

An Introduction to

Nuclear Physics

Second edition

W. N. COTTINGHAM
University of Bristol

D. A. GREENWOOD
University of Bristol

 **CAMBRIDGE**
UNIVERSITY PRESS

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York, NY 10011-4211, USA
10 Stamford Road, Oakleigh, VIC 3166, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa

<http://www.cambridge.org>

© Cambridge University Press 1986, 2001

This book is in copyright. Subject to statutory exception
and to the provisions of relevant collective licensing agreements,
no reproduction of any part may take place without
the written permission of Cambridge University Press.

First published 1986
Reprinted 1987 (with corrections and additions), 1988, 1990, 1992, 1998
Second edition 2001

Printed in the United Kingdom at the University Press, Cambridge

Typeface Times 10/13pt 3B2 [KB]

A catalogue record for this book is available from the British Library

Library of Congress Cataloguing in Publication data

Cottingham, W. N.
An introduction to nuclear physics / W.N. Cottingham and
D.A. Greenwood – 2nd ed. p. cm.
Includes bibliographical references and index.
ISBN 0 521 65149 2 (hardback) – ISBN 0 521 65733 4 (paperback)
I. Nuclear physics. I. Greenwood, D. A. II. Title.
QC776.C63 2001
539.7–dc21 00-059885

ISBN 0 521 65149 2 hardback
ISBN 0 521 65733 4 paperback

Contents

<i>Preface to the second edition</i>	ix
<i>Preface to the first edition</i>	x
<i>Constants of nature, conversion factors and notation</i>	xii
<i>Glossary of some important symbols</i>	xiii
1 Prologue	1
1.1 Fermions and bosons	2
1.2 The particle physicist's picture of nature	2
1.3 Conservation laws and symmetries: parity	3
1.4 Units	4
Problems	5
2 Leptons and the electromagnetic and weak interactions	7
2.1 The electromagnetic interaction	7
2.2 The weak interaction	9
2.3 Mean life and half life	12
2.4 Leptons	13
2.5 The instability of the heavy leptons: muon decay	15
2.6 Parity violation in muon decay	16
Problems	17
3 Nucleons and the strong interaction	19
3.1 Properties of the proton and the neutron	19
3.2 The quark model of nucleons	21
3.3 The nucleon–nucleon interaction: the phenomenological description	22
3.4 Mesons and the nucleon–nucleon interaction	26
3.5 The weak interaction: β -decay	28
3.6 More quarks	29
3.7 The Standard Model of particle physics	31
Problems	31

4	Nuclear sizes and nuclear masses	33
4.1	Electron scattering by the nuclear charge distribution	33
4.2	Muon interactions	36
4.3	The distribution of nuclear matter in nuclei	37
4.4	The masses and binding energies of nuclei in their ground states	39
4.5	The semi-empirical mass formula	41
4.6	The β -stability valley	44
4.7	The masses of the β -stable nuclei	48
4.8	The energetics of α -decay and fission	50
4.9	Nuclear binding and the nucleon–nucleon potential	52
	Problems	52
5	Ground-state properties of nuclei: the shell model	56
5.1	Nuclear potential wells	56
5.2	Estimates of nucleon energies	58
5.3	Energy shells and angular momentum	60
5.4	Magic numbers	65
5.5	The magnetic dipole moment of the nucleus	66
5.6	Calculation of the magnetic dipole moment	67
5.7	The electric quadrupole moment of the nucleus	68
	Problems	72
6	Alpha decay and spontaneous fission	74
6.1	Energy release in α -decay	74
6.2	The theory of α -decay	75
6.3	Spontaneous fission	83
	Problems	87
7	Excited states of nuclei	89
7.1	The experimental determination of excited states	89
7.2	Some general features of excited states	93
7.3	The decay of excited states: γ -decay and internal conversion	97
7.4	Partial decay rates and partial widths	99
7.5	Excited states arising from β -decay	100
	Problems	101
8	Nuclear reactions	103
8.1	The Breit–Wigner formula	103
8.2	Neutron reactions at low energies	107
8.3	Coulomb effects in nuclear reactions	109
8.4	Doppler broadening of resonance peaks	111
	Problems	113
9	Power from nuclear fission	115
9.1	Induced fission	115

