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0521637694 - New Approaches to Macroeconomic Modeling: Evolutionary
Stochastic Dynamics, Multiple Equilibria, and Externalities as Field Effects

Masanao Aoki

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