## Metal Forming Analysis

The introduction of numerical methods, particularly finite-element (FE) analysis, represents a significant advance in metal forming operations. Numerical methods are used increasingly to optimize product design and deal with problems in metal forging, rolling, and extrusion processes. *Metal Forming Analysis* describes the latest and most important numerical techniques for simulating metal forming operations. The first part of the book describes principles and procedures and includes numerous examples and worked problems. The remaining chapters focus on applications of numerical analysis to specific forming operations. Most of these results are drawn from the authors' research in the areas of metal testing, sheet-metal forming, forging, extrusion, and similar operations. Sufficient information is presented so that readers can understand the nonlinear finite-element method as applied to forming problems without a prior background in structural finite-element analysis. Graduate students, researchers, and practicing engineers will welcome this thorough reference to state-of-the-art numerical methods used in metal forming analyses.

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# METAL FORMING ANALYSIS

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# Preface

*Metal Forming Analysis* has two purposes: (a) to acquaint the advanced graduate student with numerical principles and procedures used in the modern analysis of industrial forming operations, and (b) to provide reference material for those performing such an analysis in industrial settings, government laboratories, and academia. In both cases, an understanding of the most important methods and their respective characteristics is the goal.

The first seven chapters focus on principles and procedures, which are derived and presented in an intuitive, informal manner. Exercises appear throughout these chapters, proposing and then solving illuminating problems related to the subject. Extensive problems are provided in three categories at the end of each chapter: proficiency, depth, and numerical, to solidify the information presented.

The last five chapters focus on applications of the numerical analysis to specific forming operations in order to illustrate the lessons learned from these simulations. Most of these results are drawn from the authors' research in this area, using programs developed over many years at their laboratories. Exercises are presented where appropriate and practical, and a limited number of problems are provided at the end of some chapters.

It should be noted that this advanced text and reference volume does not provide a detailed treatment of the underlying physical equations or principles necessary to understand metal deformation itself. This material is limited to Chapter 1, which is a very brief review of the physical descriptions and equations. For a thorough treatment of the physical fundamentals leading to the numerical treatment, we recommend that the interested reader refer to *Fundamentals of Metal Forming* (Wiley, 1997, ISBN 0-471-57004-4), which we wrote for this purpose.

Because of the nature of metal forming, the challenges to a numerical analysis lie predominantly in the large deformation experienced by materials and the nonlinear aspects of the finite-element method. With this required focus, it is impossible to introduce in any systematic way the broad field of linear finite-element modeling for structural applications. There are many excellent and exhaustive texts on these subjects that may be consulted. However, sufficient information is provided to understand the nonlinear finite-element method as applied to forming problems without this breadth of background.

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This book is made possible by the selfless contributions of close friends, whom we imposed upon to read, write, edit, criticize, and give up other activities so that we could do these same things. Robyn K. Wagoner, without an iota of technical training, typed much of this manuscript and made nearly all of the many changes required in figures, equations, and text. Robert H. Wagoner (senior), also without technical training, read and edited every chapter several times, finding all of the inevitable misreferenced equations and figures, the improper English, and the many subtle errors that occur when technical people from two countries try to write. Beatrice Chenot and Robyn Wagoner gave up weekends with their husbands over a period of years in order to make this book possible.

The first author would like to acknowledge the support of The Ohio State University and the Ecole des Mines de Paris, both of which provided support for a sabbatical leave during which this book began to take shape. Some of the travel facilitated the collaboration of the authors and was provided by the National Science Foundation (DMR-8814926).

The second author would like to acknowledge the Ecole des Mines de Paris and his Director, J. Lévy, for creating the favorable conditions to develop research and engineering applications with industrial companies, upon which this work is based. Most of the computational examples in Chapters 9, 10, and 12 were kindly provided by colleagues and students of the CEMEF laboratory: they will find here my deep gratitude.

Chapters 8 and 11 were drafted by Dr. Dajun Zhou and the equations were proofed by Dr. Kaiping Li. Many of the original figures were drafted by Weili Wang, wife of Dajun Zhou, who provided criticism for the technical content, especially in the area of numerical methods.

Without these many services, provided without complaint or compensation, this book would have not existed, or would have been even later, by years, than it is now.

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