9.2	Neutron cross-sections for ^{235}U and ^{238}U	116
9.3	The fission process	118
9.4	The chain reaction	119
9.5	Nuclear fission reactors	121
9.6	Reactor control and delayed neutrons	122
9.7	Production and use of plutonium	124
9.8	Radioactive waste	125
9.9	The future of nuclear power	126
	Problems	127
10	Nuclear fusion	130
10.1	The Sun	130
10.2	Cross-sections for hydrogen burning	132
10.3	Nuclear reaction rates in a plasma	135
10.4	Other solar reactions	139
10.5	Solar neutrinos	140
10.6	Fusion reactors	143
10.7	Muon-catalysed fusion	146
	Problems	148
11	Nucleosynthesis in stars	151
11.1	Stellar evolution	151
11.2	From helium to silicon	155
11.3	Silicon burning	156
11.4	Supernovae	157
11.5	Nucleosynthesis of heavy elements	160
	Problems	161
12	Beta decay and gamma decay	163
12.1	What must a theory of β -decay explain?	163
12.2	The Fermi theory of β -decay	166
12.3	Electron and positron energy spectra	168
12.4	Electron capture	171
12.5	The Fermi and Gamow–Teller interactions	173
12.6	The constants V_{ud} and g_{A}	177
12.7	Electron polarisation	178
12.8	Theory of γ -decay	179
12.9	Internal conversion	184
	Problems	185
13	Neutrinos	186
13.1	Neutrino cross-sections	186
13.2	The mass of the electron neutrino	188
13.3	Neutrino mixing and neutrino oscillations	189

13.4	Solar neutrinos	193
13.5	Atmospheric neutrinos	195
	Problems	196
14	The passage of energetic particles through matter	199
14.1	Charged particles	199
14.2	Multiple scattering of charged particles	206
14.3	Energetic photons	207
14.4	The relative penetrating power of energetic particles	211
	Problems	212
15	Radiation and life	214
15.1	Ionising radiation and biological damage	214
15.2	Becquerels (and curies)	215
15.3	Grays and sieverts (and rads and rems)	216
15.4	Natural levels of radiation	217
15.5	Man-made sources of radiation	218
15.6	Risk assessment	219
	Problems	220
	Appendix A: Cross-sections	222
A.1	Neutron and photon cross-sections	222
A.2	Differential cross-sections	224
A.3	Reaction rates	225
A.4	Charged particle cross-sections: Rutherford scattering	226
	Appendix B: Density of states	227
	Problems	230
	Appendix C: Angular momentum	230
C.1	Orbital angular momentum	230
C.2	Intrinsic angular momentum	232
C.3	Addition of angular momenta	233
C.4	The deuteron	234
	Problems	235
	Appendix D: Unstable states and resonances	235
D.1	Time development of a quantum system	236
D.2	The formation of excited states in scattering: resonances and the Breit–Wigner formula	241
	Problems	244
	<i>Further reading</i>	245
	<i>Answers to problems</i>	246
	<i>Index</i>	267

Prologue

More than 100 elements are now known to exist, distinguished from each other by the electric charge Ze on the atomic nucleus. This charge is balanced by the charge carried by the Z electrons which together with the nucleus make up the neutral atom. The elements are also distinguished by their mass, more than 99% of which resides in the nucleus. Are there other distinguishing properties of nuclei? Have the nuclei been in existence since the beginning of time? Are there elements in the Universe which do not exist on Earth? What physical principles underlie the properties of nuclei? Why are their masses so closely correlated with their electric charges? Why are some nuclei radioactive? Radioactivity is used to man's benefit in medicine. Nuclear fission is exploited in power generation. But man's use of nuclear physics has also posed the terrible threat of nuclear weapons.

This book aims to set out the basic concepts which have been developed by nuclear physicists in their attempts to understand the nucleus. Besides satisfying our appetite for knowledge, these concepts must be understood if we are to make an informed judgment on the benefits and problems of nuclear technology.

After the discovery of the neutron by Chadwick in 1932, it was accepted that a nucleus of atomic number Z was made up of Z protons and some number N of neutrons. The proton and neutron were then thought to be elementary particles, although it is now clear that they are not but rather are themselves structured entities. We shall also see that in addition to neutrons and protons several other particles play an important, if indirect, role in the physics of nuclei. In this and the following two chapters, to provide a background to our subsequent study of the

nucleus, we shall describe the elementary particles of nature, and their interactions, as they are at present understood.

1.1 Fermions and bosons

Elementary particles are classified as either *fermions* or *bosons*. Fermions are particles which satisfy the Pauli exclusion principle: if an assembly of identical fermions is described in terms of single-particle wave-functions, then no two fermions can have the same wave-function. For example, electrons are fermions. This rule explains the shell structure of atoms and hence underlies the whole of chemistry. Fermions are so called because they obey the Fermi–Dirac statistics of statistical mechanics.

