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978-0-521-64337-5 - Mechanical Response of Polymers: An Introduction

Alan S. Wineman and K. R. Rajagopal

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## Mechanical Response of Polymers

The use of polymers has become so commonplace that it would be nearly impossible to pass a day without coming into contact with polymer-based products. For example, automobiles, household appliances, and electronic devices all make use of polymeric materials. As polymers are used increasingly in sophisticated industrial applications, it has become essential that mechanical engineers, who have traditionally focused on the behavior of metals, become as capable and adept with polymers.

This text provides a thorough introduction to the subject of polymers from a mechanical engineering perspective, treating stresses and deformations in structural components made of polymers. Three themes are developed. First, the authors discuss the time-dependent response of polymers and its implications for mechanical response. Secondly, descriptions of mechanical response are presented for both time-dependent and frequency-dependent material properties. Finally, the stress–strain–time relation is applied to determine stresses and deformations in structures.

With numerous examples and extensive illustrations, this book will help advanced undergraduate and graduate students, as well as practicing mechanical engineers, make optimal and effective use of polymeric materials.

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## AN INTRODUCTION

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UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.

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[www.cambridge.org](http://www.cambridge.org)

Information on this title: [www.cambridge.org/9780521643375](http://www.cambridge.org/9780521643375)

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First published 2000

*A catalogue record for this publication is available from the British Library*

*Library of Congress Cataloguing in Publication data*

Wineman, A. S.

Mechanical response of polymers : an introduction / Alan S.

Wineman, K. R. Rajagopal.

p. cm.

ISBN 0-521-64337-6 (hb). – ISBN 0-521-64409-7 (pb)

1. Polymers - Mechanical properties. I. Rajagopal, K. R.

(Kumbakonam Ramamani) II. Title.

TA455.P58W55 2000

620.1'9292-DC21

ISBN 978-0-521-64337-5 Hardback

ISBN 978-0-521-64409-9 Paperback

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Contents

<i>Preface</i>	<i>page</i>	ix
<b>1 Discussion of Response of a Viscoelastic Material</b>		<b>1</b>
1.1 Comparison with the Response of Classical Elastic and Classical Viscous Materials		1
1.2 Response of a Classical Elastic Solid		1
1.3 Response of a Classical Viscous Fluid		3
1.4 Comments on Material Microstructure		6
1.5 Response of a Viscoelastic Material		7
1.6 Typical Experimental Results		10
1.7 Material Properties		15
1.8 Linearity of Response		17
1.9 Aging Materials		21
PROBLEMS		24
<b>2 Constitutive Equations for One-Dimensional Response of Viscoelastic Materials: Mechanical Analogs</b>		<b>28</b>
2.1 Maxwell Model		28
2.2 Kelvin–Voigt Model		35
2.3 Three-Parameter Solid or Standard Linear Solid		40
2.4 <i>N</i> Maxwell Elements in Parallel		45
2.5 <i>N</i> Kelvin–Voigt Elements in Series		50
2.6 Relaxation and Creep Spectra		51
PROBLEMS		52
<b>3 Constitutive Equations for One-Dimensional Linear Response of a Viscoelastic Material</b>		<b>54</b>
3.1 General Restrictions on the Constitutive Equation		54
3.2 Linearity of Response: Superposition of Step Increments		58
3.3 Linearity of Response: Superposition of Pulses		62
3.4 Creep Forms of the Constitutive Equation		64
3.5 Summary of Forms of the Constitutive Equation		64
PROBLEMS		65

