

Group Theory With Applications in Chemical Physics

Group theory is widely used in many branches of physics and chemistry, and today it may be considered as an essential component in the training of both chemists and physicists. This book provides a thorough, self-contained introduction to the fundamentals of group theory and its applications in chemistry and molecular and solid state physics. The first half of the book, with the exception of a few marked sections, focuses on elementary topics. The second half (Chapters 11–18) deals with more advanced topics which often do not receive much attention in introductory texts. These include the rotation group, projective representations, space groups, and magnetic crystals. The book includes numerous examples, exercises, and problems, and it will appeal to advanced undergraduates and graduate students in the physical sciences. It is well suited to form the basis of a two-semester course in group theory or for private study.

PROFESSOR P. W. M. JACOBS is Emeritus Professor of Physical Chemistry at the University of Western Ontario, where he taught widely in the area of physical chemistry, particularly group theory. He has lectured extensively on his research in North America, Europe, and the former USSR. He has authored more than 315 publications, mainly in solid state chemistry and physics, and he was awarded the Solid State Medal of the Royal Society of Chemistry.

Cambridge University Press
0521642507 - Group Theory with Applications in Chemical Physics
P. W. M. Jacobs
Frontmatter
[More information](#)

Group Theory With Applications in Chemical Physics

P. W. M. JACOBS
The University of Western Ontario



Cambridge University Press
0521642507 - Group Theory with Applications in Chemical Physics
P. W. M. Jacobs
Frontmatter
[More information](#)

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge, CB2 2RU, UK
Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521642507

© P. W. M. Jacobs 2005

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2005

Printed in the United Kingdom at the University Press, Cambridge

Typeface Times 10/13pt. and Frutiger *System* Advent 3B2 8.07f [PND]

A catalog record for this publication is available from the British Library

ISBN-13 978-0-521-64250-7 hardback
ISBN-10 0-521-64250-7 hardback

The publisher has used its best endeavors to ensure that the URLs for external websites referred to in this publication are correct and active at the time of going to press. However, the publisher has no responsibility for the websites and can make no guarantee that a site will remain live or that the content is or will remain appropriate.

Cambridge University Press
0521642507 - Group Theory with Applications in Chemical Physics
P. W. M. Jacobs
Frontmatter
[More information](#)

**To MFM
and to all those who love group theory**

Contents

<i>Preface</i>	<i>page</i> xi
<i>Notation and conventions</i>	xiii
1 The elementary properties of groups	1
1.1 Definitions	1
1.2 Conjugate elements and classes	5
1.3 Subgroups and cosets	6
1.4 The factor group	8
1.5 Minimal content of Sections 1.6, 1.7, and 1.8	12
1.6 Product groups	15
1.7 Mappings, homomorphisms, and extensions	17
1.8 More about subgroups and classes	18
Problems	22
2 Symmetry operators and point groups	23
2.1 Definitions	23
2.2 The multiplication table – an example	32
2.3 The symmetry point groups	36
2.4 Identification of molecular point groups	48
Problems	50
3 Matrix representatives	53
3.1 Linear vector spaces	53
3.2 Matrix representatives of operators	55
3.3 Mappings	60
3.4 Group representations	62
3.5 Transformation of functions	62
3.6 Some quantum mechanical considerations	67
Problems	68
4 Group representations	70
4.1 Matrix representations	70
4.2 Irreducible representations	72
4.3 The orthogonality theorem	73
4.4 The characters of a representation	74
4.5 Character tables	80
4.6 Axial vectors	82

