

HIDDEN UNITY IN NATURE'S LAWS

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CONTENTS

Preface xi

1	Motion on Earth and in the Heavens	1
1.1	Galileo's Telescope	1
1.2	The Old Astronomy	2
1.3	Aristotle and Ptolemy: Models and Mathematics	5
1.4	Copernicus: Getting Behind Appearances	9
1.5	Galileo	11
1.6	Kepler: Beyond Circles	14
1.7	Newton	19
1.8	Conclusion	30
2	Energy, Heat and Chance	32
2.1	Introduction	32
2.2	Temperature and Thermometers	33
2.3	Energy and Its Conservation	34
2.4	Heat as Energy	42
2.5	Atoms and Molecules	43
2.6	Steam Engines and Entropy	50
2.7	Entropy and Randomness	58
2.8	Chaos	62
2.9	Conclusion	69
3	Electricity and Magnetism	70
3.1	Electric Charges	70
3.2	Magnets	77
3.3	Electric Currents and Magnetism	80
3.4	Faraday and Induction of Electricity by Magnetism	88

3.5	Maxwell's Synthesis: Electromagnetism	91
3.6	Conclusion	97
4	Light	99
4.1	Waves	99
4.2	Sound	102
4.3	Light	104
4.4	The Principle of Least Time	108
4.5	What Is Light?	113
4.6	Light Waves	119
4.7	Waves in What?	127
4.8	Light Is Electromagnetism	129
4.9	Conclusion	136
5	Space and Time	137
5.1	Electrons	137
5.2	Is the Speed of Light Always the Same?	139
5.3	The Unity of Space and Time	143
5.4	Space, Time and Motion	144
5.5	The Geometry of Spacetime	147
5.6	Lorentz Transformations	151
5.7	Time Dilation and the "Twin Paradox"	155
5.8	Distances and the Lorentz-Fitzgerald Contraction	158
5.9	How Can We Believe All This?	164
5.10	4-Vectors	165
5.11	Momentum and Energy	165
5.12	Electricity and Magnetism in Spacetime	170
5.13	Conclusion	173
6	Least Action	175
6.1	What This Chapter Is About	175
6.2	Action	176
6.3	Minimum or Just Stationary?	178
6.4	Why Is the Action Least?	179
6.5	The Magnetic Action	180
6.6	Time-Varying Fields and Relativity	184
6.7	Action for the Electromagnetic Field	185
6.8	Momentum, Energy and the Uniformity of Spacetime	187
6.9	Angular Momentum	188
6.10	Conclusion	189

7	Gravitation and Curved Spacetime	191
7.1	The Problem	191
7.2	Curvature	194
7.3	Gravity as Curvature of Spacetime	199
7.4	Maps and Metrics	201
7.5	The Laws of Einstein's Theory of Gravity	203
7.6	Newton and Einstein Compared	207
7.7	Weighing Light	209
7.8	Physics and Geometry	211
7.9	General "Relativity"?	212
7.10	Conclusion	213
8	The Quantum Revolution	214
8.1	The Radiant Heat Crisis	214
8.2	Why Are Atoms Simple?	219
8.3	Niels Bohr Models the Atom	221
8.4	Heisenberg and the Quantum World	226
8.5	Schrödinger Takes Another Tack	228
8.6	Probability and Uncertainty	231
8.7	Spin	234
8.8	Feynman's All Histories Version of Quantum Theory	239
8.9	Which Way Did It Go?	242
8.10	Einstein's Revenge: Quantum Entanglement	245
8.11	What Has Happened to Determinism?	249
8.12	What an Electron Knows About Magnetic Fields	253
8.13	Which Electron Is Which?	255
8.14	Conclusion	258
9	Quantum Theory with Special Relativity	260
9.1	Einstein Plus Heisenberg	260
9.2	Fields and Oscillators	261
9.3	Lasers and the Indistinguishability of Particles	266
9.4	A Field for Matter	268
9.5	How Can Electrons Be Fermions?	271
9.6	Antiparticles	274
9.7	QED	275
9.8	Feynman's Wonderful Diagrams	277
9.9	The Perils of Point Charges	282
9.10	The Busy Vacuum	287
9.11	Conclusion	289

10	Order Breaks Symmetry	290
10.1	Cooling and Freezing	290
10.2	Refrigeration	292
10.3	Flow without Friction	294
10.4	Superfluid Vortices	298
10.5	Metals	300
10.6	Conduction without Resistance	301
10.7	Conclusion	307
11	Quarks and What Holds Them Together	309
11.1	Seeing the Very Small	309
11.2	Inside the Atomic Nucleus	310
11.3	Quantum Chromodynamics	316
11.4	Conclusion	323
12	Unifying Weak Forces with QED	324
12.1	What Are Weak Forces?	324
12.2	The Looking-Glass World	330
12.3	The Hidden Unity of Weak and Electromagnetic Forces	339
12.4	An Imaginary, Long-Range Electroweak Unification	341
12.5	The Origin of Mass	343
12.6	GUTs	348
12.7	Conclusion	351
13	Gravitation Plus Quantum Theory – Stars and Black Holes	352
13.1	Black Holes	352
13.2	Stars, Dwarves and Pulsars	360
13.3	Unleashing Gravity's Power: Black Holes at Large	366
13.4	The Crack in Gravity's Armour	367
13.5	Black Hole Entropy: Gravity and Thermodynamics	371
13.6	Quantum Gravity: The Big Challenge	372
13.7	Something from Nothing	377
13.8	Conclusion	378
14	Particles, Symmetries and the Universe	379
14.1	Cosmology	380
14.2	The Hot Big Bang	388
14.3	The Shape of the Universe in Spacetime	391

