CHAPTER ONE

Discussion of Response of a Viscoelastic Material

1.1 Comparison with the Response of Classical Elastic and Classical Viscous Materials

As the word "viscoelasticity" suggests, the kind of mechanical response under consideration involves aspects of familiar types of material response – those of elastic solids and viscous fluids. In order to compare viscoelastic response with that of elastic solids and viscous fluids it is necessary to account for time as an explicit physical parameter. This approach is introduced by first discussing the response of linear elastic solids and linear viscous (Newtonian) fluids using time as an explicit parameter. This will set the stage for a similar discussion for viscoelastic materials.

Consider one-dimensional stress–strain states, the material being either in uniaxial extension or in simple shear. The material is in an undeformed state for times $t < 0$. As Figure 1.1 shows, σ denotes either a normal or a shear stress and ε denotes a normal strain or a shear strain. Figure 1.1 shows an extension state and a shear state at a typical time t. The mechanical response is discussed by considering variations of stress and strain with time, by means of plots of stress versus time and strain versus time. It is then possible to determine the conditions under which it is reasonable to eliminate time as an explicit parameter and plot stress versus strain. Attention will be confined to materials which are initially undeformed and unstressed, that is, $\sigma(t) = 0$ and $\varepsilon(t) = 0$ for $t < 0$.

1.2 Response of a Classical Elastic Solid

The one-dimensional mechanical response of a linear elastic solid is often represented by a mechanical analog – a linear spring, as shown in Figure 1.2. The response of the spring is characterized by the force–deformation relation $F = k\Delta$, in which F is the force, Δ is the elongation, and k is the spring constant. This relation is assumed to be valid under all conditions. The purpose of the mechanical analog is as an aid in visualizing the material response described below. The mechanical analog is also used in developing a stress–strain relation. This is done by associating force F with the stress σ and elongation Δ with the strain ε.

A number of stress–time and strain–time experiments are now considered which lead to the conclusion that $\sigma(t) = E\varepsilon(t)$, where E is the Young's modulus. In the following discussion, it is assumed that the specimen has no mass, so that there are no inertial effects.

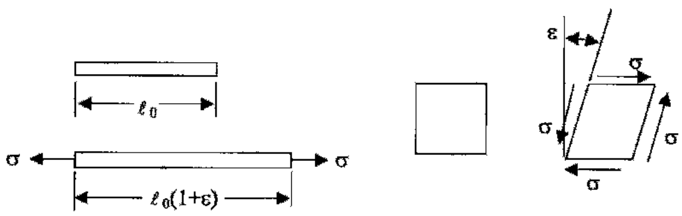

Figure 1.1. One-dimensional stress-strain states. Left: uniaxial extension. Right: simple shear.

STRESS CONTROL TEST, RESPONSE TO STEP STRESS

If the stress is instantaneously increased to σ_0 at $t = 0$ and then held constant, an elastic solid instantaneously deforms to a fixed state at some strain ε_0 which does not vary with t. This is shown in curves (a) of Figure 1.3.

RELEASE OF STRESS

If the stress is instantaneously removed at time t_2, the strain instantaneously returns to zero. That is, the material instantaneously and completely recovers its original shape (springiness).

STRAIN CONTROL TEST, RESPONSE TO STEP STRAIN

If the strain is suddenly increased to ε_0 at $t = 0$ and then held constant, the stress instantaneously increases to σ_0 and stays constant.

EFFECT OF DIFFERENT HISTORIES

If strain ε_0 is reached at time t_1 by distinct strain histories (b) and (c) as well as (a), as shown in Figure 1.3, the same stress is required at time t_1 and is independent of the strain rate or how the value at time t_1 is reached. The same statements would hold if stress σ_0 were reached at time t_1 by different stress histories.

The above behavior suggests that for each value of strain ε there corresponds a unique value of stress σ. Whenever the strain is ε_0, the corresponding stress at that instant is always σ_0. It is then possible to eliminate t between the ε–t and the σ–t plots and produce the unique stress–strain plot shown in Figure 1.4. The stress–strain relation becomes $\sigma(t) = E\varepsilon(t)$.