vi CONTENTS

<b>4</b>	<b>Some Features of the Linear Response of Viscoelastic Materials</b>	<b>67</b>
4.1	Relation Between Relaxation and Creep Functions	67
4.2	Characteristic Creep and Relaxation Times	72
4.3	Characteristic Relaxation, Creep, and Process Times	75
4.4	Some Examples Illustrating Implications of Fading Memory	80
	PROBLEMS	83
<b>5</b>	<b>Histories with Constant Strain or Stress Rates</b>	<b>88</b>
5.1	Constant Strain Rate Deformation	88
5.2	Constant Strain Rate Deformation and Recovery	92
5.3	Influence of Rise Time $T^*$ or Strain Rate $\alpha$	97
5.4	Work Done in a Constant Strain Rate Deformation and Recovery Test	98
5.5	Repeated Cycles	100
5.6	Step Strain and Recovery	100
5.7	Ramp Strain Approximation to a Step Strain History	103
5.8	Constant Stress Rate Loading and Unloading History	105
	PROBLEMS	109
<b>6</b>	<b>Sinusoidal Oscillations</b>	<b>115</b>
6.1	Sinusoidal Strain Histories	115
6.2	Sinusoidal Stress Histories	120
6.3	Relation Between $G^*(\omega)$ and $J^*(\omega)$	123
6.4	Work per Cycle During Sinusoidal Oscillations	124
6.5	Complex Viscosity	125
6.6	Examples of Calculation of $G^*(\omega)$ and $J^*(\omega)$	125
6.7	Low and High Frequency Limits of $G^*(\omega)$ and $J^*(\omega)$	130
6.8	Fourier Integral Theorem, Fourier Transform	135
6.9	Expressions for $G(t)$ and $J(t)$ in Terms of $G^*(\omega)$ and $J^*(\omega)$	137
6.10	Work Done During a General Deformation History	140
	PROBLEMS	142
<b>7</b>	<b>Constitutive Equation for Three-Dimensional Response of Linear Isotropic Viscoelastic Materials</b>	<b>148</b>
7.1	Introduction	148
7.2	Linearity	149
7.3	Uniaxial Extension, Poisson's Ratio, Isotropy	150
7.4	Uniaxial Extension Along the $x_2$ and $x_3$ Directions	153
7.5	Shear Response	153
7.6	Constitutive Equation for Three-Dimensional Response	154
7.7	A Relation Between Poisson's Ratio and the Extensional and Shear Material Properties	155

	CONTENTS	vii
7.8	Volumetric and Pure Shear Response	157
7.9	Stress in Terms of Strain History	160
7.10	Sinusoidal Oscillations	160
7.11	Laplace Transformation of the Constitutive Equations	163
7.12	Effect of Viscoelasticity on Principal Directions of Stress and Strain	164
7.13	Summary of Constitutive Relations	166
7.14	Relations for Special Cases of Volumetric Response	168
	PROBLEMS	168
<b>8</b>	<b>Axial Load, Bending, and Torsion</b>	<b>172</b>
8.1	Introduction	172
8.2	Structural Components Under Axial Load	172
8.3	Pure Bending of Viscoelastic Beams	176
8.4	Kinematics of Deformation	177
8.5	Constitutive Equation	179
8.6	Force Analysis	180
8.7	Stress, Bending Moment, and Curvature Relations	181
8.8	Deformation of Beams Subjected to Transverse Loads	182
8.9	Beams on Hard Supports, Correspondence Principle	185
8.10	Delayed Contact, Direct Method of Solution	190
8.11	Interaction of Polymeric Structural Components, a Viscoelastic Beam on a Viscoelastic Support	195
8.12	Extrusion of a Bar, Tracking the History of a Material Element	199
8.13	Traveling Concentrated Load on a Beam	202
8.14	Torsion of Circular Bars	207
8.15	Analysis of Viscoelastic Structures	212
	PROBLEMS	212
<b>9</b>	<b>Dynamics of Bodies with Viscoelastic Support</b>	<b>219</b>
9.1	Introduction	219
9.2	Comparison of Spring–Damper and Viscoelastic Supports	219
9.3	Forced Oscillations	221
9.4	Free Oscillations	228
	PROBLEM	231
<b>10</b>	<b>Boundary Value Problems for Linear Isotropic Viscoelastic Materials</b>	<b>232</b>
10.1	Introduction	232
10.2	Governing Equations	232
10.3	Correspondence Theorem for Quasi-Static Motion	234
10.4	Breakdown of the Correspondence Principle	237
10.5	Application of the Correspondence Principle: Pressure Loading of a Viscoelastic Cylinder	238

viii CONTENTS

10.6	Application of the Correspondence Principle: Torsion of Bars of Non-Circular Cross-Section	240
10.7	Direct Solution Methods	243
	PROBLEM	246
<b>11</b>	<b>Influence of Temperature</b>	<b>247</b>
11.1	Introduction	247
11.2	Thermally Induced Dimensional Changes	247
11.3	Mechanical Response at Different Temperatures	248
11.4	Time–Temperature Superposition	251
11.5	Experimental Support for Time–Temperature Superposition	253
11.6	General Comments	254
11.7	Effect of Temperature on Characteristic Stress Relaxation Time	256
11.8	Other Material Property Functions	257
11.9	Implications of Time–Temperature Superposition for Processes	258
11.10	Rate of Work	259
11.11	An Experimental Study	260
11.12	Extension to Time-Varying Temperature Histories	262
11.13	Constitutive Equation for Time-Varying Temperature Histories	268
11.14	Thermo-Viscoelastic Response of a Three Bar Structure: Formulation	269
11.15	Thermo-Viscoelastic Response of a Three Bar Structure: Development of Frozen-in Deformation	271
11.16	Thermo-Viscoelastic Response of a Three Bar Structure: Frozen-in Forces	276
11.17	Thermo-Viscoelastic Response of a Three Bar Structure: Cooling Induced Warping	285
11.18	Thermo-Viscoelastic Response of a Three Bar Structure: Comments	292
<b>Appendix A</b>	<b>Operator Notation for Time Derivatives</b>	293
<b>Appendix B</b>	<b>Laplace Transform</b>	295
<b>Appendix C</b>	<b>Volterra Integral Equations</b>	299
<b>Appendix D</b>	<b>Formal Manipulation Methods</b>	305
<b>Appendix E</b>	<b>Field Equations in Cartesian and Cylindrical Coordinates</b>	308
	<i>References</i>	311
	<i>Index</i>	313



