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Introduction

It is, perhaps, trivial to remark of Greek, Roman, Byzantine, Romanesque and Gothic buildings that some of them still exist. The observation has force, however, when placed in a structural context. A masonry structure – a cathedral from the High Gothic period, for example – may be viewed in many ways: from the liturgical aspect, or the cultural, the historical, or the aesthetic, all of which may give rise to disputes of one sort or another. There remains one viewpoint which seems to engender an unequivocal statement: the large masonry building is clearly a feat of structural engineering. Moreover, the mere survival of ancient buildings implies an extreme stability of their structure.

Minor failures have, of course, occurred, and there have been major catastrophes. The fact remains that two severe earthquakes only slightly damaged Hagia Sofia, and the bombardments of the Second World War often resulted in a medieval cathedral left standing in the ruins of a modern city. At a much less severe level of disturbance, the continual shifts and settlements of foundations experienced over the centuries seem to cause the masonry structure no real distress, although, as will be seen, there may be an initial high-risk period of about a generation after completion of the building. It is the intention of this book to explain this extraordinary stability. A discussion of the actual structural behaviour of masonry is necessarily involved, and some of the history of structural analysis will be touched on, since it may help to deepen understanding.

The general principles which will be established apply to any form of masonry construction, but they will be developed by reference to the simple voussoir arch, whose behaviour is particularly easy to describe. Examples will be drawn from Gothic, because it is in Gothic that the problems of structural engineering applied to masonry are encountered in their most critical form. Only domes from the pre-Gothic period (and

running through Gothic into Renaissance) present any structural problem not otherwise met in Gothic itself. Indeed the collapse from Gothic to Renaissance (an aesthetic view which may perhaps be disputed) is reflected in the absence of structural interest in any Renaissance building (an engineering view which is unequivocal). It is not until the end of the nineteenth century that the gradual introduction of iron and then steel, to be followed by reinforced concrete, prestressed concrete and shell structures, has led to a renewed interest in structural analysis.

A byproduct of the structural examination of masonry is the light thrown on the activities of medieval architects. A Gothic cathedral was designed by a man who was both architect and engineer – or, of course, by a succession of such men, if the building campaigns spread over decades. The 'master of the work' had survived the long training of apprentice to journeyman to the career grade of master, and had been one of those few outstanding masters who were put to school again in the design office, before finally achieving control of a major work. This educational path contrasts strongly with that of modern Western European practice, which is based upon the Renaissance concept of the 'gentleman' architect, for whom considerations of history and aesthetics are divorced to some extent from those of engineering structure. If the building is complex, then the architect must work hand in hand with a technical adviser. The evolutionary tree of the modern structural engineer has its roots in Gothic, and earlier; that of the modern architect in Renaissance.

There was not this division in the thirteenth century (or in the sixth, when Justinian employed two outstanding Greeks, Anthemios and Isidorus, to design Hagia Sofia). The architect then knew, in the fullest technical sense, how to build, as well as how to give his building an 'architectural' design. The question seems to be obscured by the technical evidence – for example, that there are immense differences in the visual aspects of each one of the great ring of High Gothic cathedrals round Paris. Or again, the records of the late Gothic expertise at Milan (of which more will be said in Chapter 8) led Ackerman (1949) to the tempting conclusion '...that structure plays a secondary role in the process of creation'. This conclusion may just be true for part of the work at Milan; once it is believed to be true for any cathedral, then it is clear that the views of the 'architect' override those of the 'engineer', and Gothic is being thrust aside by Renaissance. Harvey (1958) has discussed this: 'The Gothic rules were so complicated that no one who had not served a long apprenticeship and spent years of practice could master them; whereas the rules of Vitruvius were so easy to grasp that even bishops

could understand them, and princes could try their hand at design on their own.'