viii Contents

4.7	Cyclic groups	86
4.8	Induced representations	88
	Problems	95
5	Bases of representations	96
5.1	Basis functions	96
5.2	Construction of basis functions	97
5.3	Direct product representations	99
5.4	Matrix elements	101
	Problems	105
6	Molecular orbitals	106
6.1	Hybridization	106
6.2	π Electron systems	109
6.3	Equivalent bond orbitals	114
6.4	Transition metal complexes	117
	Problems	129
7	Crystal-field theory	131
7.1	Electron spin	131
7.2	Spherical symmetry	132
7.3	Intermediate crystal field	134
7.4	Strong crystal fields	139
	Problems	146
8	Double groups	148
8.1	Spin-orbit coupling and double groups	148
8.2	Weak crystal fields	152
	Problems	154
9	Molecular vibrations	156
9.1	Classification of normal modes	156
9.2	Allowed transitions	158
9.3	Inelastic Raman scattering	161
9.4	Determination of the normal modes	162
	Problems	168
10	Transitions between electronic states	171
10.1	Selection rules	171
10.2	Vibronic coupling	173
10.3	Charge transfer	178
	Problems	181
11	Continuous groups	182
11.1	Rotations in \mathfrak{R}^2	182
11.2	The infinitesimal generator for SO(2)	183
11.3	Rotations in \mathfrak{R}^3	184
11.4	The commutation relations	187
11.5	The irreducible representations of SO(3)	192

 Contents

ix

11.6	The special unitary group $SU(2)$	200
11.7	Euler parameterization of a rotation	205
11.8	The homomorphism of $SU(2)$ and $SO(3)$	208
	Problems	216
12	Projective representations	218
12.1	Complex numbers	218
12.2	Quaternions	220
12.3	Geometry of rotations	222
12.4	The theory of turns	225
12.5	The algebra of turns	228
12.6	Projective representations	232
12.7	Improper groups	240
12.8	The irreducible representations	243
	Problems	250
13	Time-reversal symmetry	252
13.1	Time evolution	252
13.2	Time reversal with neglect of electron spin	253
13.3	Time reversal with spin–orbit coupling	254
13.4	Co-representations	257
	Problems	264
14	Magnetic point groups	265
14.1	Crystallographic magnetic point groups	265
14.2	Co-representations of magnetic point groups	267
14.3	Clebsch–Gordan coefficients	277
14.4	Crystal-field theory for magnetic crystals	280
	Problems	281
15	Physical properties of crystals	282
15.1	Tensors	282
15.2	Crystal symmetry: the direct method	286
15.3	Group theory and physical properties of crystals	288
15.4	Applications	293
15.5	Properties of crystals with magnetic point groups	303
	Problems	305
16	Space groups	307
16.1	Translational symmetry	307
16.2	The space group of a crystal	314
16.3	Reciprocal lattice and Brillouin zones	324
16.4	Space-group representations	331
16.5	The covering group	336
16.6	The irreducible representations of G	337
16.7	Herring method for non-symmorphic space groups	344
16.8	Spinor representations of space groups	351
	Problems	355

x Contents

17	Electronic energy states in crystals	357
17.1	Translational symmetry	357
17.2	Time-reversal symmetry	357
17.3	Translational symmetry in the reciprocal lattice representation	358
17.4	Point group symmetry	359
17.5	Energy bands in the free-electron approximation: symmorphic space groups	365
17.6	Free-electron states for crystals with non-symmorphic space groups	378
17.7	Spinor representations	383
17.8	Transitions between electronic states	384
	Problems	390
18	Vibration of atoms in crystals	391
18.1	Equations of motion	391
18.2	Space-group symmetry	394
18.3	Symmetry of the dynamical matrix	398
18.4	Symmetry coordinates	401
18.5	Time-reversal symmetry	404
18.6	An example: silicon	406
	Problems	412
	Appendices	
A1	Determinants and matrices	413
A2	Class algebra	434
A3	Character tables for point groups	447
A4	Correlation tables	467
	<i>References</i>	476
	<i>Index</i>	481