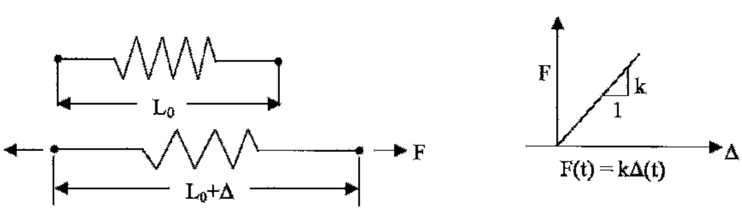

Figure 1.2. Linear elastic solid. Left: mechanical analog – linear spring. Right: force–elongation relation for the spring.

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ENERGY DISSIPATION

If an elastic specimen is deformed and then returned to its original shape, the work done is zero. No energy is dissipated.

EFFECT OF SINUSOIDAL OSCILLATIONS

If $\varepsilon(t) = \varepsilon_0 \sin \omega t$ then $\sigma(t) = \sigma_0 \sin \omega t$. Stress and strain are in phase and their amplitude ratio does not vary with ω, as shown in Figure 1.5.

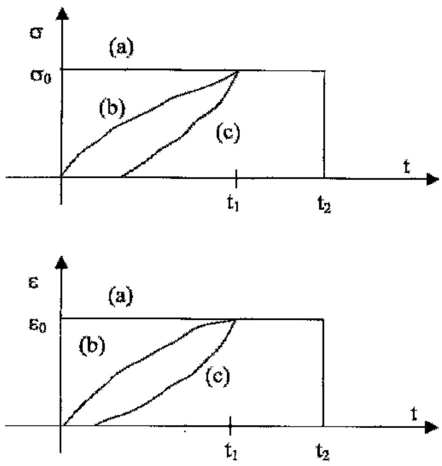


Figure 1.3. Mechanical response of a linear elastic solid. Top: several stress histories. Bottom: corresponding strain histories.

1.3 Response of a Classical Viscous Fluid

The one-dimensional mechanical response of a linear viscous fluid is often represented by a mechanical analog, the viscous damper (a piston in an oil bath in a cylinder) shown in Figure 1.6. The response of the viscous damper is characterized by a relation between force F and elongation rate, denoted by $d\Delta/dt = \dot{\Delta}$. The force–elongation rate relation for a linear viscous damper is then $F = c\dot{\Delta}$, where c is the viscosity. This relation is assumed to be valid under all conditions. This suggests that the linear viscous fluid is described by a relation between stress and strain rate of the form $\sigma = \mu\dot{\varepsilon}$, in which μ represents a fluid material property, its viscosity. We now consider a number of stress–time and strain–time experiments which enable us to see the implications of this relation.

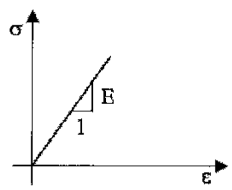
STRESS CONTROL TEST, RESPONSE TO STEP STRESS

If the stress is increased to σ_0 at time zero and then held constant, a linear viscous fluid does not reach a fixed deformed state. There is continued straining in time, that is, the material flows. At constant stress σ_0 the strain rate $\dot{\varepsilon}$ becomes constant, as shown in curves (a) of Figure 1.7.

RELEASE OF STRESS

If the stress σ is released at time t_2, the strain ε does not change. No strain is recovered. The strain stays constant, and the strain rate reduces instantaneously to zero. There is no tendency for the material to return to a previous shape.

Figure 1.4. Stress–strain plot for a linear elastic solid.



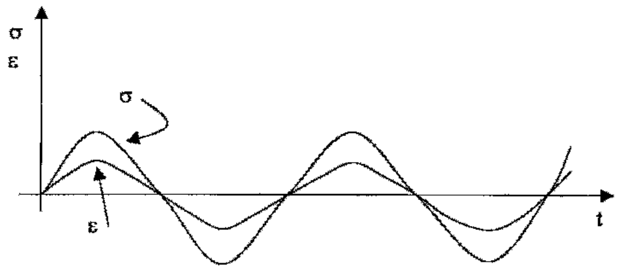


Figure 1.5. Sinusoidal stress and strain histories for a linear elastic solid.