Vitruvian rules, however, find no place for the flying buttress or for the rib vault. These two structural elements, which may perhaps be thought to represent the essence of Gothic (see e.g. Choisy 1899), would seem to demand a long apprenticeship indeed for the mastery of their design. The medieval rules, the secrets of the lodges, ensured that the structure was effective; 'decorative' developments could then take place safely. The rules shaped the skeleton, and were themselves subject to evolutionary change; the skeleton once fixed, however, could be fleshed in a wide variety of forms.

To take a single example, the great range of vaults, from the simple quadripartite through the lierne to the fan vault, have a skeletal 'shell' very much in common, and their basic structural action is the same. This basic action stems from the nature of masonry as a material, and to understand the action it is necessary to construct a structural theory which incorporates the curious properties of masonry. Above all, it is necessary to state clearly the question that is being posed when an engineer undertakes a structural analysis: what is the problem that requires solution?

1.1 Structural criteria

In recent years structural design has come to be viewed in terms of limit states, and indeed the use of these ideas does remind the engineer that a structure must satisfy several of perhaps a large number of criteria. For example, a limit to permitted corrosion, or a restriction of crack width, may play leading roles in the design of a steel or concrete frame, respectively. These two particular criteria may also play a part in the design of masonry, although it seems reasonable to suppose them to be of secondary importance, to be reviewed finally by the designer but not likely to dictate the design.

The three main structural criteria are those of strength, stiffness and stability. The structure must be strong enough to carry whatever loads are imposed, including its own weight; it must not deflect unduly; and it must not develop large unstable displacements, whether locally or overall. If these three criteria can be satisfied, then the designer can run through a check list of secondary limit states to make sure that the structure is otherwise serviceable.

An immediate, and paradoxical, difficulty arises when the ideas of

strength, stiffness and stability are applied to masonry. Ancient structures – the Roman Pantheon, for example, or a Greek temple – seem intuitively to be strong enough; they are still standing, and evidently the loading (self-weight, wind, earthquake) has not, over the centuries, caused failure to occur by fracture of the material. This matter will be discussed further, but it is a fact that mean stresses are low in a typical masonry structure; cracking and local spalling may be seen, but these seem hardly to affect the structural integrity of the whole.

Similarly, the engineer is unlikely to worry, in the first instance, about unduly large working deflexions of the vault of a Gothic cathedral. Strength and stiffness do not lie in the foreground of masonry design. Further, the engineer usually encounters instability as a local phenomenon; a slender steel column must be designed not to buckle, for example, whereas the masonry pier in a nave arcade is 'stocky'. Nevertheless, it is the third major criterion, that of stability, that is relevant for masonry, albeit in a curious form. As an illustration, the masonry arch of fig. 1.1 may be perfectly comfortable under the action of its own weight and at a certain intensity of the superimposed load P. The stresses are low and the deflexions negligible, and both will remain so as the value of P is increased. However, at a certain value of P a sudden change puts an end to this stability. As will be seen, a point is reached at which the structural forces can no longer be contained within the arch; stresses remain low but an unstable mechanism of collapse is formed (the four-bar chain of fig. 1.1(b)).

The semicircular arch of fig. 1.1 will carry a given load P provided that the arch ring has a certain minimum thickness; the design of the arch consists in the process of assigning this thickness for a given span and a given load. Once the design has been made and the arch constructed, then it will at once, and ever after, satisfy the criteria of strength (it will not crush), of stiffness (deflexions will be negligible) and of overall stability (a four-bar chain will never develop). The design consists, somewhat strangely to the mind of the modern engineer, in assigning correct proportions to the arch.

