PRINCIPLES OF MAGNETOHYDRODYNAMICS
With Applications to Laboratory and Astrophysical Plasmas

This textbook provides a modern and accessible introduction to magnetohydrodynamics (MHD). It describes the two main applications of plasma physics – laboratory research on thermonuclear fusion energy, and plasma-astrophysics of the solar system, stars and accretion discs – from the single viewpoint of MHD. This approach provides effective methods and insights for the interpretation of plasma phenomena on virtually all scales, from the laboratory to the Universe. It equips the reader with the necessary tools to understand the complexities of plasma dynamics in extended magnetic structures. The classical MHD model is developed in detail without omitting steps in the derivations, and problems are included at the end of each chapter. This text is ideal for senior-level undergraduate and graduate courses in plasma physics and astrophysics.

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To Antonia and Micheline

Contents

Preface

This book describes the two main applications of plasma physics, laboratory research on thermonuclear fusion energy and plasma-astrophysics of the solar system, stars, accretion discs, etc., from the single viewpoint of magnetohydrodynamics (MHD). This provides effective methods and insights for the interpretation of plasma phenomena on virtually all scales, ranging from the laboratory to the Universe. The key issue is understanding the complexities of plasma dynamics in extended magnetic structures.

The book starts with an exposition of the elements of plasma physics, followed by an in-depth derivation of the MHD model. By means of the conservation laws, different model problems for laboratory and astrophysical plasmas are formulated. The spectral theory of MHD waves and instabilities is then developed in analogy with quantum mechanics. The centrepiece is the analysis of inhomogeneous plasmas with intricate spectral structures that provide a unified view of waves and instabilities in plasmas as different as tokamaks and coronal flux tubes. This is illustrated by the magnetic structures and dynamics observed in the solar system, and analysed in detail for cylindrical flux tubes. Advanced chapters on wave damping and resonant heating expose the wonderful interplay of physics and mathematics.

In order to provide the student with all the tools that are necessary to understand plasma dynamics, the classical MHD model is developed in great detail without omitting steps in the derivations. The necessary restriction to ideal dissipationless plasmas, in static equilibrium and with inhomogeneity in one direction, is more than compensated by the insight gained in the intricacies of magnetized plasmas. With this objective the size of the original manuscript, including advanced topics of magnetohydrodynamics, became impractical so that we decided to split it into two volumes.

In the companion volume *Advanced Magnetohydrodynamics*, that will appear later, the restrictions of the classical theory are relaxed one by one: introducing stationary background flows, resistivity and reconnection, two-dimensional toroidal geometry, linear and nonlinear computational techniques, and transonic flows and shocks. These topics transform the subject into a vital new area with many applications in laboratory (thermonuclear fusion), space (space weather), and astrophysical plasmas (stellar winds, accretion discs and jets).

This book (Volume 1) and its companion (Volume 2) consist of three parts:

- *Plasma Physics Preliminaries* (Volume 1, Chapters 1–3),
- *Basic Magnetohydrodynamics* (Volume 1, Chapters 4–11),
- *Advanced Magnetohydrodynamics* (Volume 2).

Inevitably, with the chosen distinction between topics for Volume 1 (mostly ideal linear phenomena described by self-adjoint linear operators) and Volume 2 (mostly non-ideal and nonlinear phenomena), the difference between 'basic' and 'advanced' levels of magnetohydrodynamics could not be strictly maintained. The logical order required a quite advanced derivation of the MHD equations from kinetic theory (Chapter 3) at an early stage, different sections on advanced topics interspersed throughout the book, and a rather complete discussion of the initial value problem (Chapter 10) at the end. These parts are marked by a star (*) and can be skipped on a first study of the book. The same applies to text put in small print, in between triangles ($\triangleright \cdots \triangleleft$), usually containing tedious derivations or advanced material. The serious student is advised though not to skip the Exercises, which are also put in small print for typographical reasons only. In particular, frequent use of the vector expressions and tables of the appendices is encouraged. The subject of magnetohydrodynamics can only be mastered through extensive practice.

An overview of the subject matter of the different chapters of this volume may help the reader to find his way:

- Chapter 1 gives an introduction to laboratory fusion and astrophysical plasmas, and formulates provisional microscopic and macroscopic definitions of the plasma state.
- Chapter 2 discusses the three complementary points of view of single particle motion, kinetic theory and fluid description. The corresponding theoretical models provide the opportunity to introduce some of the basic concepts of plasma physics.
- Chapter 3 gives the 'derivation' of the macroscopic equations from the kinetic (Boltzmann) equation. The quotation marks because a fully satisfactory derivation cannot be given at present in view of the largely unknown contribution of turbulent transport processes. The presentation given is meant to provide some idea on the limitations of the macroscopic viewpoint.
- Chapter 4 defines the MHD model and introduces the concept of scale independence. The central importance of the conservation laws is discussed at length. Based on this, the similarities and differences of laboratory and astrophysical plasmas are articulated in terms of a number of generic boundary value problems.

- Chapter 5 derives the basic MHD waves and describes their properties, with an eye on their important role in spectral analysis and computational MHD. The theory of characteristics is introduced as a vehicle for the propagation of nonlinear disturbances.
- Chapter 6 treats the subject of waves and instabilities from the unifying point of view of spectral theory. The force operator formulation and the energy principle are extensively discussed. The analogy with quantum mechanics is pointed out and exploited. The difficult extension to interface systems is treated in detail.
- Chapter 7 applies the spectral analysis developed in Chapter 6 to inhomogeneous plasmas in a plane slab. The wave equation for gravito-MHD waves is derived and solved in various limits. Here, all the intricacies of the subject enter: continuous spectra, damping of Alfvén waves, local instabilities, etc. The analogy between helioseismology and MHD spectroscopy in tokamaks is shown to hold great promise for the investigation of plasma dynamics.
- Chapter 8 introduces the enormous variety of magnetic phenomena in astrophysics, in particular the solar system (dynamo, solar wind, magnetospheres, etc.), and provides basic examples of plasma dynamics worked out in later chapters.
- Chapter 9 is the cylindrical counterpart of Chapter 7, with a wave equation describing the various waves and instabilities. It presents the stability analysis of diffuse cylindrical plasmas (classical pinches and present tokamak models) from the spectral perspective.
- Chapter 10 solves the initial value problem for one-dimensional inhomogeneous MHD and the associated damping due to the continuous spectrum.
- Chapter 11 discusses resonant absorption and phase mixing in the context of heating mechanisms of solar and stellar coronae. Anticipating Volume 2, numerical methods to solve these problems are indicated. Sunspot seismology is introduced as another example of MHD spectroscopy.

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