The Geometry of Ecological Interactions:
Simplifying Spatial Complexity

The concept of invasion fitness, defined as the initial per capita growth rate of a rare mutant in the environment set by the resident types, lies at the heart of adaptive dynamics theory. Current research seeks to provide techniques for determining measures of invasion fitness in different ecological settings. These measures are well established for populations without spatial structure. However, for spatially heterogeneous populations, the patterns that typically arise from short-range ecological interactions often decisively influence invasion fitness. This first volume of the Cambridge Studies in Adaptive Dynamics provides systematic introductions to the modern tools available for describing ecological and evolutionary change in spatially structured populations.

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Cambridge Studies in Adaptive Dynamics

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The modern synthesis of the first half of the twentieth century reconciled Darwinian selection with Mendelian genetics. However, it failed to incorporate ecology and hence did not develop into a predictive theory of long-term evolution. It was only in the 1970s that evolutionary game theory allowed the consequences of frequency-dependent ecological interactions to be analyzed. Adaptive Dynamics extends evolutionary game theory by describing the dynamics of adaptive trait substitutions and by analyzing the evolutionary implications of complex ecological settings.

The Cambridge Studies in Adaptive Dynamics highlight these novel concepts and techniques for ecological and evolutionary research. The series is designed to help graduate students and researchers to use the new methods for their own studies. Volumes in the series provide coverage of both empirical observations and theoretical insights, offering natural points of departure for various groups of readers. If you would like to contribute a book to the series, please contact Cambridge University Press or the series editors.

1. *The Geometry of Ecological Interactions: Simplifying Spatial Complexity*
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The Geometry of Ecological Interactions: 
Simplifying Spatial Complexity

Edited by 
Ulf Dieckmann, Richard Law, and Johan A.J. Metz
Contents

Contributing Authors

1 Introduction
Richard Law, Ulf Dieckmann, and Johan A.J. Metz

A Empirical and Statistical Background:
A Plant Ecological Perspective

2 A Neighborhood View of Interactions among Individual Plants
Peter Stoll and Jacob Weiner
2.1 Introduction .................................................. 11
2.2 Competition Mechanisms .................................. 12
2.3 Moving from the Population to the Individual Level .... 18
2.4 What is a Plant’s Neighborhood? .......................... 19
2.5 Challenges for a Neighborhood Perspective of
Plant Interactions ............................................. 24
2.6 Suggestions for Modelers ................................. 26

3 Spatial Interactions among Grassland Plant Populations
Jonathan Silvertown and J. Bastow Wilson
3.1 Introduction ................................................... 28
3.2 Methods for Measuring Competition in the Field ....... 29
3.3 Results of Field Experiments ............................. 32
3.4 Competition Matrices ....................................... 38
3.5 Community Consequences of Spatial Interactions .... 42
3.6 Concluding Comments ...................................... 46

4 Spatio-temporal Patterns in Grassland Communities
Tomáš Herben, Heinjo J. During, and Richard Law
4.1 Introduction .................................................... 48
4.2 Spatio-temporal Patterns in Plant Communities ....... 48
4.3 Externally versus Internally Generated Spatial Patterns .. 52
4.4 Concepts in Spatio-temporal Processes in
Plant Communities ........................................... 54
4.5 Ergodic and Non-ergodic Communities ................... 60
4.6 Concluding Comments ...................................... 64
viii

5 Statistical Modeling and Analysis of Spatial Patterns 65
   David R. Cox, Valerie Isham, and Paul Northrop
   5.1 Introduction ........................................... 65
   5.2 Descriptive Analysis ................................. 66
   5.3 Stochastic Models .................................. 70
   5.4 Model Fitting ........................................ 80
   5.5 Concluding Comments ............................... 88

8 When the Mean-field Approximation Breaks Down 89

6 Grid-based Models as Tools for Ecological Research 94
   Christian Wissel
   6.1 Introduction ......................................... 94
   6.2 Grid-based Simulation Models .................. 95
   6.3 Spread and Control of Rabies .................... 97
   6.4 Dynamics of a Dwarf Shrub Community .......... 104
   6.5 A Generic Forest Fire Model .................... 109
   6.6 Concluding Comments ............................. 114