## Preface

During the past several decades, the use of polymers has become so commonplace that it would be nearly impossible to pass a day without coming into contact with a polymer-based product. The automobile, aerospace, appliance, and electronics industries use more polymers, by weight, than all the metals put together. Despite the increased use of polymers in engineering products, the stresses and deformations that these polymers undergo are generally determined as if the behavior is that of a classical elastic material. This is in part due to the traditional mechanical engineering education that has emphasized the behavior of metals rather than polymers. Polymeric materials have been studied more within the context of understanding their material properties by chemists, chemical engineers, and material scientists, rather than with a view toward understanding the stresses and deformations in structural components. However, rapid changes are occurring in current engineering practices involving polymers from the perspective of mechanical engineering. Polymers are being considered for increasingly sophisticated industrial applications. The effective and efficient use of these materials requires an understanding of their time-dependent response and energy dissipation properties. Thus, it is essential that the mechanical engineering education be expanded so that students become as capable and adept with polymers as they are with metals in determining stresses and deformations.

The authors have spent many years teaching engineering students about stresses and deformations in metallic structural components, on the one hand, and carrying out research in the mechanical response of polymers, on the other. They have also taught graduate courses in the viscoelastic response of polymers. In recognition of the increasing need that mechanical engineers be educated in the mechanics of polymers, we have used our teaching and research expertise to develop a book which is intended to serve as both a textbook and an engineering reference.

This book was prepared with several purposes in mind. The first is to instill a solid grasp of the phenomena of stress relaxation and creep in polymers, and their consequences for mechanical response. This is achieved by developing the stress-strain-time relation for the response of polymers, and then using it to explore characteristic material and process times, energy dissipation, and the effects of fading memory. The second purpose has to do with the mechanical properties of polymers. An engineer should be familiar with descriptions of mechanical response in terms of both time-dependent and frequency-dependent material properties. Thus, we develop the background necessary for this purpose. In particular, we use the stress-strain-time

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[More information](#)**x PREFACE**

relation to interpret the response in fundamental experiments, and then to develop relations between material properties in their creep, stress relaxation, time- and frequency-based forms.

Our third purpose is to show how the stress–strain–time relation is used to determine stresses and deformations in structures. We begin with a thorough treatment of polymeric beams and bars under torsion. Examples are presented which not only illustrate different aspects of the consequences of creep and stress relaxation, but also illustrate different methods for analyzing structural problems. In effect, given a textbook for an introductory solid mechanics course in the mechanical engineering curriculum, it is shown that each example can be defined for polymeric materials, and can be treated by the methods presented here. The same approach is then used to determine stresses and deformations in bodies with more complicated shapes and loadings. Instead of examples of beams and torsion bars, examples are drawn from the classical linear theory of elasticity.

To come to grips with viscoelasticity it is helpful to have a clear grasp of the response of elastic solids and viscous fluids. Here, we shall concentrate our efforts on describing the linear response of viscoelastic materials that stems from the material responding both as a linear elastic solid and a linear viscous fluid. While the linear viscous fluid is a proper model in its own right, the linear elastic solid model (linearized elastic solid to be more precise) is an approximation that has served as an indispensable model in virtue of its usefulness and applicability. The same can be said of the linear viscoelastic model; while it is not frame-invariant, its ease of use and the conformity of the predictions of the model with available experimental data have rendered it an essential tool to the practicing engineer.

It is our goal that the treatment of material modeling, formulation of the basic issues in mechanics, and methods for the calculation and solution of engineering problems presented here will enable the student or practicing engineer to make optimal and effective use of polymeric materials.

A word of caution to the reader about our notation: we follow the style of Timoshenko. While the equations are numbered sequentially during the development of the theory in each chapter, we assign equation numbers independently for the solution of the special problems that are treated in each chapter.

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