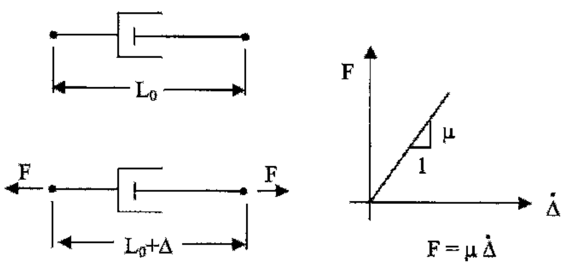


Figure 1.6. Linear viscous damper. Left: mechanical analog – linear viscous damper. Right: force–elongation rate relation.

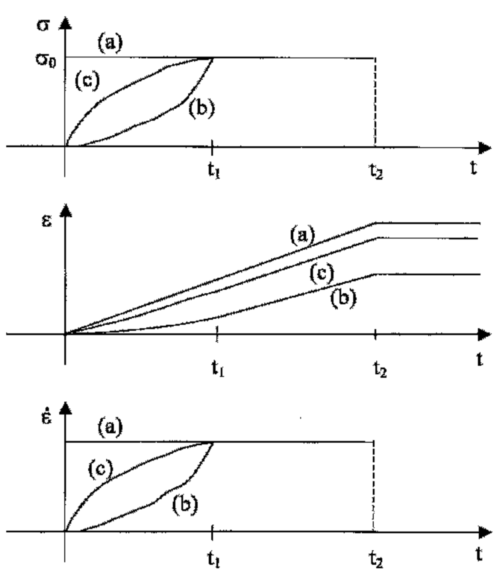


Figure 1.7. Mechanical response of a linear viscous fluid. Stress histories (top). Corresponding strain (middle) and strain rate (bottom) histories.

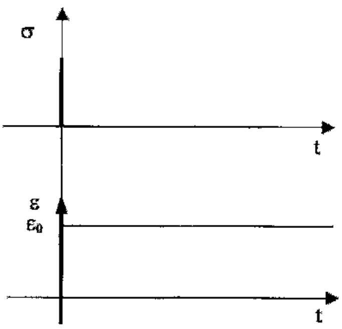


Figure 1.8. Linear viscous fluid: response to a step strain test.

STRAIN CONTROL TEST, RESPONSE TO STEP STRAIN

Suppose the strain is increased instantaneously to ε_0 and then held fixed. A very large stress is needed to produce the sudden shape change. If the strain is held constant for $t > 0$, the stress required to maintain this strain reduces immediately to zero. The stress is then zero for all times $t > 0$. This is shown in Figure 1.8.

EFFECT OF DIFFERENT STRAIN HISTORIES

Suppose that the strain rate $\dot{\varepsilon}$ is reached at time t_1 by strain sequences (b), (c) as well as (a), as shown in Figure 1.7. The same stress is required for each sequence at time t_1. Note that for each strain sequence, there is a different amount of strain at time t_1. In general, there can be any value of strain at time t_1 corresponding to the stress at time t_1. On the other hand, there appears to be only one value of strain rate at time t_1 which corresponds to this stress. We conclude that the stress at time t_1 depends neither on the strain at time t_1 nor on the previous sequence of strain values. It depends only on the strain rate at time t_1.

If time t is eliminated between the σ–t and the ε–t graphs, the single graph in Figure 1.9 is produced which is described by the relation $\sigma(t) = \mu\dot{\varepsilon}(t)$.

ENERGY DISSIPATION

If the specimen is deformed from its original shape and then restored to that shape, the work is completely converted to thermal energy.