7 Coexistence of Replicators in Prebiotic Evolution 116
   Tamás Czárán and Éörs Szathmáry
   7.1 Introduction ......................................... 116
   7.2 Metabolic Replication: A Cellular Automaton Model .. 119
   7.3 The Phenomenology of Coexistence ............... 123
   7.4 Spatial Pattern and the “Advantage of the Rare” Effect .. 127
   7.5 Resistance to Parasites and the Evolution of Community Size .......................... 129
   7.6 Toward a Dynamical Theory of Surface Metabolism ... 133

8 Games on Grids 135
   Martin A. Nowak and Karl Sigmund
   8.1 Introduction ......................................... 135
   8.2 One-round Games .................................. 137
   8.3 Repeated Games .................................... 145
   8.4 Extensions and Related Work .................... 149
   8.5 Concluding Comments ............................. 150
9 The Interplay between Reaction and Diffusion
Mikael B. Cronhjort
9.1 Introduction ............................................. 151
9.2 The Models: Cellular Automata versus Partial Differential Equations .................. 153
9.3 Spiral and Scroll Ring Patterns ......................... 159
9.4 Cluster Dynamics ...................................... 163
9.5 Concluding Comments .................................. 169

10 Spirals and Spots: Novel Evolutionary Phenomena through Spatial Self-structuring
Maarten C. Boerlijst
10.1 Introduction .......................................... 171
10.2 A Spatial Hypercycle Model ......................... 173
10.3 Spirals and Spots ...................................... 174
10.4 Local versus Global Extinction ...................... 175
10.5 Resistance to Parasites ............................... 178
10.6 Concluding Comments ............................... 180

11 The Role of Space in Reducing Predator–Prey Cycles
Vincent A.A. Jansen and André M. de Roos
11.1 Introduction ........................................ 183
11.2 Individual-based Predator–Prey Models ............ 184
11.3 A Deterministic Model of Two Coupled Local Populations .............................. 187
11.4 Larger Spatial Domains ............................... 193
11.5 The Spatial Rosenzweig–MacArthur Model .......... 196
11.6 Concluding Comments ............................... 199
11.7 Stability Analysis of a Multi-patch System ........ 200

C Simplifying Spatial Complexity: Examples

12 Spatial Scales and Low-dimensional Deterministic Dynamics
Howard B. Wilson and Matthew J. Keeling
12.1 Introduction ........................................ 209
12.2 Two Models from Evolutionary Ecology ............ 210
12.3 Identifying Spatial Scales ............................ 213
12.4 Dynamics, Determinism, and Dimensionality ........ 219
12.5 Concluding Comments ................................ 225
12.A Singular Value Decomposition ...................... 225
## 13 Lattice Models and Pair Approximation in Ecology

Yoh Iwasa

13.1 Introduction ................................................. 227
13.2 Plants Reproducing by Seed and Clonal Growth ........ 228
13.3 Forest Gaps ................................................. 236
13.4 Colicin-producing and Colicin-sensitive Bacteria ....... 243
13.5 Limitations, Extensions, and Further Applications ... 247

## 14 Moment Approximations of Individual-based Models

Richard Law and Ulf Dieckmann

14.1 Introduction ................................................. 252
14.2 Spatial Patterns and Spatial Moments .................. 253
14.3 Extracting the Ecological Signal from Stochastic Realizations ................................................. 256
14.4 Qualitative Dependencies in a Spatial Logistic Equation . 261
14.5 Exploration of Parameter Space .......................... 267
14.6 Concluding Comments ...................................... 269

## 15 Evolutionary Dynamics in Spatial Host–Parasite Systems

Matthew J. Keeling

15.1 Introduction ................................................. 271
15.2 Dynamics of the Spatial Host–Parasite Model .......... 272
15.3 A Difference Equation for the Dynamics of Local Configurations ................................................. 279
15.4 Evolution to Critical Transmissibility .................. 282
15.5 Concluding Comments ...................................... 288
15.6 A Mathematical Specification of the PATCH Model .... 289

