

## Molecular and Cellular Biophysics

This book provides advanced undergraduate and beginning graduate students with a foundation in the basic concepts of molecular and cellular biophysics. Students who have taken physical chemistry and calculus courses will find this book an accessible and valuable aid in learning how these concepts can be used in biological research. The text provides a rigorous treatment of the fundamental theories in biophysics and illustrates their application with examples. Conformational transitions of proteins are studied first using thermodynamics, and subsequently with kinetics. Allosteric theory is developed as the synthesis of conformational transitions and association reactions. Basic ideas of thermodynamics and kinetics are applied to topics such as protein folding, enzyme catalysis and ion channel permeation. These concepts are then used as the building blocks in a treatment of membrane excitability. Through these examples, students will gain an understanding of the general importance and broad applicability of biophysical principles to biological problems.

*Meyer B. Jackson* is the Kenneth Cole Professor of Physiology at the University of Wisconsin Medical School. He has been teaching graduate level biophysics for nearly 25 years.

Cambridge University Press  
0521624703 - Molecular and Cellular Biophysics  
Meyer B. Jackson  
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CAMBRIDGE UNIVERSITY PRESS  
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

CAMBRIDGE UNIVERSITY PRESS  
The Edinburgh Building, Cambridge CB2 2RU, UK

Published in the United States of America by Cambridge University Press,  
New York

[www.cambridge.org](http://www.cambridge.org)  
Information on this title: [www.cambridge.org/9780521624411](http://www.cambridge.org/9780521624411)

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First published 2006

Printed in the United Kingdom at the University Press, Cambridge

*A catalogue record for this publication is available from the British Library*

ISBN-13 978-0-521-62441-1 hardback  
ISBN-10 0-521-62441-X hardback  
ISBN-13 978-0-521-62470-1 paperback  
ISBN-10 0-521-62470-3 paperback

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## Contents

<i>Preface</i>	page xii
<i>Acknowledgements</i>	xiv
<hr/>	
<b>Chapter 1</b>   Global transitions in proteins	1
1.1 Defining a global state	2
1.2 Equilibrium between two global states	4
1.3 Global transitions induced by temperature	5
1.4 Lysozyme unfolding	7
1.5 Steepness and enthalpy	9
1.6 Cooperativity and thermal transitions	11
1.7 Transitions induced by other variables	12
1.8 Transitions induced by voltage	14
1.9 The voltage sensor of voltage-gated channels	17
1.10 Gating current	18
1.11 Cooperativity and voltage-induced transitions	19
1.12 Compliance of a global state	21
<hr/>	
<b>Chapter 2</b>   Molecular forces in biological structures	25
2.1 The Coulomb potential	25
2.2 Electrostatic self-energy	27
2.3 Image forces	29
2.4 Charge–dipole interactions	31
2.5 Induced dipoles	32
2.6 Cation– $\pi$ interactions	33
2.7 Dispersion forces	35
2.8 Hydrophobic forces	36
2.9 Hydration forces	39
2.10 Hydrogen bonds	39
2.11 Steric repulsions	43
2.12 Bond flexing and harmonic potentials	44
2.13 Stabilizing forces in proteins	46
2.14 Protein force fields	50
2.15 Stabilizing forces in nucleic acids	52
2.16 Lipid bilayers and membrane proteins	53
<hr/>	
<b>Chapter 3</b>   Conformations of macromolecules	56
3.1 <i>n</i> -Butane	56
3.2 Configurational partition functions and polymer chains	58
3.3 Statistics of random coils	60
3.4 Effective segment length	62
3.5 Nonideal polymer chains and theta solvents	63