Preface

Symmetry pervades many forms of art and science, and group theory provides a systematic way of thinking about symmetry. The mathematical concept of a group was invented in 1823 by Évariste Galois. Its applications in physical science developed rapidly during the twentieth century, and today it is considered as an indispensable aid in many branches of physics and chemistry. This book provides a thorough introduction to the subject and could form the basis of two successive one-semester courses at the advanced undergraduate and graduate levels. Some features not usually found in an introductory text are detailed discussions of induced representations, the Dirac characters, the rotation group, projective representations, space groups, magnetic crystals, and spinor bases. New concepts or applications are illustrated by worked examples and there are a number of exercises. Answers to exercises are given at the end of each section. Problems appear at the end of each chapter, but solutions to problems are not included, as that would preclude their use as problem assignments. No previous knowledge of group theory is necessary, but it is assumed that readers will have an elementary knowledge of calculus and linear algebra and will have had a first course in quantum mechanics. An advanced knowledge of chemistry is not assumed; diagrams are given of all molecules that might be unfamiliar to a physicist.

The book falls naturally into two parts. Chapters 1–10 (with the exception of a few marked sections) are elementary and could form the basis of a one-semester advanced undergraduate course. This material has been used as the basis of such a course at the University of Western Ontario for many years and, though offered as a chemistry course, it was taken also by some physicists and applied mathematicians. Chapters 11–18 are at a necessarily higher level; this material is suited to a one-semester graduate course.

Throughout, explanations of new concepts and developments are detailed and, for the most part, complete. In a few instances complete proofs have been omitted and detailed references to other sources substituted. It has not been my intention to give a complete bibliography, but essential references to core work in group theory have been given. Other references supply the sources of experimental data and references where further development of a particular topic may be followed up.

I am considerably indebted to Professor Boris Zapol who not only drew all the diagrams but also read the entire manuscript and made many useful comments. I thank him also for his translation of the line from Alexander Pushkin quoted below. I am also indebted to my colleague Professor Alan Allnatt for his comments on Chapters 15 and 16 and for several fruitful discussions. I am indebted to Dr. Peter Neumann and Dr. Gabrielle Stoy of Oxford

xii Preface

University for their comments on the proof (in Chapter 12) that multiplication of quaternions is associative. I also thank Richard Jacobs and Professor Amy Mullin for advice on computing.

Grateful acknowledgement is made to the following for permission to make use of previously published material:

The Chemical Society of Japan, for Figure 10.3;
Taylor and Francis Ltd (<http://www.tandf.co.uk/journals>) for Table 10.2;
Cambridge University Press for Figure 12.5;
The American Physical Society and Dr. C. J. Bradley for Table 14.6.

“Служенье муз не терпит суеты . . . ”

А. С. Пушкин
“19 октября”

which might be translated as:

“Who serves the muses should keep away from fuss,” or, more prosaically,

“Life interferes with Art.”

I am greatly indebted to my wife Mary Mullin for shielding me effectively from the daily intrusions of “Life” and thus enabling me to concentrate on this particular work of “Art.”

Notation and conventions

General mathematical notation

\equiv	identically equal to
\Rightarrow	leads logically to; thus $p \Rightarrow q$ means if p is true, then q follows
\sum	sum of (no special notation is used when \sum is applied to sets, since it will always be clear from the context when \sum means a sum of sets)
\forall	all
iff	if and only if
\exists	there exists
\bar{a}	the negative of a (but note $\bar{\psi} = \Theta\psi$ in Chapter 13 and $\bar{R} = \bar{E}R$, an operator in the double group \bar{G} , in Chapter 8)
C^n	n -dimensional space in which the components of vectors are complex numbers
c, s	$\cos \phi, \sin \phi$
c_2, s_2	$\cos 2\phi, \sin 2\phi$
$c x$	$x \cos \phi$
c_n^m	$\cos(m\pi/n)$
i	imaginary unit, defined by $i^2 = \sqrt{-1}$
$q_1 q_2 q_3$	quaternion units
\mathfrak{R}^n	n -dimensional space, in which the components of vectors are real numbers
\mathfrak{R}^3	configuration space, that is the three-dimensional space of real vectors in which symmetry operations are represented
$s x$	$x \sin \phi$
$T(n)$	tensor of rank n in Section 15.1

