> This book is in the tradition of non-market clearing approaches to macrodynamic analysis. It builds a series of integrated disequilibrium growth models of increasing complexity, which display the economic interaction between households, firms and government across labor, goods, money, bonds, and equities markets. Chiarella and Flaschel demonstrate how macrodynamics can be developed in a hierarchical way from economically simple structures to more advanced ones. In addition they investigate complex macrodynamic feedback mechanisms.

> The book is organized into seven chapters. Chapter 1 discusses traditional macrodynamic model building. Chapters 2–4 show how Keynesian disequilibrium growth can be obtained from Tobin and Keynes–Wicksell monetary growth models. Chapter 5 treats the cases of substitution in production, and chapter 6 provides the working model of the book. Chapter 7 discusses further extensions and gives an outlook on future work.

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The Dynamics of Keynesian Monetary Growth

The Dynamics of Keynesian Monetary Growth:

Macro Foundations

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Foreword

Richard H. Day

In his effort to reorient economic theory so that it might offer an explanation of severe and prolonged recessions and insights concerning the possibilities and limitations of fiscal and monetary policies for dealing with them, Keynes introduced two factually based assumptions: first, price and wage stickiness; second, independently determined savings and investment variables. In developing the implications of these two facts, Keynes exploited the concept of demand, not at the usual level of the market for a single good, but at the level of the entire economy for the aggregate of all goods. Thus, for example, instead of an Engle curve for a single good, which gives the demand for a good in terms of real income, he exploited the dependence of aggregate demand for all goods on income, that is, the consumption function. With the real money rate of interest as the only endogenously determined price – in this case the price (or opportunity cost) of using money as an idle balance – the money market is seen to play a potential role; potential, because its role depends on sensitivity to interest rates on the markets for goods and money.

Although his analytical derivations were static and focused on a new kind of persistent unemployment situation, Keynes had in mind a dynamic theory. He fully intended to illuminate the tendency of the market economy to fluctuate due to the interactions between the monetary and real goods sectors.

Keynes' ideas were obviously relevant. Within a decade they led to a new field of economics based on a reduction of the microeconomics of many goods and prices to a macroeconomics based on a measure of the aggregate of all goods and money. Extreme assumptions were necessary to reduce the theory to the graphical dimensions required by contemporary pedagogy. This yielded the standard by which the theory became widely known: Hicks' ingenious IS–LM framework.

The deficiencies of this static, simplified version were obvious. Instead of prices and wages that adjusted stickily, it assumed prices that did not

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adjust at all; it included investment but not capital accumulation; it treated the money supply as exogenous instead of incorporating a dependence on credit conditions and government finance. These deficiencies motivated a large body of work aimed at reducing these deficiencies, work that continues to the present as readers of this volume will come to appreciate. But, in the meantime, a quite different body of work veered off this (then) mainstream approach. Instead of building on aggregate supply and demand of heterogeneous firms and households out of equilibrium, it built on the concept of a Robinson Crusoe or a representative agent in intertemporal equilibrium. Since only a single agent is modeled, there is no problem of coordination among markets, no need to consider savings and investment out of equilibrium, and no need to consider fluctuations caused by the interaction of money markets and goods. Instead, the source of fluctuation is sought in terms of unexplained, exogenous shocks which push equilibria around.

The intellectual advantage of this approach lies in its reliance on the equilibrium assumption which is embodied in the principle of optimality. It enables the derivation of "optimal trajectories" for consumption, labor, and capital. By means of an extended duality principle, the supporting competitive equilibrium price trajectories are implied. Along such equilibrium paths involuntary unemployment and excess capacity do not occur.

This is not the occasion to address in detail the relationship between these two approaches except to emphasize that the dynamic, aggregate supply/demand approach represents the economy as one that adjusts out of perfect coordination to disequilibrium signals, in contrast to the equilibrium approach which represents the economy as a perfectly coordinated process with no need for mechanisms of adaptation to deal with discrepancies among the constituent parts.

As the American academic establishment expanded during the last half-century into a new kind of mass market for education and science, it began to exhibit a herding phenomenon not unlike fashions in consumer goods. For a time, the macroeconomic fashion leaders were centered at a Harvard/MIT/Penn nexus in the persons of Hansen, Duessenberry, Samuelson, and Klein. Out-of-equilibrium thinking ruled macroeconomic theory and econometrics until the mid 1970s. When the leading macroeconomic equilibrium pundit moved from Carnegie Mellon to Chicago, a new fashion of equilibrium macroeconomics emerged with a new center of gravity. The new fashion leaders spread in due course to Harvard and Stanford and many points in between and beyond.

In the meantime, the serious work of extending the out-of-equilibrium aspect of macroeconomics so as to remove its deficiencies, so as to improve its ability to explain real world events and so as to improve its potential for

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policy repercussion analysis has continued. In England and especially in Germany and Italy, as well as to a lesser extent in the USA, this stream of work has continued until a more general and more satisfactory theory has emerged. Its potential for illuminating macroeconomic phenomena has been enhanced and its potential for providing new understanding of fiscal and monetary policy improved.