Such a design process would not have seemed strange to an ancient or to a medieval designer. It was precisely rules of proportion that were used by classical builders to design their structures, as is at once evident, for example, from Vitruvius. The brief historical notes in Chapter 8 show that these rules were never lost; they survived the Dark Ages, built in to the secret books of the Masonic lodges, and flourished in the twelfth and thirteenth centuries in the age of High Gothic. Moreover, it will be

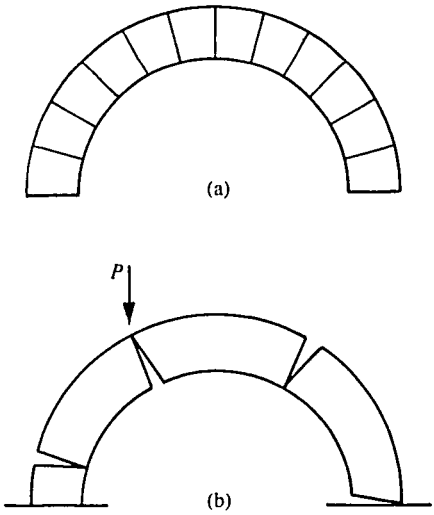

Fig. 1.1. Collapse mechanism for a masonry arch.

seen that rules of proportion give a fundamentally correct understanding of the design and behaviour of masonry.

1.2 Modern analysis

Rules of proportion, then, will be seen to lead (when correctly applied) to a masonry structure that will stand up. No statement is made about any margin of safety (if, indeed, any meaning could be attached to such a statement) nor about the loads which might cause collapse. Ancient and medieval designers did not apparently ask such questions, although they would have been well aware of structural failures – Anthemios and Isidorus only succeeded at their third attempt at a satisfactory design for the dome of Hagia Sofia.

It was Galileo who first considered the analysis of the strength of a structure, in one of his *Two new sciences* of 1638, and thereby signalled the end of medieval structural theory. He posed the problem of the assessment of the strength of a cantilever beam; what was the value of the breaking load? This sort of question may have been asked before (a tree trunk across a ditch will break under a heavy load), but not until Galileo in the context of the development of practical design rules. He wished to determine the strength of the transversely loaded beam as a function of its breadth and depth, so that a formula could be derived

from which the strength of any other (rectangular cross-section) beam could be calculated. This is an example of what a modern engineer recognises as the design process (for strength of the structure).

Galileo solved the problem, essentially correctly, and found that geometrical rules of proportion no longer applied; if the dimensions of the beam were doubled, the strength was very much more than doubled. The new science of structural mechanics was eagerly pursued in the eighteenth century, and the idea of stress slowly emerged; two centuries after Galileo, the problem of the breaking load of a cantilever beam had been transformed into a problem of the determination of the value of stress in that beam. Side by side with these advances through rational mechanics, experiments on commonly used building materials had established reference values of limiting stresses. It was a natural step to attempt to relate the two values, to try to arrange that the calculated working values of stress should have an adequate margin of safety when compared with the known limiting values for the materials used.

Navier (1826) seems to have been the first to declare that the engineer was not in fact interested in the collapse state of the structure (that is, in answering Galileo's question), because all were agreed that it was a state to be avoided. Rather, Navier effectively reasserted the medieval requirement that the building should stand. But this was now to be assured not by assigning certain geometrical proportions to the structure, but by the calculation of stresses throughout its elements. It is, implied Navier, the engineer's job to calculate the actual, or working, state of the structure, and to ensure that the associated stresses do not exceed a safe fraction of their ultimate values.

Prima facie this seems a sensible procedure, but doubts arise when the analytical process is examined in detail. The designer must as a first step find the internal forces in the structure, so that corresponding values of the stresses may be calculated. The first equations written are those of statics; the internal forces must be in equilibrium with the external imposed loads. If these equations can be solved straight away then the first step is complete (and, technically, the structure is statically determinate). Generally, however, the equilibrium equations, standing alone, are insoluble; the structure is statically indeterminate (hyperstatic). There are many possible equilibrium states, that is, there are many ways in which the structure can carry its loads, and other information must be introduced into the analysis in order to determine the actual state.