Sets and groups

$\{g_i\}$	the set of objects $g_i, i = 1, \dots, g$, which are generally referred to as ‘elements’
\in	belongs to, as in $g_i \in G$
\notin	does not belong to
$A \rightarrow B$	map of set A onto set B
$a \rightarrow b$	map of element a (the pre-image of b) onto element b (the image of a)

xiv Notation and conventions

$A \cap B$	intersection of A and B, that is the set of all the elements that belong to both A and B
$A \cup B$	the union of A and B, that is the set of all the elements that belong to A, or to B, or to both A and B
G	a group $G = \{g_i\}$, the elements g_i of which have specific properties (Section 1.1)
E , or g_1	the identity element in G
g	the order of G , that is the number of elements in G
H, A, B	groups of order h, a , and b , respectively, often subgroups of G
$H \subset G$	H is a subset of G ; if $\{h_i\}$ have the group properties, H is a subgroup of G of order h
$A \sim B$	the groups A and B are isomorphous
C	a cyclic group of order c
\mathcal{C}_k	the class of g_k in G (Section 1.2) of order c_k
C_{ij}^k	class constants in the expansion $\Omega_i \Omega_j = \sum_{k=1}^{N_c} C_{ij}^k \Omega_k$ (Section A2.2)
$g_i(\mathcal{C}_k)$	i th element of the k th class
\underline{G}	a group consisting of a unitary subgroup H and the coset AH , where A is an antiunitary operator (Section 13.2), such that $\underline{G} = \{H\} \oplus A\{H\}$
K	the kernel of G , of order k (Section 1.6)
l_i	dimension of i th irreducible representation
l_s	dimension of an irreducible spinor representation
l_v	dimension of an irreducible vector representation
N_c	number of classes in G
N_{rc}	number of regular classes
N_r	number of irreducible representations
N_s	number of irreducible spinor representations
N_v	number of irreducible vector representations
$N(H G)$	the normalizer of H in G , of order n (Section 1.7)
t	index of a coset expansion of G on H , $G = \sum_{r=1}^t g_r H$, with $g_r \notin H$ except for $g_1 = E$; $\{g_r\}$ is the set of coset representatives in the coset expansion of G , and $\{g_r\}$ is not used for G itself.
$Z(h_j G)$	the centralizer of h_j in G , of order z (Section 1.7)
Z_i	an abbreviation for $Z(g_i G)$
$\Omega_k, \Omega(\mathcal{C}_k)$	Dirac character of \mathcal{C}_k , equal to $\sum_{i=1}^{c_k} g_i(\mathcal{C}_k)$
$A \otimes B$	(outer) direct product of A and B , often abbreviated to DP
$A \boxtimes B$	inner direct product of A and B
$A \wedge B$	semidirect product of A and B
$A \overline{\otimes} B$	symmetric direct product of A and B (Section 5.3)
$A \underline{\otimes} B$	antisymmetric direct product of A and B (Section 5.3)