14.4	A Simple Recipe for the Universe	395
14.5	Why Is There Any Matter Now?	399
14.6	How Do We Tell the Future from the Past?	402
14.7	Inflation	406
14.8	Conclusion	409
15	Queries	410
15.1	Hidden Dimensions: Charge as Geometry	410
15.2	Supersymmetry: Marrying Fermions with Bosons	413
15.3	String Theory: Beyond Points	417
15.4	Lumps and Hedgehogs	423
15.5	Gravity Modified – a Radical Proposal	428
APPENDIX A	The Inverse-Square Law	431
APPENDIX B	Vectors and Complex Numbers	437
APPENDIX C	Brownian Motion	442
APPENDIX D	Units	444
	<i>Glossary</i>	450
	<i>Bibliography</i>	477
	<i>Index</i>	481

MOTION ON EARTH AND IN THE HEAVENS

How modern science began when people realized that the same laws of motion applied to the planets as to objects on Earth.

1.1 Galileo's Telescope

In the summer of 1609, Galileo Galilei, professor of mathematics at the University of Padua, began constructing telescopes and using them to look at the Moon and stars. By January the next year he had seen that the Moon is not smooth, that there are far more stars than are visible to the naked eye, that the Milky Way is made of a myriad stars and that the planet Jupiter has faint “Jovian planets” (satellites) revolving about it. Galileo forthwith brought out a short book, *The Starry Messenger* (the Latin title was *Sidereus Nuncius*), to describe his discoveries, which quickly became famous. The English ambassador to the Venetian Republic reported (I quote from Nicolson's *Science and Imagination*):

I send herewith unto his Majesty the strangest piece of news . . . ; which is the annexed book of the Mathematical Professor at Padua, who by the help of an optical instrument (which both enlargeth and approximateth the object) invented first in Flanders, and bettered by himself, hath discovered four new planets rolling around the sphere of Jupiter, besides many other unknown fixed stars; likewise the true cause of the *Via Lactae*, so long searched; and, lastly, that the Moon is not spherical but endued with many prominences. . . . So as upon the whole subject he hath overthrown all former astronomy . . . and next all astrology. . . . And he runneth

a fortune to be either exceeding famous or exceeding ridiculous.
By the next ship your Lordship shall receive from me one of these instruments, as it is bettered by this man.

Galileo's discoveries proved to be at least as important as they were perceived to be at the time. They are a convenient marker for the beginning of the scientific revolution in Europe. By 1687, Isaac Newton had published his *Mathematical Principles of Natural Philosophy and the System of the World* (often called the *Principia* from the first word of its Latin title), and the first phase of the revolution was complete. The laws of motion and of gravity were known, and they accounted for the movements of the planets as well as objects on Earth.

1.2 The Old Astronomy

Let us review what was known before the seventeenth century about motion and astronomy. I will try to describe what humankind has known for thousands of years, forgetting modern knowledge gained from telescopes, space travel and so on. I will also ignore exceptions and refinements. The basic facts are obvious, qualitatively at least, to anyone. On the Earth, these facts are simple. Solid objects (and liquids) that are free to do so fall down. Otherwise, an effort of some sort is needed to make something move. A stone, once thrown, moves through the air some distance and then falls to the ground. But also a heavy object in motion, like a drifting ship, requires effort to stop it quickly.

The facts about the motion of the stars take longer to tell. I shall describe things as they appear from the Earth, as they would have been perceived say 3,000 years ago.

Thousands of "fixed" stars are visible to the naked eye. These all rotate together through the night sky along parallel circles from east to west. It is as if there were some axis, called the *celestial axis*, about which they all turned. The Pole Star, being very near this axis, hardly moves at all. Stars near the axis appear to move in smaller circles; stars further away in larger ones. The stars that appear to move on the largest circle are said to lie near the *celestial equator* (see Figure 1.1). The time taken to complete one of these

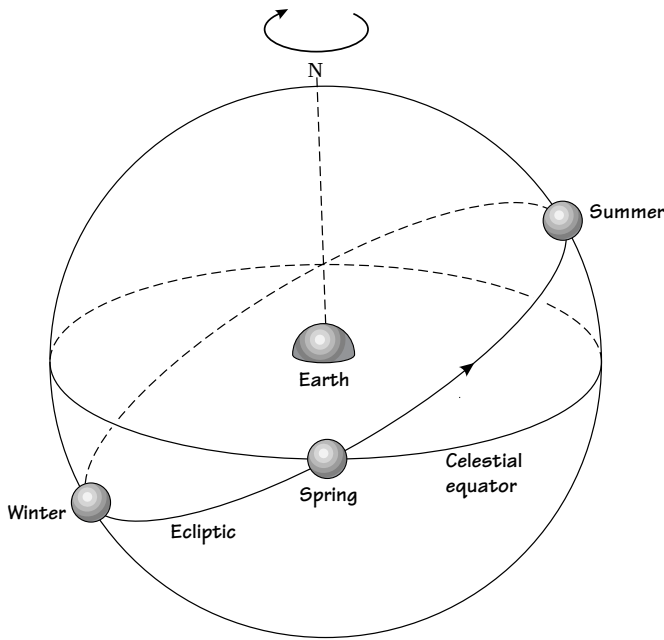


FIGURE 1.1 The “sphere of the fixed stars”, which appears to rotate westward daily (as indicated by the arrow at the top). The Sun, relative to the stars, circuits eastward annually along the ecliptic.

apparent revolutions, 23 hours, 56 minutes, 4 seconds, is called a *sidereal day*.

