PRINCIPLES OF LASERS AND OPTICS

Principles of Lasers and Optics describes both the fundamental principles of lasers and the propagation and application of laser radiation in bulk and guided wave components. All solid state, gas and semiconductor lasers are analyzed uniformly as macroscopic devices with susceptibility originated from quantum mechanical interactions to develop an overall understating of the coherent nature of laser radiation.

The objective of the book is to present lasers and applications of laser radiation from a macroscopic, uniform point of view. Analyses of the unique properties of coherent laser light in optical components are presented together and derived from fundamental principles, to allow students to appreciate the differences and similarities. Topics covered include a discussion of whether laser radiation should be analyzed as natural light or as a guided wave, the macroscopic differences and similarities between various types of lasers, special techniques, such as super-modes and the two-dimensional Green’s function for planar waveguides, and some unusual analyses.

This clearly presented and concise text will be useful for first-year graduates in electrical engineering and physics. It also acts as a reference book on the mathematical and analytical techniques used to understand many opto-electronic applications.

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PRINCIPLES OF LASERS
AND OPTICS

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When I look back at my time as a graduate student, I realize that the most valuable knowledge that I acquired concerned fundamental concepts in physics and mathematics, quantum mechanics and electromagnetic theory, with specific emphasis on their use in electronic and electro-optical devices. Today, many students acquire such information as well as analytical techniques from studies and analysis of the laser and its light in devices, components and systems. When teaching a graduate course at the University of California San Diego on this topic, I emphasize the understanding of basic principles of the laser and the properties of its radiation.

In this book I present a unified approach to all lasers, including gas, solid state and semiconductor lasers, in terms of “classical” devices, with gain and material susceptibility derived from their quantum mechanical interactions. For example, the properties of laser oscillators are derived from optical feedback analysis of different cavities. Moreover, since applications of laser radiation often involve its well defined phase and amplitude, the analysis of such radiation in components and systems requires special care in optical procedures as well as microwave techniques. In order to demonstrate the applications of these fundamental principles, analytical techniques and specific examples are presented. I used the notes for my course because I was unable to find a textbook that provided such a compact approach, although many excellent books are already available which provide comprehensive treatments of quantum electronics, lasers and optics. It is not the objective of this book to present a comprehensive treatment of properties of lasers and optical components.

Our experience indicates that such a course can be covered in two academic quarters, and perhaps might be suitable for one academic semester in an abbreviated form. Students will learn both fundamental physics principles and analytical techniques from the course. They can apply what they have learned immediately to applications such as optical communication and signal processing. Professionals may find the book useful as a reference to fundamental principles and analytical techniques.