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0521652278 - Mathematical Methods for Physicists: A Concise Introduction - Tai L. Chow

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## **Mathematical Methods for Physicists**

A concise introduction

This text is designed for an intermediate-level, two-semester undergraduate course in mathematical physics. It provides an accessible account of most of the current, important mathematical tools required in physics these days. It is assumed that the reader has an adequate preparation in general physics and calculus.

The book bridges the gap between an introductory physics course and more advanced courses in classical mechanics, electricity and magnetism, quantum mechanics, and thermal and statistical physics. The text contains a large number of worked examples to illustrate the mathematical techniques developed and to show their relevance to physics.

The book is designed primarily for undergraduate physics majors, but could also be used by students in other subjects, such as engineering, astronomy and mathematics.

TAI L. CHOW was born and raised in China. He received a BS degree in physics from the National Taiwan University, a Masters degree in physics from Case Western Reserve University, and a PhD in physics from the University of Rochester. Since 1969, Dr Chow has been in the Department of Physics at California State University, Stanislaus, and served as department chairman for 17 years, until 1992. He served as Visiting Professor of Physics at University of California (at Davis and Berkeley) during his sabbatical years. He also worked as Summer Faculty Research Fellow at Stanford University and at NASA. Dr Chow has published more than 35 articles in physics journals and is the author of two textbooks and a solutions manual.

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# Mathematical Methods for Physicists

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*A concise introduction*

TAI L. CHOW

*California State University*



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## *Preface*

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This book evolved from a set of lecture notes for a course on ‘Introduction to Mathematical Physics’, that I have given at California State University, Stanislaus (CSUS) for many years. Physics majors at CSUS take introductory mathematical physics before the physics core courses, so that they may acquire the expected level of mathematical competency for the core course. It is assumed that the student has an adequate preparation in general physics and a good understanding of the mathematical manipulations of calculus. For the student who is in need of a review of calculus, however, Appendix 1 and Appendix 2 are included.

This book is not encyclopedic in character, nor does it give in a highly mathematical rigorous account. Our emphasis in the text is to provide an accessible working knowledge of some of the current important mathematical tools required in physics.

The student will find that a generous amount of detail has been given mathematical manipulations, and that ‘it-may-be-shown-thats’ have been kept to a minimum. However, to ensure that the student does not lose sight of the development underway, some of the more lengthy and tedious algebraic manipulations have been omitted when possible.

Each chapter contains a number of physics examples to illustrate the mathematical techniques just developed and to show their relevance to physics. They supplement or amplify the material in the text, and are arranged in the order in which the material is covered in the chapter. No effort has been made to trace the origins of the homework problems and examples in the book. A solution manual for instructors is available from the publishers upon adoption.

Many individuals have been very helpful in the preparation of this text. I wish to thank my colleagues in the physics department at CSUS.

Any suggestions for improvement of this text will be greatly appreciated.

*Turlock, California*  
2000

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