EFFECT OF SINUSOIDAL OSCILLATION

If $\varepsilon(t) = \varepsilon_0 \sin \omega t$ then $\sigma(t) = \mu\varepsilon_0\omega \sin(\omega t + \pi/2)$. The stress and strain are 90° out of phase and their amplitude ratio varies with frequency (see Figure 1.10).

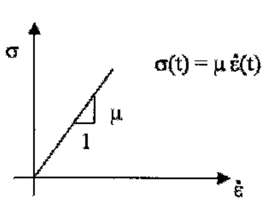


Figure 1.9. Stress–strain rate plot for a linear viscous fluid.

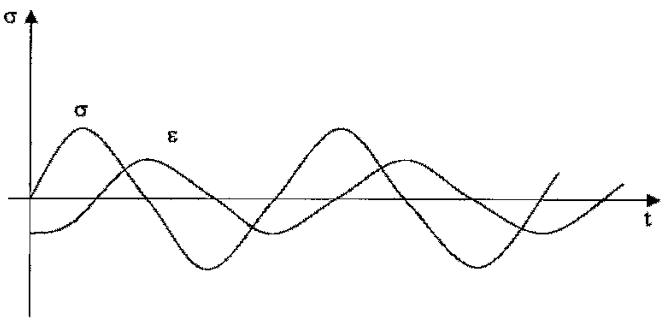


Figure 1.10. Sinusoidal stress and strain histories for a linear viscous fluid.

1.4 Comments on Material Microstructure

When an external force is applied to a piece of material, internal forces are produced. The material develops the ability to produce this internal force by distortion of its underlying physical structure. For example, metals have an atomic crystalline structure, with strong interatomic forces. Elastic response is due to large cohesive interatomic forces brought into play by small deformation of the crystalline structure. Fluids such as air and water are composed of molecules which exert weak attractive forces on each other. Internal forces are built up by the continuous movement of particles with respect to each other, which is seen as fluid flow.

Viscoelastic behavior involves qualities of both elastic solid and viscous fluid like response. This is due to the nature of the material microstructure. Viscoelastic response occurs in a variety of materials, such as soils, concrete, cartilage, biological tissue, and polymers. Soils and cartilage can be thought of as materials consisting of a porous solid material filled with fluid. Time-dependent response is due to the flow of the fluid in the pores as well as the distortion of the porous solid.

Viscoelastic phenomena in polymers and biological materials appear to be related to the movement of flexible thread-like long chain molecules, called macromolecules. They span an average volume which is much greater than atomic dimensions. In order to develop internal forces, these macromolecules must undergo changes in configuration. These shape changes involve molecular rearrangements on various scales (Figure 1.11):

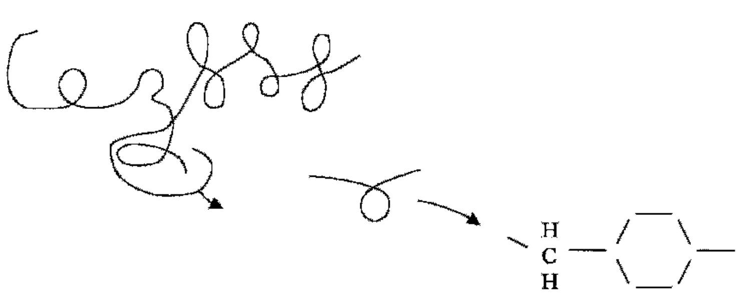


Figure 1.11. Scales of structure of macromolecules.

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1. gross long-range contour rearrangements which are slowly achieved,
2. rearrangements on a more local level, which are more rapidly achieved,
3. reorientation of bonds on the chain backbone on the atomic scale.

In other words, rearrangements occur on a broad range of time scales.

The distinction between solid and fluid response is related to the cross-linking of macromolecules (see Figure 1.12). If macromolecules are cross-linked, that is, attached to one other, they form a network in which there is a maximum possible amount of deformation. If the stress is removed, the intermolecular force caused by cross-linking causes the network to return to its original configuration. If the macromolecules are not cross-linked, they can slide over one another. Under constant stress, they continue to slid over one another and flow. If the stress is removed, there are no intermolecular forces to cause the macromolecules to return to their original arrangement.