Vectors and matrices

\mathbf{r}	a polar vector (often just a vector) which changes sign under inversion; \mathbf{r} may be represented by the directed line segment OP, where O is the origin of the coordinate system
$x\ y\ z$	coordinates of the point P and therefore the components of a vector $\mathbf{r} = OP$; independent variables in the function $f(x, y, z)$.
$\mathbf{x}\ \mathbf{y}\ \mathbf{z}$	space-fixed right-handed orthonormal axes, collinear with OX, OY, OZ
$\mathbf{e}_1\ \mathbf{e}_2\ \mathbf{e}_3$	unit vectors, initially coincident with $\mathbf{x}\ \mathbf{y}\ \mathbf{z}$, but firmly embedded in configuration space (see $R(\phi\ \mathbf{n})$ below). Note that $\{\mathbf{e}_1\ \mathbf{e}_2\ \mathbf{e}_3\}$ behave like polar vectors under rotation but are invariant under inversion and therefore they are <i>pseudovectors</i> . Since, in configuration space the vector $\mathbf{r} = \mathbf{e}_1x + \mathbf{e}_2y + \mathbf{e}_3z$ changes sign on inversion, the components of \mathbf{r} , $\{x\ y\ z\}$, must change sign on inversion and are therefore <i>pseudoscalars</i>
$\{\mathbf{e}_i\}$	unit vectors in a space of n dimensions, $i = 1, \dots, n$
$\{v_i\}$	components of the vector $\mathbf{v} = \sum_i \mathbf{e}_i v_i$
\mathbf{A}	the matrix $\mathbf{A} = [a_{rs}]$, with m rows and n columns so that $r = 1, \dots, m$ and $s = 1, \dots, n$. See Table A1.1 for definitions of some special matrices
A_{rs}, a_{rs}	element of matrix \mathbf{A} common to the r th row and s th column
\mathbf{E}_n	unit matrix of dimensions $n \times n$, in which all the elements are zero except those on the principal diagonal, which are all unity; often abbreviated to \mathbf{E} when the dimensions of \mathbf{E} may be understood from the context
$\det \mathbf{A}$ or $ a_{rs} $	determinant of the square matrix \mathbf{A}
$\mathbf{A} \otimes \mathbf{B}$	direct product of the matrices \mathbf{A} and \mathbf{B}
$C_{pr,qs}$	element $a_{pq}b_{rs}$ in $\mathbf{C} = \mathbf{A} \otimes \mathbf{B}$
$\mathbf{A}_{[ij]}$	ij th element (which is itself a matrix) of the supermatrix \mathbf{A}
$\langle a_1\ a_2 \dots a_n $	a matrix of one row containing the set of elements $\{a_i\}$
$\langle a $	an abbreviation for $\langle a_1\ a_2 \dots a_n $. The set of elements $\{a_i\}$ may be basis vectors, for example $\langle \mathbf{e}_1\ \mathbf{e}_2\ \mathbf{e}_3 $, or basis functions $\langle \phi_1\ \phi_2 \dots \phi_n $.
$ b_1\ b_2 \dots b_n \rangle$	a matrix of one column containing the set of elements $\{b_i\}$, often abbreviated to $ b\rangle$; $\langle b $ is the transpose of $ b\rangle$
$\langle a' $	the transform of $\langle a $ under some stated operation
$\langle \mathbf{e} r \rangle$	an abbreviation for the matrix representative of a vector \mathbf{r} ; often given fully as $\langle \mathbf{e}_1\ \mathbf{e}_2\ \mathbf{e}_3 x\ y\ z \rangle$

Brackets

\langle , \rangle	Dirac bra and ket, respectively; no special notation is used to distinguish the bra and ket from row and column matrices, since which objects are intended will always be clear from the context
------------------------	--

xvi Notation and conventions

$[A, B]$	commutator of A and B equal to $AB - BA$
$[a, A]$	complex number $a + iA$
$[a ; \mathbf{A}]$	quaternion (Chapter 11)
$[g_i ; g_j]$	projective factor, or multiplier (Chapter 12); often abbreviated to $[i ; j]$
$[n_1 n_2 n_3]$	components of the unit vector \mathbf{n} , usually given without the normalization factor; for example, $[1\ 1\ 1]$ are the components of the unit vector that makes equal angles with OX, OY, OZ, the normalization factor $3^{-1/2}$ being understood. Normalization factors will, however, be given explicitly when they enter into a calculation, as, for example, in calculations using quaternions