Bosons are particles which obey Bose–Einstein statistics, and are characterised by the property that *any* number of particles may be assigned the same single-particle wave-function. Thus, in the case of bosons, coherent waves of macroscopic amplitude can be constructed, and such waves may to a good approximation be described classically. For example, photons are bosons and the corresponding classical field is the familiar electromagnetic field $\mathbf{E}$ and $\mathbf{B}$, which satisfies Maxwell’s equations.

At a more fundamental level, these properties are a consequence of the possible symmetries of the wave-function of a system of identical particles when the coordinates of any two particles are interchanged. In the case of fermions, the wave-function changes sign; it is completely anti-symmetric. In the case of bosons the wave-function is unchanged; it is completely symmetric.

There is also an observed relation between the intrinsic angular momentum, or spin, of a particle and its statistics. The intrinsic spin $\mathbf{s}$ is quantised, with spin quantum number s (see Appendix C). For a fermion, s takes one of the values $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots$; for a boson, s takes one of the values $0, 1, 2, \dots$. A theoretical explanation of this relationship can be given within the framework of relativistic quantum field theory.

1.2 The particle physicist’s picture of nature

Elementary particle physics describes the world in terms of elementary fermions, interacting through fields of which they are sources. The particles associated with the interaction fields are bosons. To take the most familiar example, an electron is an elementary fermion; it carries electric charge $-e$ and this charge produces an electromagnetic field $\mathbf{E}$, $\mathbf{B}$, which

Table 1.1. *Types of interaction field*

Interaction field	Boson	Spin
Gravitational field	‘Gravitons’ postulated	2
Weak field	W^+ , W^- , Z particles	1
Electromagnetic field	Photons	1
Strong field	‘Gluons’ postulated	1

exerts forces on other charged particles. The electromagnetic field, quantised according to the rules of quantum mechanics, corresponds to an assembly of *photons*, which are bosons. Indeed, Bose–Einstein statistics were first applied to photons.

Four types of interaction field may be distinguished in nature (see Table 1.1). All of these interactions are relevant to nuclear physics, though the gravitational field becomes important only in densely aggregated matter, such as stars. Gravitational forces act on all particles and are important for the physics on the large scale of macroscopic bodies. On the small scale of most terrestrial atomic and nuclear physics, gravitational forces are insignificant and except in Chapter 10 and Chapter 11 we shall ignore them.

Nature provides an even greater diversity of elementary fermions than of bosons. It is convenient to divide the elementary fermions into two classes: *leptons*, which are not sources of the strong fields and hence do not participate in the strong interaction; and *quarks*, which take part in all interactions.

The electron is an example of a lepton. Leptons and their interactions are described in Chapter 2. Quarks are always confined in compound systems which extend over distances of about 1 fm. The term *hadron* is used generically for a quark system. The proton and neutron are hadrons, as are mesons. The proton and neutron are the subject matter of Chapter 3.

1.3 Conservation laws and symmetries: parity

The total energy of an isolated system is constant in time. So also are its linear momentum and angular momentum. These conservation laws are derivable from Newton’s laws of motion and Maxwell’s equations, or from the laws of quantum mechanics, but they can also, at a deeper

level, be regarded as consequences of ‘symmetries’ of space and time. Thus the law of conservation of linear momentum follows from the homogeneity of space, the law of conservation of angular momentum from the isotropy of space; it does not matter where we place the origin of our coordinate axes, or in which direction they are oriented.

These conservation laws are as significant in nuclear physics as elsewhere, but there is another symmetry and conservation law which is of particular importance in quantum systems such as the nucleus: reflection symmetry and parity. By reflection symmetry we mean reflection about the origin, $\mathbf{r} \rightarrow \mathbf{r}' = -\mathbf{r}$. A single-particle wave-function $\psi(\mathbf{r})$ is said to have parity $+1$ if it is even under reflection, i.e.

$$\psi(-\mathbf{r}) = \psi(\mathbf{r}),$$

and parity -1 if it is odd under reflection, i.e.

$$\psi(-\mathbf{r}) = -\psi(\mathbf{r}).$$

More generally, a many-particle wave-function has parity $+1$ if it is even under reflection of all the particle coordinates, and parity -1 if it is odd under reflection.