This book by Chiarella and Flaschel is a contribution to this out-ofequilibrium stream of macroeconomic theory. Beginning with Tobin's monetary growth analysis, it successively introduces realistic, complicating relationships that eliminate, step by step, some of the major deficiencies in the earlier Keynesian models. It gives a meticulous analysis of each model's properties and an equally meticulous explanation of each model's relationship to the contributions of other scholars. Anyone who wants to understand the development of macroeconomic thinking as a whole and who wants to see the modern development of the out-of-equilibrium approach, will want to study this volume.

The authors dramatically demonstrate the power of the dynamic point of view, and the potential for explaining apparent anomalies by endogenous economic forces. For example, the scatter of data suggesting a relationship between the rate of price changes and unemployment has usually been explained in terms of shifting Phillips curves. However, when the scattered dots are connected in a time sequence, irregular Phillips *spirals* are revealed. To theorists of dynamics, such spirals suggest an underlying endogenous mechanism, not stationary points at the intersection of exogenously shifting curves. By chapter 4 of the present volume, somewhat similar spirals are shown to emerge from endogenous, out-ofequilibrium, real/monetary interactions, a finding of great potential importance.

The authors modestly present their findings as work in progress, and so it is, but it is, nonetheless, a work of consummate scholarship. I have been fortunate in having been able to follow the gradual accumulation of the authors' and their collaborators' studies to their present state. It is an appropriate stage to present it in this integrated form. Every serious student of macroeconomic theory will want to know what they have done, for in this work they will find a comprehensive analytical exegesis of the steps by which the theory has reached its present state at the frontier, and an excellent jumping off point for further research. It seems to me likely that it is only a matter of time before empirical studies, based on models of the kind analyzed here, will achieve a new breakthrough in understanding real economic data and a new basis for predicting policy analysis.

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Preface

"Macroeconomics has never reached a consensus and probably never will. The subject is too diverse and the approaches too varied for that to become likely." (S. Turnovsky, *Methods of Macroeconomics Dynamics*)

This book provides the reader with a systematic study of macrodynamic models of monetary growth in the Tobin, the Keynes–Wicksell and the Keynesian (if it exists) tradition. Our point of departure is, therefore, the core of descriptive macrodynamic models of monetary growth of primarily traditional origin; recent contributions of neo- or post-Keynesian type as well as other schools of thought are given scant consideration in this book. Instead, we considerably extend and refine the aforementioned model types so that they give rise to a hierarchical sequence of fully integrated macrodynamic models, each providing an improvement on the shortcomings of one or more structural equations of its predecessor. In this way we arrive at the formulation of an integrated model of the Keynes–Metzler type, with both sluggish price and quantity adjustment and under- or overemployment of both labor and capital, which may be considered as the working Keynesian prototype model of IS–LM growth.

Yet, this model type also has its shortcomings, so the hierarchical structure proposed here does not end with it, but rather will be continued in future research by way of more refined treatments of asset markets, of expectations, of the role of income distribution, of international trade in goods and financial assets, of stochastic influences, and so on. In this regard we view this book as providing "macro foundations," or a systematic way to proceed from elementary studies of models of cycles and growth with a full range of markets (labor, goods, money, bonds, equities) to ever more refined and detailed ones. Partial "micro foundations" for the structures exist in the literature and, of course, must be improved as well, but this has to be done based on the knowledge of what indeed has to be micro

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Preface

founded, and thus put on a firmer basis.

This book is the product of a continuing collaboration between the authors which began in 1992 when the first author was on study leave at the University of Mannheim in Germany. At a number of meetings during that period we realized that we shared a strong, common desire to set up a framework within which the non-market-clearing approach to dynamic macroeconomics could be built in a systematic, consistent, and transparent manner, starting from mainstream contributions to disequilibrium growth developed in the sixties, seventies, and eighties. We have sought to construct a framework in which such mainstream contributions to the nonmarket-clearing paradigm could be reformulated on a common basis and extended systematically, leading successively to more and more coherent integrated models of disequilibrium growth with progressively richer interactions between markets and sectors. In this way, we sought a framework to which further refinements, in terms of more markets, more agents, more advanced behavior of agents, could be added or inserted in a natural way, far beyond even the general working model of traditional Keynesian monetary growth that is the focus of this book. Indeed, in other work we have already started the task of these further extensions in several directions, and these are alluded to in the final chapter. Of course, we must leave to the reader to judge whether we have succeeded in our aims of providing what we would call macro foundations of traditional macrodynamics on the basis of which more recent contributions to the non-market-clearing approach to economic dynamics may be reconsidered, evaluated and used as macro perspectives for the project we have begun with this book.

Acknowledgments

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We owe a particular debt of gratitude to Alexander Khomin, formerly of the School of Finance and Economics at the University of Technology, Sydney, and now at the Commonwealth Trading Bank, Australia. He designed and built the C + + computer package which we used to perform many of the simulations of the model reported both here and in our other published work.