## 16 Foci, Small and Large: A Specific Class of Biological Invasion

Jan-Carel Zadoks

16.1 Introduction ................................................. 292
16.2 Epidemic Orders ........................................... 293
16.3 A Theory of Foci ........................................... 298
16.4 Generalizations ............................................. 312
16.5 Concluding Comments ...................................... 315
16.6 Quantitative Applications of Models for Spatial Population Expansion (by Johan A.J. Metz) .......... 315
17 Wave Patterns in Spatial Games and the Evolution of Cooperation 318
Régis Ferrière and Richard E. Michod
17.1 Introduction .................................................. 318
17.2 Invasion in Time- and Space-continuous Games ........ 319
17.3 Invasion of Tit For Tat in Games with Time-limited Memory ............................................. 323
17.4 Invasion of Tit For Tat in Games with Space-limited Memory ............................................. 329
17.5 Concluding Comments ........................................... 332

D Simplifying Spatial Complexity: Techniques 337
18 Pair Approximations for Lattice-based Ecological Models 341
Kazunori Satō and Yoh Iwasa
18.1 Introduction .................................................. 341
18.2 Pair Approximation ........................................... 344
18.3 Improved Pair Approximation ......................... 349
18.4 Improved Pair Approximation with Variable Discounting . 355
18.5 Concluding Comments ........................................... 357

19 Pair Approximations for Different Spatial Geometries 359
Marius van Baalen
19.1 Introduction .................................................. 359
19.2 The Dynamics of Pair Events ......................... 364
19.3 Average Event Rates ........................................... 368
19.4 Pair Approximations for Special Geometries ........ 372
19.5 Pair Approximations versus Explicit Simulations .... 379
19.6 Invasion Dynamics ........................................... 382
19.7 Concluding Comments ........................................... 385

20 Moment Methods for Ecological Processes in Continuous Space 388
Benjamin M. Bolker, Stephen W. Pacala, and Simon A. Levin
20.1 Introduction .................................................. 388
20.2 Moment Methods ........................................... 389
20.3 A Spatial Logistic Model ........................................... 391
20.4 A Spatial Competition Model ......................... 400
20.5 Extensions and Related Work ......................... 403
20.6 Concluding Comments ........................................... 405
20.A Mean Equation ................................. 406
20.B Covariance Equation ......................... 408
20.C Analyzing the One-species System .......... 409
20.D Analyzing the Two-species System .......... 410

21 Relaxation Projections and the Method of Moments 412
Ulf Dieckmann and Richard Law
21.1 Introduction ................................... 412
21.2 Individual-based Dynamics in Continuous Space .... 418
21.3 Dynamics of Correlation Densities .......... 425
21.4 Moment Closures and their Performance .......... 438
21.5 Further Developments and Extensions .......... 447
21.A Derivation of Pair Dynamics ............... 452

22 Methods for Reaction–Diffusion Models 456
Vivian Hutson and Glenn T. Vickers
22.1 Introduction .................................. 456
22.2 Continuous Models .............................. 459
22.3 Linearized Stability and the Turing Bifurcation .... 466
22.4 Comparison Methods .......................... 471
22.5 Traveling Waves ............................... 475
22.6 The Evolution of Diffusion .................... 479
22.7 Concluding Comments ...................... 481

23 The Dynamics of Invasion Waves 482
Johan A.J. Metz, Denis Mollison, and Frank van den Bosch
23.1 Introduction .................................. 482
23.2 Relative Scales of the Process Components .......... 483
23.3 Independent Spread in Homogeneous Space: 
   A Natural Gauging Point ..................... 485
23.4 Complications ................................ 497
23.5 The Link with Reaction–Diffusion Models ....... 504
23.6 Dispersal on Reaction–Diffusion Models ......... 507
23.7 Concluding Comments ...................... 512

24 Epilogue 513
Johan A.J. Metz, Ulf Dieckmann, and Richard Law

References 517

Index 553
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