3.6	Probability distributions	65
3.7	Loop formation	66
3.8	Stretching a random coil	67
3.9	When do molecules act like random coils?	68
3.10	Backbone rotations in proteins: secondary structure	68
3.11	The entropy of protein denaturation	71
3.12	The helix-coil transition	73
3.13	Mathematical analysis of the helix-coil transition	74
3.14	Results of helix-coil theory	78
3.15	Helical propensities	80
3.16	Protein folding	82
3.17	Cooperativity in protein folding	86
<b>Chapter 4   Molecular associations</b>		89
4.1	Association equilibrium in solution	89
4.2	Cooperativity	91
4.2.1	Concerted binding	91
4.2.2	Sequential binding	93
4.2.3	Nearest neighbor interactions	94
4.3	Thermodynamics of associations	94
4.4	Contact formation	95
4.5	Statistical mechanics of association	96
4.6	Translational free energy	98
4.7	Rotational free energy	101
4.8	Vibrational free energy	102
4.9	Solvation effects	105
4.10	Configurational free energy	106
4.11	Protein association in membranes - reduction of dimensionality	107
4.12	Binding to membranes	108
<b>Chapter 5   Allosteric interactions</b>		111
5.1	The allosteric transition	112
5.2	The simplest case: one binding site and one allosteric transition	112
5.3	Binding and response	115
5.4	Energy balance in the one-site model	116
5.5	G-protein coupled receptors	117
5.6	Binding site interactions	121
5.7	The Monod-Wyman-Changeux (MWC) model	123
5.8	Hemoglobin	126
5.9	Energetics of the MWC model	127
5.10	Macroscopic and microscopic additivity	128
5.11	Phosphofructokinase	130
5.12	Ligand-gated channels	132

5.13 Subunit-subunit interactions: the Koshland-Nemethy-Filmer (KNF) model	134
5.14 The Szabo-Karplus (SK) model	137
<b>Chapter 6</b>   Diffusion and Brownian motion	142
6.1 Macroscopic diffusion: Fick's laws	142
6.2 Solving the diffusion equation	143
6.2.1 One-dimensional diffusion from a point	144
6.2.2 Three-dimensional diffusion from a point	146
6.2.3 Diffusion across an interface	146
6.2.4 Diffusion with boundary conditions	148
6.3 Diffusion at steady state	150
6.3.1 A long pipe	151
6.3.2 A small hole	152
6.3.3 A porous membrane	153
6.4 Microscopic diffusion - random walks	154
6.5 Random walks and the Gaussian distribution	156
6.6 The diffusion equation from microscopic theory	159
6.7 Friction	160
6.8 Stokes' law	162
6.9 Diffusion constants of macromolecules	163
6.10 Lateral diffusion in membranes	164
<b>Chapter 7</b>   Fundamental rate processes	167
7.1 Exponential relaxations	167
7.2 Activation energies	169
7.3 The reaction coordinate and detailed balance	170
7.4 Linear free energy relations	172
7.5 Voltage-dependent rate constants	175
7.6 The Marcus free energy relation	177
7.7 Eyring theory	179
7.8 Diffusion over a barrier - Kramers' theory	180
7.9 Single-channel kinetics	183
7.10 The reaction coordinate for a global transition	186
<b>Chapter 8</b>   Association kinetics	194
8.1 Bimolecular association	194
8.2 Small perturbations	195
8.3 Diffusion-limited association	197
8.4 Diffusion-limited dissociation	200
8.5 Site binding	201
8.6 Protein-ligand association rates	203
8.6.1 Evolution of speed	205
8.6.2 Acetylcholinesterase	205
8.6.3 Horseradish peroxidase	206
8.7 Proton transfer	207

