

Cambridge University Press

0521637694 - New Approaches to Macroeconomic Modeling: Evolutionary Stochastic Dynamics, Multiple Equilibria, and Externalities as Field Effects

Masanao Aoki

Frontmatter

[More information](#)

This book contributes substantively to the current state of art of macroeconomic modeling by providing a method for modeling large collections of possibly heterogeneous agents subject to nonpairwise externality called field effects, that is, feedback of aggregate effects on individual agents or agents using state-dependent strategies. By adopting a level of microeconomic description that keeps track of compositions of fractions of agents by types or strategies, time evolution of the microeconomic states is described by (backward) Chapman–Kolmogorov equations. Macroeconomic dynamics naturally arise from these equations by expansion of the solutions in some power series of the number of participants. Specification of the microeconomic transition rates thus leads to macroeconomic dynamic models. This approach provides a consistent way for dealing with multiple equilibria of macroeconomic dynamics by ergodic decomposition and associated calculations of mean first passage times, and stationary probabilities of equilibria provide further useful information on macroeconomic behavior.

This book ends with a set of elaborations, sketches of further topics of research, and a collection of supporting materials in the Appendix.

Cambridge University Press

0521637694 - New Approaches to Macroeconomic Modeling: Evolutionary
Stochastic Dynamics, Multiple Equilibria, and Externalities as Field Effects

Masanao Aoki

Frontmatter

[More information](#)

New Approaches to Macroeconomic Modeling

Cambridge University Press

0521637694 - New Approaches to Macroeconomic Modeling: Evolutionary
Stochastic Dynamics, Multiple Equilibria, and Externalities as Field Effects

Masanao Aoki

Frontmatter

[More information](#)

New Approaches to Macroeconomic Modeling

**Evolutionary Stochastic Dynamics, Multiple
Equilibria, and Externalities as Field Effects**

MASANAO AOKI

University of California, Los Angeles



Cambridge University Press

0521637694 - New Approaches to Macroeconomic Modeling: Evolutionary
Stochastic Dynamics, Multiple Equilibria, and Externalities as Field Effects

Masanao Aoki

Frontmatter

[More information](#)

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge CB2 1RP

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, United Kingdom
40 West 20th Street, New York, NY 10011-4211, USA
10 Stamford Road, Oakleigh, Melbourne 3166, Australia

© Cambridge University Press 1996

This book is in copyright. Subject to statutory exception
and to the provisions of relevant collective licensing agreements,
no reproduction of any part may take place without
the written permission of Cambridge University Press.

First published 1996
First paperback edition 1998

Library of Congress Cataloging-in-Publication Data is available.

A catalog record for this book is available from the British Library.

ISBN 0-521-48207-0 hardback
ISBN 0-521-63769-4 paperback

Transferred to digital printing 2004

Cambridge University Press

0521637694 - New Approaches to Macroeconomic Modeling: Evolutionary
Stochastic Dynamics, Multiple Equilibria, and Externalities as Field Effects

Masanao Aoki

Frontmatter

[More information](#)

To My Mother

Cambridge University Press

0521637694 - New Approaches to Macroeconomic Modeling: Evolutionary
Stochastic Dynamics, Multiple Equilibria, and Externalities as Field Effects

Masanao Aoki

Frontmatter

[More information](#)

CONTENTS

Preface xiii

Part I Introduction

1	Introduction	1
1.1	Multiplicity	2
1.2	Multiple equilibria	3
1.3	Jump Markov processes	4
1.4	Hierarchical state spaces	8
1.5	Scope of modeling	9
2	Simple Illustrative and Motivating Examples	10
2.1	Stochastic Descriptions of Economic Variables	10
2.1.1	Distinguishable and Exchangeable Agents	11
2.2	Entropy: A Measure of Economic Activity	14
2.2.1	Maximizing Entropy	15
2.2.2	Laplace Transform and Moment-Generating Functions	16
2.2.3	Replacing the Sum with the Maximum Term	17
2.3	Empirical Distributions	18
2.3.1	Examples	19
2.3.2	Multiplicity of Microstates	19
2.3.3	Sanov's Theorem	22
2.3.4	Conditional-Limit Theorem	23
2.4	Stochastic Dynamics and Processes	23
2.4.1	Mean First-Passage Times	23
2.4.2	Dynamics with Multiple Equilibria	25
2.4.3	Random Partition of Agents by Types	28
2.4.3.1	Pólya's Urn	28
2.4.3.2	Generalized Pólya Urn Model	28
		vii

Cambridge University Press

0521637694 - New Approaches to Macroeconomic Modeling: Evolutionary Stochastic Dynamics, Multiple Equilibria, and Externalities as Field Effects