The motions of the Sun, Moon and planets are more complicated. I shall describe their apparent motions *relative* to the fixed stars, because this is slower and somewhat simpler than the motion relative to Earth. The positions of the Moon and planets can easily be compared with those of the stars. The Sun is not usually visible at the same time as the stars, but we can work out what stars the Sun *would* be near, if only we could see them.

Relative to the stars, then, the Sun moves from west to east round a circle, called the *ecliptic*, taking $365\frac{1}{4}$ days to complete a circuit. Since

$$365\frac{1}{4} \times (24 \text{ hours}) = 366\frac{1}{4} \times (23 \text{ hours } 36 \text{ minutes } 4 \text{ seconds}),$$

this means that the Sun appears to circle the Earth in 24 hours. In

a year the Sun appears to rise and set $365\frac{1}{4}$ times, but the stars rise and set $366\frac{1}{4}$ times.

The ecliptic (the path of the Sun) is tilted at $23\frac{1}{2}$ degrees to the celestial equator, so that the Sun moves to the north of the celestial equator in summer (the summer of the northern hemisphere) and to the south in winter. (See Figure 1.1.) The ecliptic crosses the celestial equator at two points, and the Sun is at one of these points at the spring equinox and at the other at the autumn equinox.

The Moon too appears to move round from east to west, near the ecliptic, and, of course, it waxes and wanes. The interval between two new moons (when the Sun and Moon are nearly in the same direction) is $27\frac{1}{3}$ days.

Lastly there are the planets, five of which were known up to 1781: Mercury, Venus, Mars, Jupiter and Saturn. They are often brighter than the fixed stars, and they move in much more complicated ways. Like the Sun and Moon, they appear to move relative to the fixed stars in large circles. These circles are tilted relative to the ecliptic by small angles, which vary from planet to planet. But, unlike the Sun, the planets do not move at a constant rate, nor even always in the same direction. Most of the time, they appear to move, like the Sun, west to east relative to the stars, but at rates that vary greatly from time to time and from planet to planet. Sometimes they appear to slow down and stop and go east to west temporarily. As examples, as seen from Earth, Venus completes a circuit relative to the stars in 485 days and Mars in 683 days. (This apparent motion comes about from a combination of the planet's true motion with the Earth's. The true periods of Venus and Mars are 225 and 687 days.)

What was made of all this before modern times? Ancient civilizations, like the Babylonian, the Chinese and the Mayan, had officials who kept very accurate records of the movements of the heavenly bodies. They noticed regularities from which, by extrapolating to the future, they were able to predict events like eclipses. One practical motive for their interest was to construct an accurate calendar. This is a complicated matter, because there are not a whole number of days in a year or in a month, nor a whole number of months in a year. Navigation was another application of astronomy. Astrology was yet another.

Yet these peoples did not try to *explain* their astronomical observations, except in terms of what we would call myth. The first people known to have looked for an explanation were from the Greek cities bordering the Aegean in the sixth and fifth centuries B.C. The problem of decoding the (Sir Thomas Browne quoted in Nicolson's book)

Strange cryptography of his [God's] starre Book of Heaven

occupied some peoples' minds for about 2,200 years before it was solved. It needs an effort of our imagination to appreciate how difficult the problem was.

Some things were understood quite early, for example, that the Earth is round, and that the Moon shines by the reflected light of the Sun, the waxing and waning being due to the fraction of the illuminated side of the Moon that is visible from the Earth. For example, the full Moon occurs when the Earth is nearly between the Moon and the Sun, so that the whole of the illuminated side of the Moon is facing the Earth. In the fifth century B.C., Anaxagoras (who was expelled from Periclean Athens for teaching that the Sun was a red-hot rock) understood the cause of eclipses. An eclipse of the Sun is seen from a place on Earth when the Moon comes between the Earth and Sun and casts its shadow at that place. (Because the Moon is small compared to the Sun, the region in shadow on the Earth is small.) The Moon's path is tilted with respect to the ecliptic (the Sun's path), so an eclipse does not happen every month. The two paths cross each other at two points called *nodes*. An eclipse of the Sun occurs only when the Sun and Moon happen to be both simultaneously in the direction of one of these nodes. An eclipse of the Moon occurs when the Moon comes into the Earth's shadow. This happens only when, simultaneously, the Moon is in the direction of one node and the Sun in the direction of the other.

1.3 Aristotle and Ptolemy: Models and Mathematics

I will now move on to the ideas of Aristotle in the fourth century B.C. He had amongst other things a full theory of motion and of astronomy, which was (with some amendments) enormously influential

for some 2,000 years. The story of the Scientific Revolution in the seventeenth century is in some ways the story of the escape from the influence of Aristotle's physics.

Aristotle contrasted "natural" motion and "forced" motion. On Earth, the natural motion of heavy bodies (made of the elements earth and water) was towards the centre of the Earth (which was considered also to be the centre of the universe). In the heavens, the natural motion was motion in a circle at constant speed. On Earth, there were also forced departures from natural motion, caused by efforts like pushing, pulling and throwing. In the heavens, only the natural circular motion could occur, lasting eternally unchanged. Thus the heavens were perfect and the "sublunary" regions were not. Stones fall, but stars do not.