8.8	Binding to membrane receptors	208
8.9	Reduction in dimensionality	212
8.10	Binding to DNA	214
<b>Chapter 9   Multi-state kinetics</b>		216
9.1	The three-state model	216
9.2	Initial conditions	219
9.3	Separation of timescales	220
9.4	General solution to multi-state systems	221
9.5	The three-state model in matrix notation	225
9.6	Stationarity, conservation, and detailed balance	226
9.7	Single-channel kinetics: the three-state model	229
9.8	Separation of timescales in single channels: burst analysis	232
9.9	General treatment of single-channel kinetics: state counting	235
9.10	Relation between single-channel and macroscopic kinetics	236
9.11	Loss of stationarity, conservation, and detailed balance	237
9.12	Single-channel correlations: pathway counting	240
9.13	Multisubunit kinetics	242
9.14	Random walks and “stretched kinetics”	244
<b>Chapter 10   Enzyme catalysis</b>		248
10.1	Basic mechanisms – serine proteases	248
10.2	Michaelis–Menten kinetics	251
10.3	Steady-state approximations	254
10.4	Pre-steady-state kinetics	256
10.5	Allosteric enzymes	257
10.6	Utilization of binding energy	258
10.7	Kramers’ rate theory and catalysis	259
10.8	Proximity and translational entropy	260
10.9	Rotational entropy	263
10.10	Reducing $E^\ddagger$ : transition state complementarity	264
10.11	Friction in an enzyme–substrate complex	267
10.12	General-acid–base catalysis and Brønsted slopes	268
10.13	Acid–base catalysis in $\beta$ -galactosidase	270
10.14	Catalysis in serine proteases and strong H-bonds	272
10.15	Marcus’ theory and proton transfer in carbonic anhydrase	273
<b>Chapter 11   Ions and counterions</b>		276
11.1	The Poisson–Boltzmann equation and the Debye length	277
11.2	Activity coefficient of an ion	279
11.3	Ionization of proteins	283

11.4	Gouy-Chapman theory and membrane surface charge	285
11.5	Stern's improvements of Gouy-Chapman theory	288
11.6	Surface charge and channel conductance	291
11.7	Surface charge and voltage gating	293
11.8	Electrophoretic mobility	294
11.9	Polyelectrolyte solutions I. Debye-Hückel screening	297
11.10	Polyelectrolyte solutions II. Counterion-condensation	300
11.11	DNA melting	302
<b>Chapter 12</b>   Fluctuations		307
12.1	Deviations from the mean	307
12.2	Number fluctuations and the Poisson distribution	309
12.3	The statistics of light detection by the eye	311
12.4	Equipartition of energy	313
12.5	Energy fluctuations in a macromolecule	315
12.6	Fluctuations in protein ionization	317
12.7	Fluctuations in a two-state system	319
12.8	Single-channel current	320
12.9	The correlation function of a two-state system	322
12.10	The Wiener-Khinchine theorem	324
12.11	Channel noise	327
12.12	Circuit noise	329
12.13	Fluorescence correlation spectroscopy	332
12.14	Friction and the fluctuation-dissipation theorem	336
<b>Chapter 13</b>   Ion permeation and membrane potential		339
13.1	Nernst potentials	339
13.2	Donnan potentials	341
13.3	Membrane potentials of cells	343
	13.3.1 Neurons	345
	13.3.2 Vertebrate skeletal muscle	345
13.4	A membrane permeable to Na <sup>+</sup> and K <sup>+</sup>	347
13.5	Membrane potentials of neurons again	350
13.6	The Ussing flux ratio and active transport	351
13.7	The Goldman-Hodgkin-Katz voltage equation	352
13.8	Membrane pumps and potentials	354
13.9	Transporters and potentials	355
13.10	The Goldman-Hodgkin-Katz current equation	357
13.11	Divalent ions	360
13.12	Surface charge and membrane potentials	361
13.13	Rate theory and membrane potentials	362
<b>Chapter 14</b>   Ion permeation and channel structure		367
14.1	Permeation without channels	367
14.2	The Ohmic channel	370

14.3	Energy barriers and channel properties	371
14.4	Eisenman selectivity sequences	374
14.5	Forces inside an ion channel	376
14.6	Gramicidin A	378
14.7	Rate theory for multibarrier channels	380
14.8	Single-ion channels	384
14.9	Single-file channels	390
14.10	The KcsA channel	394
<b>Chapter 15   Cable theory</b>		400
15.1	Current through membranes and cytoplasm	401
15.2	The cable equation	403
15.3	Steady state in a finite cable	406
15.4	Voltage steps in a finite cable	408
15.5	Current steps in a finite cable	411
15.6	Branches and equivalent cylinder representations	412
	15.6.1 Steady state	413
	15.6.2 Time constants	415
15.7	Cable analysis of a neuron	418
15.8	Synaptic integration in dendrites: analytical models	422
	15.8.1 Impulse responses	423
	15.8.2 Realistic synaptic inputs	425
15.9	Compartmental models and cable theory	428
15.10	Synaptic integration in dendrites: compartmental models	430
<b>Chapter 16   Action potentials</b>		434
16.1	The action potential	434
16.2	The voltage clamp and the properties of Na <sup>+</sup> and K <sup>+</sup> channels	439
16.3	The Hodgkin–Huxley equations	442
16.4	Current–voltage curves and thresholds	447
16.5	Propagation	450
16.6	Myelin	453
16.7	Axon geometry and conduction	455
16.8	Channel diversity	457
16.9	Repetitive activity and the A-current	458
16.10	Oscillations	461
16.11	Dendritic integration	466
<b>Appendix I   Expansions and series</b>		470
A1.1	Taylor series	470
A1.2	The binomial expansion	471
A1.3	Geometric series	471