Before examining this other information, however, it may be noted

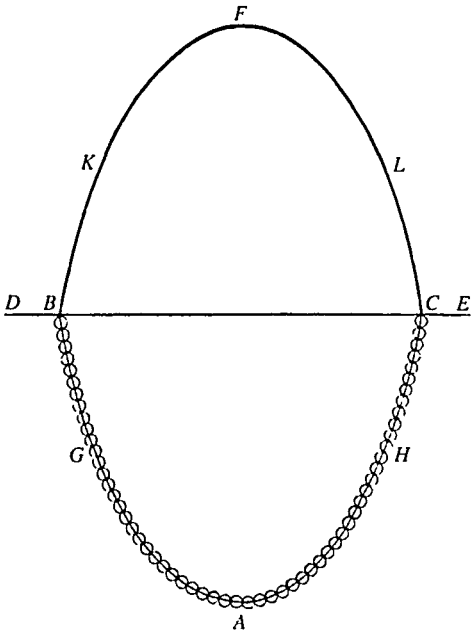

Fig. 1.2. Hooke's hanging chain.

that the arch of fig. 1.1 has infinitely many equilibrium configurations. Robert Hooke had concerned himself with 'the true Mathematical and Mechanical form of all manner of Arches for Building', and he published an anagram in 1675 (inserted in a book on helioscopes) which, rightly read and translated from the Latin, gives the statement: 'As hangs the flexible line, so but inverted will stand the rigid arch.' Hooke was unable to add the mathematics to this powerful theorem, which is illustrated in fig. 1.2. This sketch is due to Poleni (1748), whose work on the dome of St Peter's is discussed later; the shape of the chain hanging in tension loaded by its own weight is the same as that of the arch which will carry the loads in compression.

The geometry of this thrust line, that is, the actual shape of the ideal arch to carry the specified loads, will depend on the length of the equivalent chain and the distance apart of the supports. Thus a possible inverted chain is shown in fig. 1.3 lying within the boundaries of the semicircular arch; this represents one of the infinite number of ways in which the arch can carry its own weight. For masonry, as will be seen, thrusts must lie within the boundaries of the construction, and it is clear

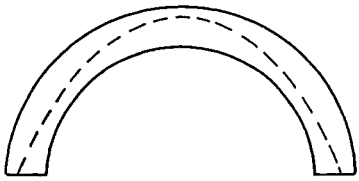

Fig. 1.3. Line of thrust in a circular arch.

that many other catenaries could have been drawn between the extrados and the intrados of the arch of fig. 1.3.

The equations of equilibrium, used alone, do not give enough information to determine the actual position of the thrust line in fig. 1.3. The other two statements of structural analysis must be used. The first of these is a statement of material properties, so that internal deformations of the arch may be related to the internal forces. The second is a geometrical statement, and usually involves some internal or external constraint on the structure; in fig. 1.3, for example, the arch rests firmly on rigid foundations, and the internal deformations of the arch must be such that they are compatible with such imposed boundary conditions – in this case, that the displacements of the arch are zero at the abutments.

1.3 The elastic solution

Navier's design philosophy involves, then, the postulation of an (elastic) law of deformation and the assumption of certain boundary conditions that arise in the solution of the problem. If the arch of fig. 1.3 is supposed to be completely rigid (as, for all practical purposes, it is) then the position of the thrust line cannot be calculated. If, however, the arch is allowed to deform slightly (and the essence of the theory of structures is that it deals with the mechanics of slightly deformable bodies), then enough equations can be made available to solve the hyperstatic problem. If the material of the arch of fig. 1.3 is linear-elastic (or obeys any other known deformation characteristic), and if the abutments are rigid, then a unique position can be calculated for the thrust line which equilibrates the given loads (self-weight and any others specified).

It is at this point that doubts about the procedure arise. Examination of the equations shows that their solution, for a hyperstatic structure, is extraordinarily sensitive to very small variations in the boundary conditions. If one of the supposedly fixed abutments of the arch in fig. 1.3 should suffer a small displacement, this would be accompanied

by a large shift in the position of the thrust line. (Small in this context implies a displacement containable within the thickness of the lines of fig. 1.3. The eye would detect no difference between drawings of the arch in the originally perfect and in the displaced states.)