To explain the complicated motions of the heavenly bodies, Aristotle invoked a system of great invisible spheres, nested inside each other, and each with its centre at the Earth. The spheres were made of a fifth element ("quintessential") different from the four "elements" (earth, water, air and fire), which he supposed to make up everything sublunary. Each sphere was pivoted to the one just outside it at an axis, about which it spun at a constant rate. The axes were not all in the same direction. The fixed stars were attached to the outermost sphere. Next inside was a system of four spheres designed to get right the motion of Saturn, the planet attached to the innermost of these four spheres. Aristotle, careful to be consistent, then put three spheres inside just to cancel out Saturn's motion. Then more spheres gave successively the motion of Jupiter, Mars, the Sun, Venus, Mercury and the Moon. He ended up with a total of 55 spheres. With this wonderful machinery, Aristotle could get the observed motions roughly right.

This theory may seem far-fetched to us. We do not find it easy to visualize these great, transparent, unalterable spheres. However the ancients thought about this cosmology, by the middle ages people had begun to envisage the celestial spheres as solid things. One then had an example of what we may call a *mechanical model*. We shall meet several such in the course of this book. It is an explanation based upon imagining a system built like a machine or a mechanical toy. It does nearly all that such a machine would do, except that some properties are pushed to extremes. The fifth element is a bit

different from anything we know on Earth: more transparent than glass, and no doubt perfectly rigid.

Aristotle's model of planetary motion did not fit all the observations, and, by the second century A.D., it had been superseded by a synthesis due to Ptolemy of Alexandria. The Earth was still fixed at the centre, and motion in circles was still assumed to be the right thing in the heavens. But, to get the motions right, Ptolemy (following Apollonius and Hipparchus) took the planets to revolve in small circles ("epicycles") whose centres were themselves rotating about the Earth in bigger circles. (It is easy to see how, for example, a planet could sometimes reverse the direction of its apparent motion when the motion in the small circle was taking it backwards with respect to the motion in the large one.) There were other complications. The centres of the larger circles were not quite at the position of the Earth, and the circles were not traversed at quite constant speed (as viewed from their centres, at any rate). With a sufficient number of such devices, Ptolemy was able to fit the observed motions very accurately. Even his system did not get everything right at the same time. For example, the Moon's epicycle would make the apparent *size* of the Moon vary much too much, because its distance from the Earth varied too much.

Ptolemy provided no mechanical mechanism for the motions. His was more of a *mathematical* (specifically, geometrical) theory than a mechanical model. This too is something we will meet again. When people despair of imagining a physical model, they fall back on mathematics, saying: "Well the mathematics fits the facts, and maybe it is not possible to do better. Maybe we are not capable of understanding more than that".

Before leaving the ancient world, we should note one more piece of knowledge that had been gained. This was some idea of size. In the third century B.C., Eratosthenes, librarian at Alexandria, had determined the radius of the Earth from a measurement of the direction of the Sun at Alexandria at noon on midsummer day. It was $7\frac{1}{2}$ degrees from being vertically overhead. On the Tropic, 500 miles south, the Sun would be overhead at the same time. From this it follows that the circumference of the Earth is

$$\frac{360}{7.5} \times (500 \text{ miles}) = 24,000 \text{ miles.}$$

Ptolemy later made an estimate of the distance of the Moon, using its different apparent positions (parallax) as viewed from different places on the Earth. The distance of the Sun could be inferred from the extent of the Sun's shadow at a solar eclipse and the extent of the Earth's shadow at a lunar eclipse, but the ancient estimates were badly out.

Aristotle and Ptolemy had these beliefs in common: that the Earth was at rest, that the motion of the heavenly bodies had to be constructed out of unchanging *circular motion* but that the motion of bodies on Earth was of a quite different nature. These beliefs dominated scientific thought, first in Arab lands from the eighth to the twelfth century, then in medieval Europe until the sixteenth century.

The ancient world became aware that the Moon had weight, like objects on Earth, and there had to be a reason why it did not fall out of the sky. For example, Plutarch wrote,

Yet the Moon is saved from falling by its very motion and the rapidity of its revolution, just as missiles placed in slings are kept from falling by being whirled around in a circle.

People were certainly aware of the shortcomings of the Aristotelian and Ptolomaic views. There were some strange coincidences in Ptolemy's theory. The periods of revolution were about one year in the *large* circles of the inner planets (Mercury and Venus) and also about one year in the *small* circles of the outer planets. Aristarchus in the third century (quoted by Archimedes) had suggested that everything would be simpler if the Sun, not the Earth, was at rest.

As regards motion on Earth, Aristotle's doctrine had great difficulties with something as simple as the flight of an arrow. This was not a "natural" motion towards the centre of the Earth (except perhaps at the end of its flight), so what was the effort keeping it in motion after it had left the bow? Aristotle said that a circulation of the air followed it along and kept it going. It is not hard to think of objections to this idea. In the sixth century, the Christian Philoponus of Alexandria made a particularly effective critique of Aristotle's physics. (See Lloyd's book *Greek Science after Aristotle*.)

In the middle ages, several attempts were made to improve on Aristotle's account of motion. Nevertheless, in the thirteenth century

Thomas Aquinas argued that Aristotelian physics was compatible with Christian theology, and the two systems of thought got locked together. When Galileo published his dialogues in the 1630s, it was still the Aristotelian viewpoint he was combating (represented in the dialogues by one of the disputants, Simplicio).