Angular momenta

$\mathbf{L}, \mathbf{S}, \mathbf{J}$	orbital, spin, and total angular momenta
$\hat{\mathbf{L}}, \hat{\mathbf{S}}, \hat{\mathbf{J}}$	quantum mechanical operators corresponding to \mathbf{L}, \mathbf{S} , and \mathbf{J}
L, S, J	quantum numbers that quantize $\mathbf{L}^2, \mathbf{S}^2$, and \mathbf{J}^2
$\hat{\mathbf{j}}$	operator that obeys the angular momentum commutation relations
$\mathbf{j} = \mathbf{j}_1 + \mathbf{j}_2$	total (\mathbf{j}) and individual ($\mathbf{j}_1, \mathbf{j}_2, \dots$) angular momenta, when angular momenta are coupled

Symmetry operators and their matrix representatives

A	antiunitary operator (Section 13.1); A, B may also denote linear, Hermitian operators according to context
E	identity operator
\bar{E}	operator $R(2\pi \mathbf{n})$ introduced in the formation of the double group $\bar{G} = \{R \bar{R}\}$ from $G = \{R\}$, where $\bar{R} = \bar{E}R$ (Section 8.1)
I	inversion operator
$I_1 I_2 I_3$	operators that generate infinitesimal rotations about $\mathbf{x}, \mathbf{y}, \mathbf{z}$, respectively (Chapter 11)
$\hat{I}_1 \hat{I}_2 \hat{I}_3$	function operators that correspond to $I_1 I_2 I_3$
Γ_3	matrix representative of I_3 , and similarly (note that the usual symbol $\Gamma(R)$ for the matrix representative of symmetry operator R is not used in this context, for brevity)
\mathbf{I}	generator of infinitesimal rotations about \mathbf{n} , with components I_1, I_2, I_3
$\Gamma_{\mathbf{n}}$	matrix representative of $I_{\mathbf{n}} = \mathbf{n} \cdot \mathbf{I}$
$\mathcal{J}_x \mathcal{J}_y \mathcal{J}_z$	matrix representatives of the angular momentum operators $\hat{J}_x, \hat{J}_y, \hat{J}_z$ for the basis $\langle m = \langle 1/2, -1/2 $. Without the numerical factors of $1/2$, these are the Pauli matrices $\sigma_1 \sigma_2 \sigma_3$

$R(\phi \mathbf{n})$	rotation through an angle ϕ about an axis which is the unit vector \mathbf{n} ; here $\phi \mathbf{n}$ is not a product but a single symbol $\phi\mathbf{n}$ that fixes the three independent parameters necessary to describe a rotation (the three components of \mathbf{n} , $[n_1 n_2 n_3]$, being connected by the normalization condition); however, a space is inserted between ϕ and \mathbf{n} in rotation operators for greater clarity, as in $R(2\pi/3 \mathbf{n})$. The range of ϕ is $-\pi < \phi \leq \pi$. R acts on configuration space and on all vectors therein (including $\{\mathbf{e}_1 \mathbf{e}_2 \mathbf{e}_3\}$) (but not on $\{\mathbf{x} \mathbf{y} \mathbf{z}\}$, which define the space-fixed axes in the active representation)
$\hat{R}(\phi \mathbf{n})$	function operator that corresponds to the symmetry operator $R(\phi \mathbf{n})$, defined so that $\hat{R}f(\mathbf{r}) = f(R^{-1}\mathbf{r})$ (Section 3.5)
R, S, T	general symbols for point symmetry operators (point symmetry operators leave at least one point invariant)
$\hat{s}_x \hat{s}_y \hat{s}_z$	spin operators whose matrix representatives are the Pauli matrices and therefore equal to $\hat{J}_x, \hat{J}_y, \hat{J}_z$ without the common factor of $1/2$
T	translation operator (the distinction between T a translation operator and T when used as a point symmetry operator will always be clear from the context)
U	a unitary operator
\mathcal{U}	time-evolution operator (Section 13.1)
$\Gamma(R)$	matrix representative of the symmetry operator R ; sometimes just R , for brevity
$\Gamma(R)_{pq}$	pq th element of the matrix representative of the symmetry operator R
Γ	matrix representation
$\Gamma_1 \approx \Gamma_2$	the matrix representations Γ_1 and Γ_2 are equivalent, that is related by a similarity transformation (Section 4.2)
$\Gamma = \sum_i c^i \Gamma_i$	the representation Γ is a direct sum of irreducible representations Γ_i , and each Γ_i occurs c^i times in the direct sum Γ ; when specific representations (for example T_{1u}) are involved, this would be written $c(T_{1u})$
$\Gamma \supset \Gamma_i$	the reducible representation Γ includes Γ_i
$\Gamma_i = \sum_j c_{i,j} \Gamma_j$	the representation Γ_i is a direct sum of irreducible representations Γ_j and each Γ_j occurs $c_{i,j}$ times in the direct sum Γ_i
$\Gamma_{ij} = \sum_k c_{ij,k} \Gamma_k$	Clebsch–Gordan decomposition of the direct product $\Gamma_{ij} = \Gamma_i \boxtimes \Gamma_j$; $c_{ij,k}$ are the Clebsch–Gordan coefficients
$\sigma_{\mathbf{n}}$	reflection in the plane normal to \mathbf{n}
$\sigma_1 \sigma_2 \sigma_3$	the Pauli matrices (Section 11.6)
Θ	time-reversal operator