1.5 Response of a Viscoelastic Material

STRESS CONTROL TEST, RESPONSE TO STEP STRESS

Let the stress be instantaneously increased to σ_0 at $t = 0$ and then be held fixed. The typical response, as shown in Figure 1.13, consists of:

1. an instantaneous increase in strain *OA*,
2. continued straining in time at a non-constant rate, *ABC*.

The strain *OA* is thought of as an instantaneous elastic response. The strain sequence *ABC* is a combination of elastic and viscous effects. If the material is solid-like, the strain asymptotically approaches a constant value ε_0. If the material is fluid-like, the strain rate $\dot{\varepsilon}$ asymptotically approaches a constant value.

RELEASE OF STRESS

If the stress is reduced to zero at time t_1, there is typically:

1. some instantaneous strain recovery *CD*,
2. delayed recovery *DEF*.

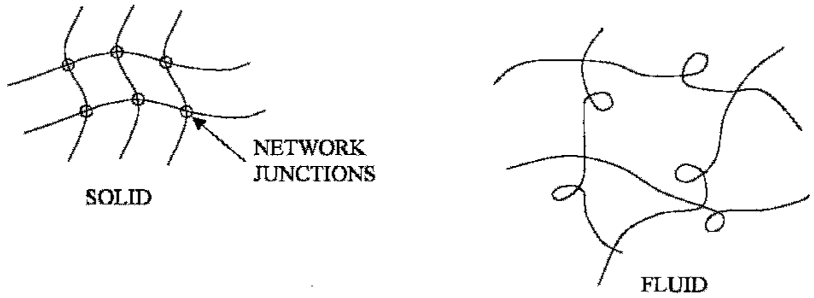

Figure 1.12. Solid: macromolecular network with junctions. Fluid: macromolecular network with no junctions.

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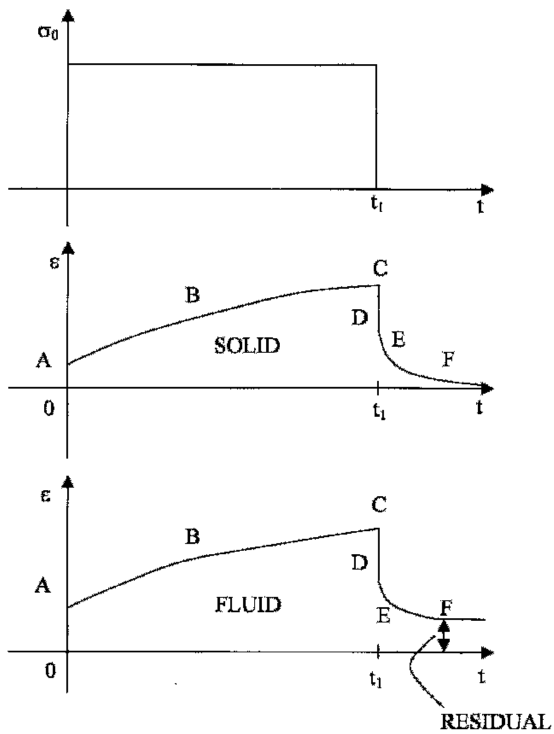

Figure 1.13. Top: step stress load and unload history. Mechanical response of a viscoelastic material; middle: solid; bottom: fluid.

If the material is solid-like, generally all the strain is recovered. If the material is fluid-like, only part of the strain is recovered, and there is some residual strain. The continued straining or flow under constant stress is called *creep*. (Elastic solids do not creep, linear viscous or Newtonian fluids have a constant rate of creep.)

STRAIN CONTROL TEST, RESPONSE TO STEP STRAIN

Suppose the strain ε is instantaneously increased to ε_0 at $t = 0$ and then held fixed. Then in the typical response shown in Figure 1.14:

1. the stress instantaneously jumps to some value OA,
2. the stress required to maintain the constant strain ε_0 decreases gradually with time, ABC.