1.4 Copernicus: Getting Behind Appearances

Nicolas Copernicus, born in 1473, was a Polish canon who worked at the University of Cracow and later in Italy. He developed a Sun-centred theory of the Solar System, in which the Earth was just another planet, circulating the Sun yearly between Venus and Mars. (Actually, the centre of the planetary motions was taken to be slightly displaced from the Sun.) He assumed that the planetary motions had to be built up out of circular motions, and so he had a system of epicycles and so on, not much less complicated than Ptolemy's. Copernicus also assumed that the "fixed" stars were indeed fixed, their apparent daily motion being due to the Earth's spinning on its axis. He nursed his ideas for some 40 years and published his complete theory (in *De Revolutionibus Orbium Coelestium*) only in the year of his death, 1543. Copernicus dedicated his book to Pope Paul III, but a colleague, Andreas Osiander, added a cautious preface saying that the Sun-centred system was not to be taken as the literal physical truth, but only as a geometrical device for fitting the observations.

In a Sun-centred system, many things fall into place. The reason that planets sometimes appear to reverse their motion relative to the stars and move "backwards" (that is, east to west instead of west to east) is that the forward motion of the Earth can, at certain times, make a planet appear, by contrast, to go backwards. In Ptolemy's system, the order of planets from the Earth was to some extent arbitrary, but in Copernicus' system there is a natural order of planets from the Sun, with the periods of revolution increasing with distance: Mercury (88 days), Venus (225 days), Earth (1 year, i.e., 365 days), Mars (1.9 years), Jupiter (11.9 years) Saturn (29.5 years). The fact that Mercury and Venus never appear far from the Sun is explained because they really *are* nearer the Sun than the Earth and other planets.

But what were the drawbacks of the Copernican system, given that it seems (to us) so much more natural than Ptolemy's picture? There were two objections, each of which might be thought to be fatal. Since the Earth moves (roughly) in a circle of radius 150,000,000 kilometres, we ought to be seeing the fixed stars from a different standpoint at different times of the year, and this should be evident. This effect is called *annual parallax*. The only way to avoid it is to assume, as Copernicus did, that the stars are at distances very large compared to this 150,000,000 kilometres, so that the annual parallax was too small to be seen. This seems a rather weak excuse: the effect is there, but unfortunately it is too small for you to see it. But it turned out to be true. The nearest star has an annual parallax of only a few hundred thousandths of a degree (i.e., its apparent direction varies by this amount at different times of the year). This is much too small to see without a good telescope. As we shall see in later chapters, very large numbers *do* turn up in nature, and as a result some things are very nearly hidden.

In fact, people had already used a weaker version of this argument, also concerned with a parallax effect. The view of the stars ought to be slightly different at different places on the Earth. For this effect to be unobservable, one must assume that the stars are very far away compared to the Earth's radius (6,378 kilometres).

The second argument against the Copernican system is this. The rotation of the Earth about the Sun gives it a speed of about 100,000 kilometres per hour, and the daily spin of the Earth gives a point on the equator a speed of 1,670 kilometres per hour. Why do we not feel these speeds? Why is the atmosphere not left behind? Why is a projectile not "left behind"? It appears to us obvious that the Earth is at rest. Copernicus of course recognized the difficulties with his theory:

Though these views of mine are difficult and counter to expectation and certainly to common sense . . .

Galileo was the first to understand fairly clearly the fallacy underlying the second objection to Copernicanism.

It happened that some natural events occurred in the latter half of the sixteenth century that challenged the Aristotelian view. In 1572 there appeared a supernova, that is, a "new" star, which

rapidly became very bright (visible in the daytime) and then faded in a few months. There was another in 1604. The Chinese had recorded another in 1054 (whose remnant now is probably the Crab Nebula), but for some reason there was no record of this in the West. A comet was seen in 1577. The Danish astronomer Tycho Brahe, for example, demonstrated that both the supernova and the comet were farther from Earth than the Moon (because they exhibited no observable parallax), contradicting Aristotle's belief that the heavens above the Moon were unchanging.

1.5 Galileo

Galileo Galilei was born in Pisa in 1564 (the same year as Shakespeare). In 1592 he became professor of mathematics at the University of Padua (part of the Republic of Venice). An unsuccessful attempt was made to patent the telescope (two lenses used together to view distant objects) in the Netherlands in 1608. Galileo heard of this in the summer of 1609 and immediately began to make, and improve, telescopes for himself. By the autumn, he had one magnifying 20 times and began making astronomical observations. Like Newton and like Enrico Fermi in our own time, Galileo must have combined theoretical genius with a flare for experiment. He saw that the Moon was rough, just like the Earth. He saw that Jupiter had four satellites (which Galileo tactfully named "Medician stars"), so the Earth was not unique in having a Moon. He saw that Venus waxes and wanes just as the Moon does. Venus is "full" when it is on the opposite side of the Sun from the Earth, but on the Ptolemaic system it would never be "full" since it would stay between the Sun and the Earth.

As mentioned at the beginning of this chapter, Galileo immediately published his *Sidereal Messenger* to report what he had seen. He became mathematician in Florence to Cosimo de Medici, grand duke of Tuscany. Although some people were convinced of the truth of Copernicanism, the universities remained Aristotelian. Twenty-two years later, in 1632, Galileo published *Dialogue on the Great World Systems* to make the case for the Copernican system. This work was written in Italian, in the form of a dialogue among three characters, and designed to be widely understood. A papal decree

of 1616 had declared Copernicanism to be “erroneous” (not as bad as being heretical); but the new pope, Urban VIII, gave Galileo leave to write about it. However that may be, Galileo was brought before the Inquisition in 1633, made to abjure his “errors and heresies”, and he spent the remaining nine years of his life effectively under house arrest.