Masanao Aoki

Frontmatter

[More information](#)

viii	Contents	
2.5	Hierarchical State Spaces	29
2.5.1	Examples of Hierarchically Structured State Spaces	32
2.5.1.1	Coin-Tossing Spaces	32
2.5.1.2	K -Level Classification	33
2.5.1.3	Pattern Classification	34
2.5.2	Tree Metric and Martingales	35
2.5.3	Markov Chains on Binary Trees	36
2.5.4	Aggregation of Dynamics on Trees	38
3	Empirical Distributions: Statistical Laws in Macroeconomics	40
3.1	Model Descriptions	40
3.1.1	Micro and Macro Descriptions of Models	40
3.1.2	Multiplicity of Microstates	42
3.2	Entropy and Relative Entropy	45
3.2.1	Kullback–Leibler Divergence Measure	46
3.2.2	Boltzmann and Shannon Entropies	47
3.3	Gibbs Distributions	50
3.3.1	Discrete-Choice Models	50
3.3.2	Detailed Balance and Gibbs Distributions	52
3.3.3	Conditional-Limit Theorems: Gibbs Conditioning Principle	53
3.3.3.1	Example of Asymptotic Independence	55
3.4	Maximizing Equilibrium Probabilities or Minimizing Potential	55
3.4.1	Optimization and Gibbs Distributions	57
3.5	Finite-State Markov Chains	58
3.5.1	Entropy Maximization	59
3.5.2	Cost Minimization with Markov-Chain Dynamics	61
3.6	Large Deviations	62
3.6.1	Example of Asset Returns	63
3.6.2	Chernoff Bounds	64
3.6.3	Tilted Distribution and Lower Chernoff Bound	68
3.6.4	Example of Large-Deviation Analysis	70
3.6.4.1	Sample Mean Greater than the Expected Value	71
3.6.5	Gärtner–Ellis Theorem	72
3.6.5.1	Examples of Dependent Asset Returns	73
3.7	Sanov’s Theorem	73
3.7.1	From Sanov’s Theorem to Cramér’s Theorem	76
3.8	Conditional-Limit Theorem	79

Part II Modeling Interactions

4	Modeling Interactions I: Jump Markov Processes	81
4.1	Market Participation and Other Discrete Adjustment Behavior	81
4.2	Construction and Infinitesimal Parameters	83

Cambridge University Press

0521637694 - New Approaches to Macroeconomic Modeling: Evolutionary Stochastic Dynamics, Multiple Equilibria, and Externalities as Field Effects

Masanao Aoki

Frontmatter

[More information](#)

Contents	ix
4.3 Examples	88
4.3.1 Birth-and-Death Processes	88
4.3.2 Poisson Processes	88
4.4 Equations for Averages: Aggregate Dynamics	89
4.4.1 Example	92
4.5 Multidimensional Birth-and-Death Processes	93
4.5.1 Epidemic Model Interpreted Economically	95
4.5.2 Open Air Market Linear Model I	95
4.5.3 Open Air Market Linear Model II	97
4.5.4 Nonlinear Birth-and-Death Models	99
4.5.5 Birth-and-Death Processes For Partition Patterns	101
4.6 Discrete Adjustment Behavior	102
4.6.1 Example of Employment Adjustment Processes I	104
4.6.2 Example of Employment Adjustment Processes II	107
4.6.3 Example of Inferring Microstate Distribution	108
4.7 Generalizations	109
4.7.1 Hazard Functions: Age-Specific Transition Rates	110
5 Modeling Interactions II: Master Equations and Field Effects	113
5.1 Master Equations	115
5.1.1 A Collection of Independent Agents	118
5.2 Structure of Transition Rates	120
5.3 Approximate Solutions of the Master Equations	121
5.3.1 Power-Series Expansion	122
5.3.2 The Method of Kubo	124
5.4 Macroeconomic Equation	127
5.5 Specifying Transition Rates: Examples	128
5.5.1 Two-Sector Capital Reallocation Dynamics	129
5.5.2 Exchange-Rate Pass-Through	133
5.6 Field Effects: Stochastic Nonlocal and Diffuse Externalities	134
5.7 Generalized Birth-and-Death Models	135
5.7.1 Mean Field Approximation of Transition Rates	137
5.8 Expressing Relative Merits of Alternative Decisions	138
5.9 Equilibrium Probability Distributions	139
5.10 Example of Multiple Equilibria	143
5.10.1 Model	143
5.10.2 Solutions	144
5.10.3 Some Simulation Results	145
5.11 First Passage Times	147
5.11.1 First-Passage Times of Unstable Dynamics	150
5.12 The Master Equation for Hierarchical Dynamics	154
5.13 The Fokker–Planck Equation	157
5.13.1 Power Series Expansion	157

Cambridge University Press

0521637694 - New Approaches to Macroeconomic Modeling: Evolutionary Stochastic Dynamics, Multiple Equilibria, and Externalities as Field Effects