For solid-like materials, the stress decreases asymptotically to some non-zero residual value. For fluid-like materials, the stress decreases asymptotically to zero. No stress is then required to maintain the material in its new shape. The decrease of stress at constant strain is called *stress relaxation*. (Elastic solids show no stress relaxation, linear viscous or Newtonian fluids show instantaneous stress relaxation.)

Some insight into the difference in the behavior between viscoelastic solids and fluids can be obtained by considering the discussion on molecular structure in Section 1.4. Any deformation of the material distorts the underlying molecular structure from the original configuration and gives rise to intermolecular forces. Some reorientation of this structure occurs which allows the intermolecular forces to reduce or relax. For viscoelastic solids, because of the cross-linking, this reorientation is limited. The material will always require some force to maintain it in a distorted state. When the external force is removed, the cross-linking causes the network to return to its original shape so that internal forces can reduce to zero. For viscoelastic fluids, there is no cross-linking. When deformation occurs, there can be reorientation of the macromolecules as well as relative motion between them. The intermolecular forces can eventually reduce to zero, and no external force will be required to maintain the new state. There will then be nothing to cause the system to return to its initial state. We see then that a viscoelastic solid has a preferred reference state toward which it tends on removal of the load, while viscoelastic fluids have none. For fluids, any new state can become a reference state.

EFFECT OF DIFFERENT HISTORIES

Suppose the material is subjected to several stress "histories" each arriving at the same value σ_0 at time t_1, as shown in Figure 1.15. The corresponding strains at time t_1 will all differ. The strain ε at time t_1 depends on the entire sequence of stresses up to time t_1, that is, the stress history.

Suppose that time t is eliminated between the σ–t graph and the ε–t graph for the corresponding stress–strain histories. A different stress–strain graph is constructed for each case. There is no longer a unique curve relating to σ and ε. Note the absurdity of the σ_0–ε_0 graph. A similar statement can be made for σ and the strain rate $\dot{\varepsilon}$.

This points out one of the main goals of viscoelasticity, to determine a physically reasonable relation involving stress, strain, and time so that we can find $\sigma(t)$ if the sequence of values of ε up to time t are known, or $\varepsilon(t)$ if the sequence of values of σ up to time t are known. In other terms, we can find $\sigma(t)$ knowing the preceding strain history, or compute $\varepsilon(t)$ knowing the preceding stress history.

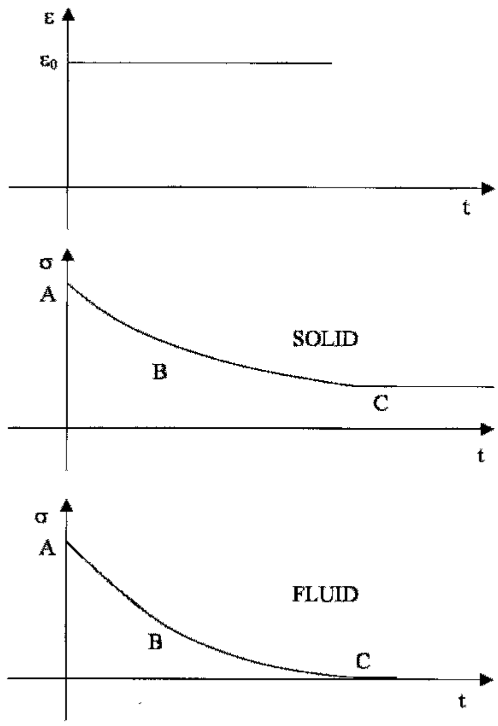

Figure 1.14. Top: step strain history. Mechanical response of a viscoelastic material; middle: solid; bottom: fluid.

ENERGY DISSIPATION

If the specimen is deformed from its original shape and then returned to that shape, some but not all of the work done is converted to heat energy.

SINUSOIDAL OSCILLATIONS

If $\varepsilon(t) = \varepsilon_0 \sin \omega t$, then $\sigma(t) = \varepsilon_0 G(\omega) \sin(\omega t + \delta(\omega))$. The amplitude ratio and the phase lag both vary with the frequency.