xviii Notation and conventions

Bases

$\langle \mathbf{e}_1 \mathbf{e}_2 \mathbf{e}_3 $	basis consisting of the three unit vectors $\{\mathbf{e}_1 \mathbf{e}_2 \mathbf{e}_3\}$ initially coincident with $\{\mathbf{x} \mathbf{y} \mathbf{z}\}$ but embedded in a unit sphere in configuration space so that $R\langle \mathbf{e}_1 \mathbf{e}_2 \mathbf{e}_3 = \langle \mathbf{e}'_1 \mathbf{e}'_2 \mathbf{e}'_3 = \langle \mathbf{e}_1 \mathbf{e}_2 \mathbf{e}_3 \Gamma(R)$. The 3×3 matrix $\Gamma(R)$ is the matrix representative of the symmetry operator R . Note that $\langle \mathbf{e}_1 \mathbf{e}_2 \mathbf{e}_3 $ is often abbreviated to $\langle \mathbf{e} $. If $\mathbf{r} \in \mathfrak{R}^3$, $R \mathbf{r} = R\langle \mathbf{e} r \rangle = \langle \mathbf{e}' r \rangle = \langle \mathbf{e} \Gamma(R) r \rangle = \langle \mathbf{e} r' \rangle$, which shows that $\langle \mathbf{e} $ and $ r \rangle$ are <i>dual bases</i> , that is they are transformed by the same matrix $\Gamma(R)$
$\langle R_x R_y R_z $	basis comprising the three <i>infinitesimal</i> rotations R_x, R_y, R_z about OX, OY, OZ respectively (Section 4.6)
$\langle u_{-j}^i \dots u_j^j $	basis consisting of the $2j + 1$ functions, u_m^j , $-j \leq m \leq j$, which are eigenfunctions of the z component of the angular momentum operator \hat{J}_z , and of \hat{J}^2 , with the Condon and Shortley choice of phase. The angular momentum quantum numbers j and m may be either an integer or a half-integer. For integral j the u_m^j are the spherical harmonics $Y_l^m(\theta \varphi)$; $y_l^m(\theta \varphi)$ are the spherical harmonics written without normalization factors, for brevity
$\langle u_m^j $	an abbreviation for $\langle u_{-j}^i \dots u_j^j $, also abbreviated to $\langle m $
$\langle u \nu $	spinor basis, an abbreviation for $\langle u_{\frac{1}{2}}^{\frac{1}{2}} = \langle \frac{1}{2} \frac{1}{2} $, $ \frac{1}{2} -\frac{1}{2}\rangle$, or $\langle \frac{1}{2} -\frac{1}{2} $ in the $\langle m $ notation
$\langle u' \nu' $	transform of $\langle u \nu $ in C^2 , equal to $\langle u \nu \mathbb{A}$
$ u \nu \rangle$	dual of $\langle u \nu $, such that $ u' \nu' \rangle = \mathbb{A} u \nu \rangle$
$ U_{-1} U_0 U_1 \rangle$	matrix representation of the spherical vector $\mathbf{U} \in C^3$ which is the dual of the basis $\langle y_1^{-1} y_1^0 y_1^1 $
N	normalization factor