Masanao Aoki

Frontmatter

[More information](#)

x	Contents	
	5.13.2 The Kubo Method	160
5.14	The Diffusion-Type Master Equation	163
	5.14.1 Ornstein–Uhlenbeck Model	164
	5.14.2 Wright–Fisher Model: A Binary-Choice Model	166
	5.14.2.1 Unemployment-Rate Model	167
	5.14.3 Logistic Model	170
6	Modeling Interactions III: Pairwise and Multiple-Pair Interactions	171
6.1	Pairwise or Multiple-Pair Interactions	171
	6.1.1 Ising Model	171
	6.1.1.1 Long-Range Pairwise Interactions	172
6.2	A Model of Pairwise Externality	173
	6.2.1 Potentials and Equilibrium Probability Distributions	174
	6.2.2 Analogy Between Economic Agents and Neurons	176
6.3	Example of Information-Exchange Equilibrium Distribution	177
6.4	Time Evolution of Patterns of Interaction	181
 Part III Hierarchical Dynamics and Critical Phenomena		
7	Sluggish Dynamics and Hierarchical State Spaces	184
7.1	Examples of Hierarchically Structured State Spaces	184
	7.1.1 Correlated Patterns of Macroeconomic Activity	184
	7.1.2 Vectors in an Infinite Dimensional Space	185
	7.1.3 Cost Barriers as Ultrametrics	186
	7.1.4 Voter Preference Patterns Among Alternatives	187
	7.1.5 Champernowne’s Income Distribution Model	188
	7.1.6 Random Walks on a Logarithmic Scale	190
	7.1.7 Random Multicomponent Cost	191
	7.1.7.1 Example of a Two-Level Tree	192
	7.1.8 Martingales	194
	7.1.9 Branching Processes	195
7.2	Dynamics on Hierarchical State Spaces	197
	7.2.1 Ultrametrics: Hierarchical Distance	198
	7.2.2 Ogielski–Stein Model	200
	7.2.2.1 An Example of Aggregation of Hierarchical Dynamics	203
	7.2.2.2 A Nonsymmetric Tree	204
	7.2.3 Schreckenberg’s Model	205
7.3	Pruning Trees: Aggregation of Hierarchical Dynamics	207
	7.3.1 Renormalization Group Theory	208
	7.3.2 Collet–Eckman Model	210
	7.3.3 Idiart–Theumann Model	211

Cambridge University Press

0521637694 - New Approaches to Macroeconomic Modeling: Evolutionary Stochastic Dynamics, Multiple Equilibria, and Externalities as Field Effects

Masanao Aoki

Frontmatter

[More information](#)

Contents

xi

8 Self-organizing and Other Critical Phenomena in Economic Models	213
8.1 Sudden Structural Changes	213
8.2 Piling-Up, or Dense Occupancy, of a Common State	215
8.3 Phase Transitions in the Ising Tree Model	219
8.3.1 Phase Transitions in a Random Cost Model	221
8.4 Example of a Two-Level Model	222
8.5 Random Cost Model	224
Elaborations and Future Directions of Research	226
E.1 Zipf's Distribution in Economic Models	227
E.1.1 Bose–Einstein Allocations	229
E.1.1.1 Two-Level Hierarchy	230
E.1.1.2 Three-Level Hierarchical Classification	231
E.1.2 Dirichlet–Multinomial Model of Chen	232
E.1.3 Applications	236
E.2 Residual Fraction Model	238
E.3 Frequency Spectrum	239
E.4 Statistical Distribution of Sizes of Attractive Basins	240
E.5 Transient Distributions	245
Appendix	246
A.1 The Method of Laplace	246
A.1.0.1 Example of Error Integral	247
A.1.0.2 Example of Stirling's Formula	248
A.1.0.3 Example of the Partition-Function Evaluation	248
A.1.1 Rate Functions	249
A.1.1.1 Example of Partition Function Evaluation	250
A.2 First-Passage Times	251
A.2.1 Discrete-Time Markov Chains	251
A.2.2 Example of a Random-Walk Model	252
A.2.3 Continuous-Time Markov Chains	253
A.2.3.1 Simple Random Walks	253
A.2.4 A Standard Wiener Process	254
A.2.5 First-Passage Times to Absorbing Barrier	254
A.3 Exchangeable Processes	255
A.4 Low-frequency Behavior	258
A.5 Lyapunov Functions	261
A.6 Fokker–Planck Equations and Detailed Balance	264
A.6.1 Fokker–Planck Equations and Stochastic Dynamics	264
A.6.2 Detailed Balance and the Fokker–Planck Equation	265
References	268
Index	281

Cambridge University Press

0521637694 - New Approaches to Macroeconomic Modeling: Evolutionary Stochastic Dynamics, Multiple Equilibria, and Externalities as Field Effects

Masanao Aoki

Frontmatter

[More information](#)

PREFACE

This book is an attempt to reformulate macroeconomic modeling as currently practiced by the macroeconomic profession.

