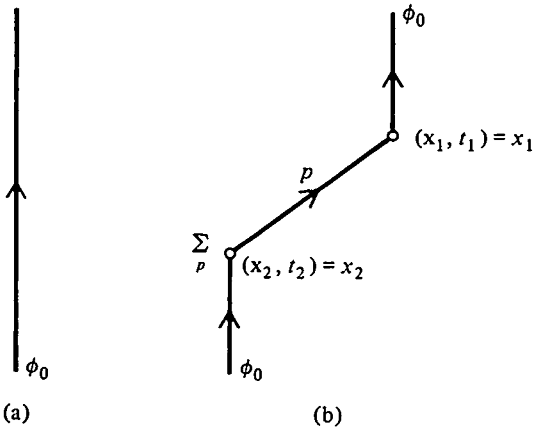

Fig. 1 Diagrammatic representation of two contributions to the amplitude for the transition $\phi_0 \rightarrow \phi_0$. (a) Direct; (b) indirect.

to examine some very simple examples and then proceed a little more generally. Hopefully you will understand the simple examples, because if you do you will understand the generalities at once–that's the way *I* understand things anyway.

In the indirect amplitude the particle is scattered from $\mathbf{x}_1$ to $\mathbf{x}_2$ and the intermediate states are particles with momentum $\mathbf{p}$ and energy E_p. We are going to suppose something: that all the energies are positive. If the energies were negative we know that we could solve all our energy problems by dumping particles into this pit of negative energy and running the world with the extra energy.

Now here is a surprise: if we evaluate the amplitude for any $a(\mathbf{x}_1)$ and $b(\mathbf{x}_2)$ (we could even arrange for $a(\mathbf{x}_1)$ and $b(\mathbf{x}_2)$ to depend on $\mathbf{p}$) we find that it cannot be zero when $\mathbf{x}_2$ is outside the light cone of $\mathbf{x}_1$. This is very surprising: if you start a series of waves from a particular point they cannot be confined to be inside the light cone if all the energies are positive. This is the result of the following mathematical theorem:

If a function $f(t)$ can be Fourier decomposed into positive frequencies only, i.e. if it can be written

$$f(t) = \int_0^\infty e^{-i\omega t} F(\omega)\, d\omega, \qquad (4)$$

then f cannot be zero for any finite range of t, unless trivially it is zero everywhere. The validity of this theorem depends on $F(\omega)$ satisfying certain properties, the details of which I would prefer to avoid.

You may be a bit surprised at this theorem because you know you can take a function which is zero over a finite range and Fourier analyze it, but

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then you get positive *and* negative frequencies. I am insisting that the frequencies be positive only.

To apply this theorem to the case at hand, we fix $\mathbf{x}_1$ and $\mathbf{x}_2$ and rewrite the integral over $\mathbf{p}$ in terms of the variable $\omega = E_p$. The integral is then of the form (4) with $F(\omega)$ zero for $\omega < m$; $F(\omega)$ will depend on $\mathbf{x}_1$ and $\mathbf{x}_2$. The theorem applies directly; we see that the amplitude cannot be zero for any finite interval of time. In particular, it cannot be zero outside the light cone of $\mathbf{x}_1$. In other words, there is an amplitude for particles to travel faster than the speed of light and no arrangement of superposition (with only positive energies) can get around that.

Therefore, if t_2 is later than t_1 we get contributions to the amplitude from particles traveling faster than the speed of light, for which $\mathbf{x}_1$ and $\mathbf{x}_2$ are separated by a spacelike interval ('spacelike-separated').

Now with a spacelike separation the order of occurrence of U_1 and U_2 is frame-dependent: if we look at the event from a frame moving sufficiently quickly relative to the original frame, t_2 is earlier than t_1 (Fig. 2).

What does this process look like from the new frame? Before time t'_2, we have one particle happily traveling along, but at time t_2' something seemingly very mysterious happens: at point $\mathbf{x}_2$, a finite distance from the original particle, the disturbance creates a pair of particles, one of which is apparently moving backwards in time. At time t_1', the original particle and that moving backwards in time disappear. So the requirements of positive energies and relativity force us to allow creation and annihilation of pairs of particles, one of which travels backwards in time. The physical interpretation of a particle traveling backwards in time can most easily be appreciated if we temporarily give our particle a charge. In Fig. 2b, the particle travels from $\mathbf{x}_1$ to $\mathbf{x}_2$, bringing, say, positive charge from $\mathbf{x}_1$ to $\mathbf{x}_2$, yet since $\mathbf{x}_2$ occurs first it is seen as negative charge flowing from $\mathbf{x}_2$ to $\mathbf{x}_1$.

In other words, *there must be antiparticles*. In fact, because of this frame-dependence of the sequence of events we can say that one man's virtual particle is another man's virtual antiparticle.

To summarize the situation, we can make the following statements:

(1) Antiparticles and pair production and destruction must exist.

(2) Antiparticle behavior is completely determined by particle behavior.