Now it is certain that one or both of the abutments of the arch will in fact suffer (unpredictable) small displacements, and this then puts in question the whole of this analytical procedure. The 'actual' state can indeed be determined, but only by taking account of the material properties (which may not be well-defined for an assemblage of say stones and mortar), and by making some assumptions about compatibility of deformation – for example, the boundary conditions at the abutments of the arch. Even then, it must be recognized that the 'actual' state of the structure is ephemeral; it could in theory be determined if all the conditions affecting the solution were known exactly, but a severe gale, a slight earth tremor, a change in water table will produce a small change in the way the structure rests on its foundations, and this will produce an entirely different equilibrium state for the structure.

An analogy may help to clarify the discussion. The forces in the legs of a three-legged table are statically determinate; three equations of equilibrium may be written from which the forces in the legs may be evaluated to support a given weight placed in a given location on the table. The addition of a fourth leg makes the problem very much more difficult, since the table is now hyperstatic; the same three equations may be written, but four forces have to be evaluated. The solution to the problem requires a knowledge of the flexibility of the table top, details of its connexion to the legs, the compressibility of the legs themselves, and so on. A computer program could perhaps be used, and the four required values of the forces in the legs can be determined. However, hidden in this program, and perhaps not noticed by the user of the program, there will be implicit boundary conditions; it will be assumed, for example, that the table is standing on a rigid level floor.

Now a real stiff table on a rigid floor will rock; if one leg is clear of the ground by only one millimetre, the force in that leg will be zero, and the forces in the other three legs are then uniquely determined. A moment later the table may be jolted and will shift to another part of the floor; the table is the same, and it is carrying the same load with comfort, but a different leg is now off the floor, with corresponding unique forces in the remaining three. Both of these states of the table are possible equilibrium states, as indeed is the computer-generated state, but none represent the 'actual' state.

1.4 Plastic theory

These conclusions are not academic abstractions. A series of tests carried out by the Steel Structures Research Committee in the early 1930s showed that the stresses (actually strains) measured in practice, in office blocks and hotels, for example, bore almost no relation to those confidently calculated by the designers. Moreover the Committee concluded that there was a problem incapable of solution – practical imperfections of construction and behaviour were inevitable, and would always lead to an unpredictable working state of a structure (in this case, a steel frame). What had gone wrong was the attempt to base the design of a structure on a knowledge of its 'actual' state. If progress were to be made, then this philosophy of design would have to be abandoned.

Common sense would seem to indicate that a severe gale, easily survived but leading to a completely different state of the structure, cannot really have weakened the structure – the four-legged table, accidentally knocked by the waitress, continues to serve its function. Common sense is in this case supported by both theory and experiment. If two seemingly identical structures, but actually with different small imperfections so that they are in very different states of initial stress, are loaded slowly to collapse, then the collapse loads (that is, the strength of the structures) will be found to be the same. It was this observation which led to the development of the so-called plastic theory of structures, applicable to any case where collapse is a ductile quasi-stable (plastic) process. The theory applies therefore to steel and to reinforced-concrete frames and, as will be seen, to masonry, or to any building type using a structurally common material (timber, wrought iron, aluminium alloy), but not to materials like cast iron or glass, which are brittle.

Thus the plastic designer abandons the quest for the actual state of a structure and, instead, examines the way in which that structure might collapse. It is not envisaged that the structure will actually collapse, however. Rather a calculation is based upon loads increased by a hypothetical factor; the master theorem of plastic analysis, the key tool of the designer, then states that the real structure acted upon by the smaller working loads will never collapse.

In the course of the calculation of hypothetical collapse the plastic designer generates an equilibrium state for the structure under its real working loads. Elastic designers believe that they have generated the 'actual' equilibrium state, whereas plastic designers know only that they have generated one particular state out of the infinitely many that are