In the *Dialogue*, the Aristotelian character Simplicio is subjected to a Socratic type of cross-examination (which he bears with cheerfulness and resilience). Sometimes Aristotle is criticized for not having done experiments, but it is not always clear whether Galileo has done them either. Sometimes the argument is about “thought experiments”, such as were used in the twentieth century by Einstein and Heisenberg, for example. Aristotle said that heavy bodies move towards the centre of the Earth. What would happen if there were a hole right through the Earth to the antipodes and you dropped a stone down it? Would it come to rest at the centre? Aristotle said that heavier objects fall faster than lighter ones. What would happen if you tied two cannon balls together to make an object of twice the weight? Would that fall faster than each separately? What happens if you release a stone from the top of a mast of a moving ship? Is it left behind so that it hits the deck behind the mast?

Here is another of Galileo’s thought experiments:

Shut yourself up with some friend in the largest room below decks of some large ship and there procure gnats, flies, and such other small winged creatures. Also get a great tub full of water and within it put certain fishes; let also a certain bottle be hung up, which drop by drop lets forth its water into another narrow-necked bottle placed underneath. Then, the ship lying still, observe how these small winged animals fly with like velocity towards all parts of the room; how the fishes swim indifferently to all sides; and how the distilling drops all fall into the bottle placed underneath. And casting anything towards your friend, you need not throw it with more force one way than another, provided the distances be equal; and jumping abroad, you will reach as far one way as another. Having observed all these particulars, though no man doubt that, so long as the vessel stands still, they ought to take place in this manner, make the ship move with what velocity

you please, so long as the motion is uniform and not fluctuating this way and that. You shall not be able to discern the least alteration in the forenamed effects, nor can you gather by any of them whether the ship moves or stands still.

By considerations like this, Galileo disposes of the argument that a moving Earth would leave things near it behind. Provided everything moves uniformly together, we notice nothing.

Galileo emphasized a new idealized state of motion (he is talking about a ball rolling along a sloping flat surface – we might think of a billiard ball on a table):

But take notice that I gave as an example a ball exactly round, and a plane exquisitely polished, so that all external and accidental impediments may be taken away. Also I would have you remove all obstruction caused by the air's resistance and any other causal obstacles, if any other there can be.

In other words, Galileo is imagining motion in the absence of friction or resistance. This is a state of affairs that can perhaps never be achieved *exactly*, but by taking careful precautions we can get nearer and nearer to such an ideal situation. Aristotle would have probably dismissed it as being hopelessly unrealistic, but in science half the battle seems to be to find the right simplified starting point, then perhaps build on it by adding complications (like friction in the present case) later.

Galileo asserted that, in this ideal frictionless situation, motion with constant speed (along a straight line in a fixed direction) persists unchanged without the application of any force or effort. No force is required to keep a billiard ball moving with constant speed. A force is needed to start it (applied by the billiard cue perhaps) or to change it (by impact on the edge of the table perhaps). So far as it is not negligible, the force of friction changes (reduces) this constant velocity.

Actually, Galileo did not get it quite right. Instead of motion in a straight line, he thought that the natural thing was motion in a great circle (that is, a circle whose centre is at the centre of the Earth) on the Earth's surface. For motion on a scale small compared to the size of the Earth, this is almost the same as motion in a straight line. Thus Galileo had not thrown off the Greek belief in the importance

of circular motion. (It seems to have been René Descartes who first got it quite right.) However, Newton attributed the correct law (Newton's first law of motion) to Galileo.

Now apply Galileo's idea to the moving Earth. If the Earth, the atmosphere and all the things on the Earth are moving with the same constant velocity, they will all continue to do so. Everything will go on moving together, and no one on the Earth will notice anything. Thus there is nothing against the Copernican system on this account.

(The previous paragraph is a slight oversimplification. Take the velocity of, say, Singapore (near the equator) due to the spin of the Earth on its axis. This is a constant 1,670 kilometres per hour, but its *direction* is changing. In fact, the rate of change of direction is

$$\frac{360 \text{ degrees}}{24 \times 60 \text{ minutes}} = 2\frac{1}{2} \text{ degrees per minute.}$$

To produce this change of direction, a force directed towards the centre of the Earth would be needed, but this force is only a small percentage of the force due to the Earth's gravity, so it is not a very noticeable effect.)

Galileo formulated another law of supreme importance. In direct opposition to Aristotle, he said that, in the absence of resistance due to the air, all bodies would fall downwards under gravity in identical ways. As mentioned, he produced some thought experiments (cannon balls tied together) in support of this claim. But he is known to have done much real experimentation too.

This law of Galileo's was to wait three centuries before being explained by Einstein.

1.6 Kepler: Beyond Circles

After Copernicus and Galileo, one feature of Aristotle's physics remained. That was the belief in the naturalness of circular motion. In fact, Johann Kepler had already shown that the planetary motions could be better understood (without epicycles and so on) if the planets moved on *ellipses* not circles. Kepler (1571–1630) in 1600 became assistant to, then succeeded, the great Danish astronomer Tycho Brahe, as mathematician to the German emperor Rudolph II

in Prague until 1612. (Brahe had previously been granted, by King Frederick II, the Danish island of Hven for his magnificent observatory complex Uraniborg.)