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Notation

The notation employed throughout this book is subdivided into statically or dynamically endogenous variables and parameters, a subdivision which is here presented from the perspective of chapter 7:¹

A Statically or dynamically endogenous variables ²			
V <i>p</i>	Potential output		
Y	Output ($\neq Y^p$ = potential output in general)		
$Y_c^D, Y_{c,e}^D$	Disposable income (index c: of capitalists, index e:		
1	perceived)		
Y^d	Aggregate demand $C + I + \delta K + G$		
Y^e	Expected aggregate demand		
L^w, L^d	Employed workforce, employment of the employed		
	workforce ($L^d = L^w$ with the exception of chapter 7)		
С	Consumption		
Ι	Investment		
I^p	Planned investment $I + \mathscr{I} (I^a = I + \dot{N} \text{ actual})$		
	investment)		
r	Nominal rate of interest (price of bonds $p_B = 1$)		
p_e	Price of equities		
S_p	Private savings		
S_p S_f S_g	Firms' savings		
	Government savings		
$S = S_p + S_f + S_g$	Total savings		
T	Real taxes $(T_w, T_c \text{ of workers and capitalists})$		
G	Government expenditure		

¹ The NAIRU-employment rate – denoted by \overline{V} in the following is the Non-Accelerating-Inflation-Rate-of-Utilization (here of the labor force), i.e., the employment-complement of the NAIRU of the literature. Starting with chapter 4 we shall make use in addition of a NAIRU concept \overline{U} with respect to the rate of capacity utilization U of the capital stock K.

 2 Some of these variables will be given parameters in the earlier chapters of this book.

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ho	Rate of profit (expected rate of profit ρ^e)	
$V = L^w/L$	Rate of employment (\overline{V} the NAIRU employment rate)	
$V^w = L^d / L^w$	Utilization rate of the employed	
$U = Y/Y^p$	Rate of capacity utilization (\overline{U} the NAIRU rate of	
$\mathbf{c} = 1/1$	capacity utilization)	
V		
K	Capital stock	
W	Nominal wages	
р	Price level	
p^*	p-star price level of the FED/German Bundesbank	
\overline{v}	Velocity of money circulation	
π	Expected rate of inflation (medium run average)	
M	Money supply (index d: demand, growth rate μ_0)	
L	Normal labor supply	
В	Bonds (index d: demand)	
E	Equities (index d: demand)	
W	Real wealth	
Ν	Stock of inventories	
N^d	Desired stock of inventories	
I	Desired rate of inventory change	
γ	Trend growth rate of the capital stock	
•	Natural growth rate	
$n = n_1$		
n_2	Rate of Harrod neutral technical change	
v = N/K	Inventory–capital ratio	
ω	Real wage ($u = \omega/x$ the wage share)	
$u = \omega / x$	Wage share (x is labor productivity, see below)	
y = Y/K	Output–capital ratio	
B Parameters (all parameters represent positive scalars)		
y ^p	Potential output-capital ratio	
x	Output–labor ratio (labor productivity)	
δ	Depreciation plus inventory rate	
	Investment parameters	
i, i_1, i_2	Money demand parameters	
h_1, h_2		
μ_0	Steady growth rate of money supply	
$\beta_w, \beta_{w_1}, \beta_{w_2}$	Wage adjustment speed parameters	
β_p	Price adjustment speed parameter	
$\hat{\beta_{\pi_1}}, \hat{\beta_{\pi_2}}$	Inflationary expectations adjustment speed	
	parameters	
β_v	Rate of employment adjustment parameter	
$\beta_{\overline{v}}$	NAIRU adjustment parameter	
β_{γ}^{v}	Trend growth adjustment parameter	
β_{v}^{γ} β_{v}	Demand adjustment parameter	
Py	Domand adjustment parameter	

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β_n^{d}		Desired inventory output ratio
β_n		Inventory or natural growth rate adjustment
		parameter
β_y^e		Demand expectations adjustment parameter
β_k		Accumulation regime parameter
κ_w, κ_p		Weights of short- and medium-run inflation
-		$(\kappa = (1 - \kappa_w \kappa_p)^{-1})$
α		Weight with respect to backward and forward
		looking expectations
τ, τ_c, τ_w		Tax rate (of capitalists and workers) $=$ const. (or
		$t^n = (T - rB/p)/K = \text{const.},$
		$t_c^n = (T_c - rB/p)/K = \text{const.})$
S _c		Savings ratio (out of profits and interest)
S _w		Savings ratio (out of wages, $= 0$ in this book)
μ_2		Fiscal policy parameter
C Mathematical notation		
ż		Time derivative of a variable x
ŵ		Growth rate of x
l', l_w		Total and partial derivatives
$y_w = y'$	$l(l)l_w$	Composite derivatives
r_0 , etc.		Steady state values (\bar{r} a parameter which may differ
		from r_0)
l = L/R	K, etc.	Real variables in intensive form

m = M/(pK), etc. Nominal variables in intensive form