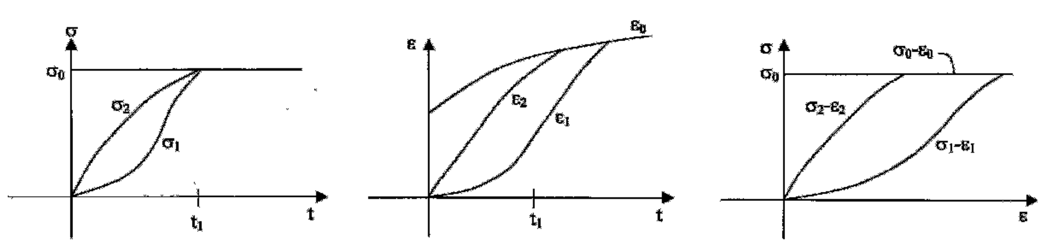

Figure 1.15. Left: several stress histories. Middle: corresponding strain histories. Right: corresponding stress–strain plots.

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MECHANICAL ANALOG

One-dimensional viscoelastic response can be interpreted as a combination of elastic solid and viscous fluid responses. It is natural to attempt to represent viscoelastic properties by combining the mechanical analogs for these simpler responses into more complicated mechanisms. In fact, viscoelastic mechanical response is usually simulated by different spring–viscous damper combinations as shown in Figure 1.16. These models are intended to be used to simulate only *macroscopic* behavior, as might be observed in a finite-sized specimen. They do not necessarily provide insight into the molecular basis for viscoelastic response. They also cannot be thought of as providing a rigorous mathematical foundation for the study of the response of viscoelastic materials.

In the next chapter, we will show how these mechanical analogs lead to a relation between the stress history and the strain history for one-dimensional response under certain conditions. It turns out that there is no unique combination of springs and viscous dampers which will simulate a specific viscoelastic response. Different combinations of springs and viscous dampers can provide the same simulation.

1.6 Typical Experimental Results

Typical experimental data are shown in Figures 1.17a–1.17g. The extensional creep strain response to various step stress levels is shown in Figure 1.17a for rayon laminate (Findley and Worley 1948: Figure 27). Note that the strains do not exceed 2.4 percent and the time scale is measured in hundreds of hours. Also note that if the stress level σ_0 is doubled to $2\sigma_0$ or tripled to $3\sigma_0$, the corresponding creep strain is approximately doubled or tripled.

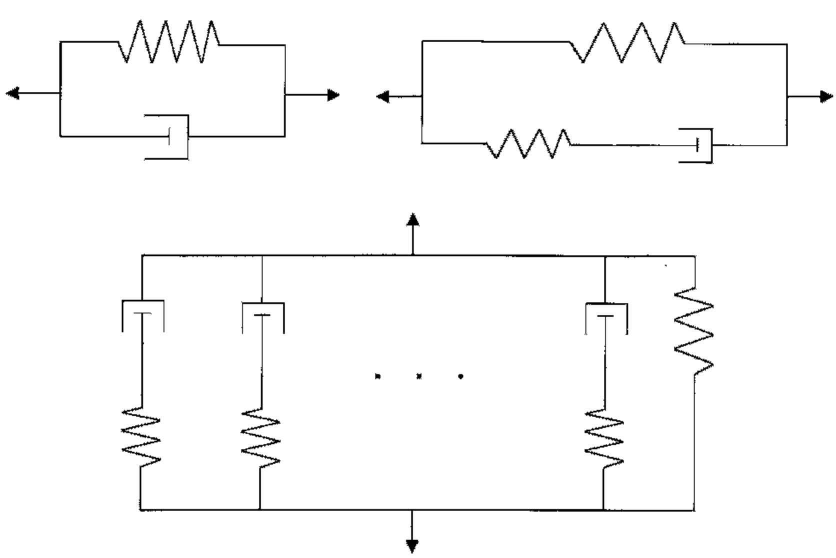

Figure 1.16. Some mechanical analogs used to represent viscoelastic response.