An ellipse is a curve got by slicing through a cone. In a perspective drawing, a circle is represented by an ellipse. (Steeper slices through a cone produce parabolas and hyperbolas, which, unlike ellipses, extend indefinitely rather than closing up.) After the geometry of points, lines and circles, the Greeks also studied ellipses. It is somewhat ironic that the most important Greek writer on ellipses, Apollonius, also introduced epicycles (see Section 1.3) into astronomy, and the epicycles were needed on the assumption that *circles* were the things out of which to build planetary motion. An ellipse has two important points inside it called *foci*. For all points on an ellipse, the sums of the distances to the two foci are the same.

Kepler proposed a modification of the Copernican system embodying three principles, which have come to be called *Kepler's three laws*. Kepler had at his disposal Tycho Brahe's and his own detailed observations. He believed that astronomy should be part of physics, and that the motions of the planets should somehow be caused by the influence (perhaps magnetic in origin) of the Sun. His theoretical arguments were erroneous, but nevertheless they inspired him in his struggle to understand planetary motions (especially that of Mars) consistently with the observational data.

Kepler's first law is that each planet moves in an ellipse with the Sun at one of its foci. These ellipses replace all the circles of Aristotle, Ptolemy and Copernicus.

The second law replaces the Ptolemaic idea that the circles should be traversed at constant rates (an assumption that had been qualified anyway). Kepler said instead that the line joining the planet to the Sun should sweep out *area* at a constant rate. What this means is illustrated by Figure 1.2. Thus the old assumption of traversing at a fixed *distance* in a given time is replaced by a fixed area in a given time. It is clear from the diagram that the second law implies that a planet moves faster when it is nearer the Sun and slower when it is farther from the Sun. This is for the same cause that a spinning ice-skater speeds up when she draws in her arms.

A circle can be thought of as a special case of an ellipse, and in that special case Kepler's laws reduce to the assumptions of the old

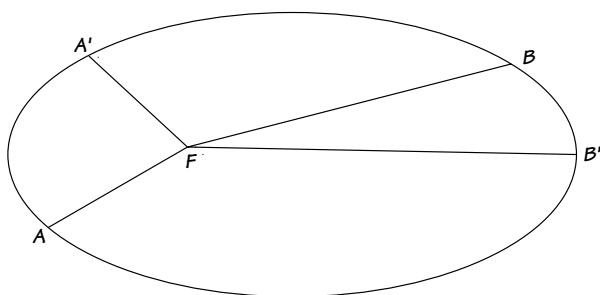


FIGURE 1.2 An example of Kepler's third law. The Sun is at the focus, F , of the elliptical orbit. The areas FAA' and FBB' are equal. The planet takes the same time from A to A' as from B to B' .

astronomy. For example, the orbit of Venus deviates from being a circle by less than 1 percent, but other planets deviate more, up to 25 percent in the case of Pluto. It is often the case that a new scientific theory contains an old one within it as a special case. Looking back from the vantage point of the new theory, things are clear. But, locked within the old theory (as humankind had been for some 2,000 years in the present example), it requires someone of immense imagination to glimpse the new one.

Kepler's third law had no counterpart in the old astronomy. It connects the average distance of a planet from the Sun and the period of its revolution (its "year").

The third law states that, for any two planets, call them P and Q , in the Solar System,

$$\frac{(\text{average distance of } P \text{ from the Sun})^3}{(\text{average distance of } Q \text{ from the Sun})^3} = \frac{(\text{period of } P)^2}{(\text{period of } Q)^2}.$$

As an example, the average distance of Pluto from the Sun is about 100 times that of Mercury, and Pluto's period is about 1,000 times Mercury's. These numbers agree with the law because $100^3 = 1,000^2$.

An equivalent way to state the Kepler's third law is:

$$\begin{aligned} & \frac{(\text{average distance of a planet from the Sun})^3}{(\text{period of this planet})^2} \\ &= \text{a fixed value for all planets of the Solar System.} \end{aligned}$$

The value of this “fixed quantity” is actually

$$3.24 \times 10^{24} \text{ (kilometres)}^3 \text{ per (year)}^2,$$

as can be inferred from the the size of the Earth’s orbit.

Kepler published his first two laws in 1609 and his third in 1619. Kepler and Galileo corresponded, and these two great and likeable men held each other in much esteem. Like Galileo, Kepler wrote in favour of Copernicanism (*Epitome astronomiae Copernicanae*). It is strange that Galileo’s *Dialogue on the Great World Systems* (1632) makes no mention of Kepler’s laws, or even of ellipses. The two men had very different scientific styles. Galileo was down-to-earth, and an exceptional communicator of science (writing often in Italian not Latin). Kepler (quoted in Baumgardt’s book) had a more unworldly attitude:

It may be that my book will have to wait for its reader for a hundred years. Has not God himself waited for six thousand years for someone to contemplate his work with understanding?

We should remember too that even the greatest of scientists get some things wrong. Galileo had a theory of the tides, with which he was very pleased. He thought it gave the most decisive argument for Copernicanism. It was wrong. Kepler thought that magnetism kept the planets moving in their orbits. He tried (like Pythagoras before him) to connect the planetary orbits with musical harmony. Also, earlier, his *Mysterium Cosmographicum* (1596) contained a beautiful explanation of the relative sizes of the planetary orbits. The Greeks had proved that there are exactly five regular solids (the “Platonic solids”). A regular solid has edges that all have the same length, faces that are all the same and corners that are all the same (with the same angles at them). Kepler, at that time thinking of the orbits as being on spheres, assumed that a regular solid was nested in between each neighbouring pair of planetary spheres, so that the faces touched the sphere inside and the corners lay on the sphere outside. The sequence went

Mercury (octahedron), Venus (icosahedron), Earth
(dodecahedron), Mars (tetrahedron), Jupiter (cube), Saturn.