Crystals

$\mathbf{a}_n = \langle \mathbf{a} n \rangle$	lattice translation vector; $\mathbf{a}_n = \langle \mathbf{a}_1 \mathbf{a}_2 \mathbf{a}_3 n_1 n_2 n_3 \rangle$ (Section 16.1) (n is often used as an abbreviation for the \mathbf{a}_n)
$\mathbf{b}_m = \langle \mathbf{b} m \rangle$	reciprocal lattice vector; $\mathbf{b}_m = \langle \mathbf{b}_1 \mathbf{b}_2 \mathbf{b}_3 m_1 m_2 m_3 \rangle = \langle \mathbf{e}_1 \mathbf{e}_2 \mathbf{e}_3 m_x m_y m_z \rangle$ (Section 16.3); m is often used as an abbreviation for the components of \mathbf{b}_m

Abbreviations

1-D	one-dimensional (etc.)
AO	atomic orbital
BB	bilateral binary
<i>bcc</i>	body-centered cubic
CC	complex conjugate
CF	crystal field

 Notation and conventions

xix

CG	Clebsch–Gordan
CR	commutation relation
CS	Condon and Shortley
CT	charge transfer
DP	direct product
<i>fcc</i>	face-centered cubic
FE	free electron
FT	fundamental theorem
<i>hcp</i>	hexagonal close-packed
HSP	Hermitian scalar product
IR	irreducible representation
ITC	<i>International Tables for Crystallography</i> (Hahn (1983))
L, R	left and right, respectively, as in L and R cosets
LA	longitudinal acoustic
LCAO	linear combination of atomic orbitals
LI	linearly independent
LO	longitudinal optic
LS	left- side (of an equation)
LVS	linear vector space
MO	molecular orbital
MR	matrix representative
N	north, as in N pole
ORR	Onsager reciprocal relation
OT	orthogonality theorem
PBC	periodic boundary conditions
PF	projective factor
PR	projective representation
RS	right side (of an equation)
<i>RS</i>	Russell–Saunders, as in <i>RS</i> coupling or <i>RS</i> states
<i>sc</i>	simple cubic
SP	scalar product
TA	transverse acoustic
TO	transverse optic
ZOA	zero overlap approximation

Cross-references

The author (date) system is used to identify a book or article in the list of references, which precedes the index.

Equations in a different section to that in which they appear are referred to by eq. ($n_1 \cdot n_2 \cdot n_3$), where n_1 is the chapter number, n_2 is the section number, and n_3 is the equation number within that section. Equations occurring within the same section are referred to simply by (n_3). Equations are numbered on the right, as usual, and, when appropriate,

xx Notation and conventions

a number (or numbers) on the left, in parentheses, indicates that these equations are used in the derivation of that equation so numbered. This convention means that such phrases as “it follows from” or “substituting eq. (n_4) in eq. (n_5)” can largely be dispensed with.

Examples and Exercises are referenced, for example, as Exercise $n_1 \cdot n_2$ - n_3 , even within the same section. Figures and Tables are numbered $n_1 \cdot n_3$ throughout each chapter. When a Table or Figure is referenced on the left side of an equation, their titles are abbreviated to T or F respectively, as in F16.1, for example.

Problems appear at the end of each chapter, and a particular problem may be referred to as Problem $n_1 \cdot n_3$, where n_1 is the number of the chapter in which Problem n_3 is to be found.