The need to improve macroeconomic models certainly is felt widely by the economic profession. A short list of the defects that we recognize in macroeconomic modeling includes extensive and almost exclusive use of the assumption of representative agents, of largely deterministic dynamic models, and inadequate attention paid to off-equilibrium dynamic phenomena. More specifically, we do not have a satisfactory model for explaining sluggish responses of macroeconomic phenomena and the distributional effects of policy actions. We lack adequate treatments of the dynamic adjustment behavior of microeconomic units in the presence of externalities of the kind designated “field effects” in this book, and are known variously in the economic literature as social influence, social consumption, or group sentiments or pressure.

This book collects my recent investigations to provide an alternative manner for building and analyzing models in macroeconomics; it is addressed to macroeconomists and advanced graduate students in macroeconomics. The book is arranged in three parts. Part I consists of three chapters. After a short introductory discussion of motivation for developing a new way to construct and analyze macroeconomic models in Chapter 1, Chapter 2 provides some simple, motivating examples of the proposed approaches. An explicitly stochastic or statistical approach is taken in Chapter 3. It collects some material that I use in the remainder of the book, since this material is not usually in the toolkit of practicing macroeconomists and is not taught in traditional economics graduate courses.

Part II, consisting of Chapters 4 through 6, presents models of interacting microeconomic units via jump Markov processes and the derivation of macroeconomic dynamics by backward Chapman–Kolmogorov equations. Time evolution of a large collection of microeconomic units is described in terms of probability densities, and is governed by the master equation. Under certain

Cambridge University Press

0521637694 - New Approaches to Macroeconomic Modeling: Evolutionary Stochastic Dynamics, Multiple Equilibria, and Externalities as Field Effects

Masanao Aoki

Frontmatter

[More information](#)xiv **Preface**

conditions, the equilibrium probability distributions are Gibbs distributions. Chapter 4 addresses situations in which each of the microeconomic units has a finite number of choices, and thus extends the literature on discrete choice to include interacting agents.

Chapter 5 is a further elaboration of the ideas presented in Chapter 4. It focuses on the type of externalities that I call field effects. Field effects are such that each agent is influenced by the aggregate effects, and hence are usually weak and diffuse. These effects are in contrast with the pairwise interactions between two, possibly anonymous, agents. In Chapter 5 as well as in Chapter 4, a new way of aggregating microeconomic units to produce macroeconomic equations is illustrated. The issue associated with the existence of multiple macroeconomic equilibria naturally arises. In the explicitly stochastic framework of this book, dynamic models with well-behaved basins of attractions for locally stable equilibria behave in such a way that each basin will be visited with a positive probability, and the expected time of transition from one locally stable equilibrium to another is shown to depend on the height of a potential barrier separating the two basins. Finally, pairwise interactions are taken up in Chapter 6.

Part III, composed of Chapters 7 and 8, examines properties of dynamics with a large state space, which are organized hierarchically. The characteristic feature of such dynamics is sluggishness in the dynamic responses. Once disturbed off an equilibrium dynamic path, a hierarchical state-space model returns to it at rates slower than the usual exponential ones exhibited by dynamics with state spaces that are not similarly organized. Chapter 8 treats gathering of agents into the same state (or beliefs), as in bubbles and other critical phenomena. The book ends with a collection of supporting materials in the Appendix.

Some of the topics have been presented in my graduate course on economic modeling at the University of California, Los Angeles, as well as in special courses or sequences of seminars that I was invited to present in the past three years at the University of Tokyo, the Åbo Academy University, the European University Institute, the University of Siena, and elsewhere. I thank my former students as well as seminar participants for their reactions and comments. I thank H. Yoshikawa, K. G. Nishimura, L. Punzo, R. Fiorito, M. Salmon, and R. Östermark for arranging such special seminars or courses.

Professor Weidlich was very helpful in an early stage of the book writing as well. I have benefited from my colleagues, Axel Leijonhufvud and John McCall in particular, through their insights, suggestions, and wide knowledge of the economic and stochastic-processes literature. William McKelvey, Michael Orszag and Arthur Havenner read parts of the early drafts and made many useful comments. Max Rhee corrected my English by his careful reading of a draft. I thank them all for their help.

Cambridge University Press

0521637694 - New Approaches to Macroeconomic Modeling: Evolutionary
Stochastic Dynamics, Multiple Equilibria, and Externalities as Field Effects

Masanao Aoki

Frontmatter

[More information](#)

Preface

xv

I also acknowledge support from the Shin Nittetsu Endowed Chair to Tokyo Institute of Technology, grants-in-aid from Toshiba Corporation, and the academic senate research grants from University of California, Los Angeles.

Masanao Aoki
Los Angeles, California
July 14, 1995