Parity is an important concept because the laws of the electromagnetic and of the strong interaction are of exactly the same form if written with respect to a reflected left-handed coordinate system $(0x', 0y', 0z')$ as they are in the standard right-handed system $(0x, 0y, 0z)$ (Fig. 1.1). We shall see in Chapter 2 that this is not true of the weak interaction. Nevertheless, for many properties of atomic and nuclear systems the weak interaction is unimportant and wave-functions for such systems can be chosen to have a definite parity which does not change as the wave-function evolves in time according to Schrödinger’s equation.

1.4 Units

Every branch of physics tends to find certain units particularly congenial. In nuclear physics, the size of the nucleus makes $10^{-15} \text{ m} = 1 \text{ fm}$ (femto-metre) convenient as a unit of length, usually called a *fermi*. However, nuclear cross-sections, which have the dimensions of area, are measured in *barns*; $1 \text{ b} = 10^{-28} \text{ m}^2 = 100 \text{ fm}^2$. Energies of interest are usually of the order of MeV. Since mc^2 has the dimensions of energy, it is convenient to quote masses in units of MeV/c^2 .

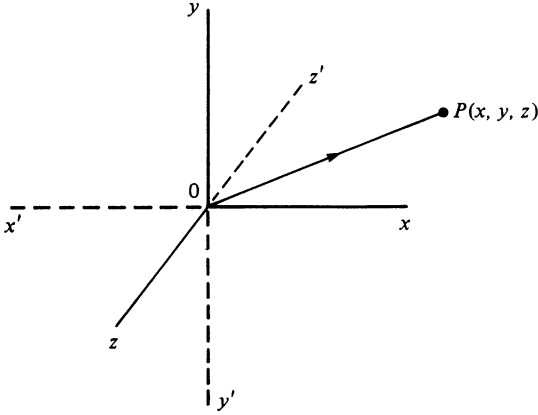


Fig. 1.1 The point P at $\mathbf{r}$ with coordinates (x, y, z) has coordinates $(-x, -y, -z)$ in the primed, reflected coordinate axes. $(0x', 0y', 0z')$ make up a *left-handed* set of axes. In the figure, the $0z$ axis is out of the plane of the page.

For order-of-magnitude calculations, the masses m_e and m_p of the electron and proton may be taken as

$$m_e \approx 0.5 \text{ MeV}/c^2$$

$$m_p \approx 938 \text{ MeV}/c^2$$

and it is useful to remember that

$$\hbar c \approx 197 \text{ MeV fm}, \quad e^2/4\pi\epsilon_0 \approx 1.44 \text{ MeV fm},$$

$$e^2/4\pi\epsilon_0 \hbar c \approx 1/137, \quad c \approx 3 \times 10^{23} \text{ fm s}^{-1}.$$

The student will perhaps be surprised to find how easily many expressions in nuclear physics can be evaluated using these quantities.

Problems

- 1.1 Show that the ratio of the gravitational potential energy to the Coulomb potential energy between two electrons is $\approx 2.4 \times 10^{-43}$.
- 1.2(a) Show that in polar coordinates (r, θ, ϕ) the reflection $\mathbf{r} \rightarrow \mathbf{r}' = -\mathbf{r}$ is equivalent to $r \rightarrow r' = r$
 $\theta \rightarrow \theta' = \pi - \theta, \phi \rightarrow \phi' = \phi + \pi$.

- (b) What are the parities of the following electron states of the hydrogen atom:

$$(i) \quad \psi_{100} = \frac{1}{\sqrt{\pi}} \left(\frac{1}{a_0} \right)^{\frac{3}{2}} e^{-r/a_0},$$

$$(ii) \quad \psi_{210} = \frac{1}{4\sqrt{(2\pi)}} \left(\frac{1}{a_0} \right)^{\frac{3}{2}} \frac{r}{a_0} e^{-r/2a_0} \cos \theta,$$

$$(iii) \quad \psi_{21-1} = \frac{1}{8\sqrt{\pi}} \left(\frac{1}{a_0} \right)^{\frac{3}{2}} \frac{r}{a_0} e^{-r/2a_0} \sin \theta e^{-i\phi}?$$

($a_0 = (4\pi\epsilon_0)\hbar^2/m_e e^2$ is the Bohr radius.)

- 1.3(a) Show that the wavelength of a photon of energy 1 MeV is ≈ 1240 fm.
- (b) The electrostatic self-energy of a uniformly charged sphere of total charge e , radius R , is $U = (3/5)e^2/(4\pi\epsilon_0 R)$. Show that if $R = 1$ fm, $U \approx 0.86$ MeV.