This construction fitted the spacings between the planets moderately

well. It was a theory that Plato and Pythagoras would have loved. Apart from anything else, it explained why there were six planets. Kepler must have been entranced by it. Of course, it was totally wrong. We know that there are more than six planets. Probably also the spacings between the planets owe a lot to accident (in the formation of the Solar System from the condensation of a cloud of gas and dust) and are not something we would expect to explain by a simple fundamental theory.

But this may not be quite right. Complicated causes can sometimes give simple results. Between the orbits of Mars and Jupiter there are a swarm of mini-planets, the asteroids. They have orbits in a spread of different sizes, and a corresponding distribution of orbital periods. But there are gaps in this distribution where, for example, the period is two-fifths or one-third of the period of Jupiter. How do these simple numbers get into such a complex dynamical system? Take an asteroid with the two-fifths period, for example. Suppose at some time it and Jupiter were at points on their orbits where they were as near as they could be. Then five orbits of the asteroid later and thus two orbits of Jupiter later they would be in just the same situation. At this position of closest approach, the gravitational force exerted by Jupiter on the asteroid (which is a small addition to the Sun's gravitational force on the asteroid) is at its greatest. It is likely that such a regular series of gravitational perturbations of the same kind would have been enough to throw the asteroid out of this particular orbit. This effect, of achieving a big result by a timed series of small impulses, is called *resonance*. It is like getting someone swinging on a garden swing by giving a series of little pushes each timed to occur at the same moment in the swing cycle.

There is another example in the Solar System in which simple ratios may be significant. The orbit of Pluto is quite eccentric and, although lying mainly outside that of Neptune, sometimes crosses inside. There are other small "Plutinos" in similar orbits. How have they avoided being ejected by gravitational tugs from (the much heavier) Neptune? Pluto and many of the Plutinos have periods close to three-halves that of Neptune. Consequently, it is possible that, everytime they cross Neptune's orbit, Neptune is at another part of its orbit.

Kepler, like Galileo again, suffered from the times he lived in. For example, between 1615 and 1621, Kepler's mother was charged with, and imprisoned for, witchcraft.

Let us return to the state of knowledge left by Galileo and Kepler. The question remained, What causes the planets to stay and move in their orbits? René Descartes (1596–1650), after a period as a professional soldier, spent 20 years in Holland and the remaining 4 years of his life in Stockholm, called there by Queen Christina. He stipulated that a body would continue with constant speed in a straight line if no force acted upon it. Therefore, a force was required to keep the planets in their curved orbits. Descartes had a mechanical explanation of this (in his *Principia Philosophiae*):

Let us assume that the material of the heaven where the planets are circulates ceaselessly, like a whirlpool with the Sun at its centre, and that the parts which are near the Sun move more quickly than those which are a certain distance from it, and that all the planets (among whose number we include from now on the Earth) always remain suspended between the same parts of this heavenly matter; for only thus, and without using any other tools, shall we find a simple explanation of all things we notice about them.

This explanation was rather persuasive, especially immersed as it was in Descartes's complete system of philosophy. It was a mechanical explanation, like Aristotle's heavenly spheres. Like all mechanical explanations in science, it pushed the problem one stage back – to the question of what gave the “material of the heaven” its properties.

1.7 Newton

Isaac Newton was born in 1642, the year of Galileo's death. He attended Trinity College, Cambridge. The plague of 1664 caused him to return to his home in Lincolnshire. In the next two years, he began to develop his ideas about motion and the Solar System. Perhaps because of his remarkably suspicious, cautious and perfectionist character, Newton wrote almost nothing of his work for some 20 years, when he was coaxed by the second Astronomer Royal, Edmund Halley. The result was *Mathematical Principles of*

Natural Philosophy (1687) – a title that makes a large, but fully justified claim.

Newton's *Principia* is a remarkable work. It is written in an austere, magisterial style, giving the reader little help and admitting no human weakness. It includes three books. Book 1, after a few definitions, begins by stating three laws. In Newton's words, these are (*Newton's laws of motion*):

- (i) Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it.
- (ii) The change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.
- (iii) To every action there is always opposed an equal reaction: or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.

Law i is Descartes's law of inertia.

In law ii the "motion" (which is now called *momentum*) is defined earlier in the *Principia* to be the "quantity of matter" (which we would call the *mass*) times the velocity. Mass ("quantity of matter") was not really well defined by Newton. For many purposes it is sufficient to say that mass is additive: that is, if two objects are put together to make a new one, the mass of the composite object is got by adding together the masses of the two original ones. In any case, law ii does not tell us anything unless we have some other method of knowing what the "motive force" is. For future reference, please note that the mass that enters in to the second law is sometimes called the *inertial mass*. This is to distinguish it from mass appearing in another context, which we shall meet shortly.

Law iii says, as an example, that if the Sun exerts a force on Jupiter, then Jupiter exerts an exactly opposite one on the Sun. Or, if two billiard balls collide, the momentary force of the first on the second is just the opposite of the force of the second on the first.

Assuming the truth of these three laws, Book 1 flows along (a bit like Euclid) with a series of mathematical proofs, giving the motions that would follow from various assumed forces. Newton shows immense mathematical power, sometimes using traditional