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<b>Appendix 2</b>	Matrix algebra	472
A2.1	Linear transforms	472
A2.2	Determinants	473
A2.3	Eigenvalues, eigenvectors, and diagonalization	474
<hr/>		
<b>Appendix 3</b>	Fourier analysis	477
<hr/>		
<b>Appendix 4</b>	Gaussian integrals	481
<hr/>		
<b>Appendix 5</b>	Hyperbolic functions	483
<hr/>		
<b>Appendix 6</b>	Polar and spherical coordinates	484
<hr/>		
<i>References</i>		486
<i>Index</i>		504

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## Preface

I have tried to present the subject of biophysics from a conceptual perspective. This needs to be stated because biophysics is too often defined as a collection of physical methods that can be used to study molecular and cellular biology. This technical emphasis often fosters narrowness, and in the worst cases leads to shallowness, where sophisticated measurements are interpreted with little consideration for the physical principles that govern the special complexities of the macromolecular world of biology.

The conceptual emphasis of this book has led to a heavy dose of theory. Theoretical analysis is essential in a conceptual approach, but I must admit that the theoretical emphasis of this book also reflects my own personal fascination with the insights that can be gained by applying physical theory to biological questions. In developing theoretical topics I have tried to be practical. I have steered toward more basic forms of mathematics wherever possible. Much of the analysis is at the level of an introductory calculus course. Where more sophisticated mathematics is involved I have tried to teach the mathematics in parallel with the development of the subject at hand. Six mathematical appendices have been added to help the reader. These may be useful guides, but are certainly not rigorous or thorough. Readers who desire a better background in mathematics will have to find appropriate texts that treat subjects such as matrices and partial differential equations. The relevant chapters in a book on mathematical methods for physics or chemistry will probably fill the gap adequately.

The level of the mathematics is not the critical issue. The most essential pre-requisite here is physical chemistry. Everything has been written with the assumption that the reader has taken an undergraduate course that introduces thermodynamics, kinetics, and statistical mechanics. Some of the essentials are reviewed but my summaries cannot substitute for some intensive study focused on these topics. I also assume that the reader has had some exposure to biochemistry.

The concepts developed here are often quite general, and illustrations with specific examples are vital. Finding suitable examples has been a challenge. I have tried to avoid excessive reliance on examples from areas closer to my own research such as membranes and ion channels, but this has been hard to avoid. The concept teaches the example as often as the example teaches the concept. In order to make this book useful to an audience beyond those who share my particular research interests, I have attempted to cast a wide net and roam far and wide to present examples from the many different fields that biophysicists study.

Much of this book presents subjects that are fundamental but have not yet found their way into textbooks. Distilling such work

and rendering it in an accessible form requires difficult decisions to be made about organization and topic selection. I can only hope that this has been successful. I am painfully aware of the many interesting and important aspects of biophysics that I have not written about. However, there is already more than enough here for a one semester course for advanced undergraduates and beginning graduate students. I can only hope that studying this book will bring the many omitted topics within reach of the initiated students.

The material covered in this book varies in difficulty. Sections that are more difficult and not essential for continuity are designated with a star (\*).

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## Acknowledgements

I owe a very special thanks to two graduate students who worked in my laboratory while I was in the final stage of writing this book. I originally asked Payne Chang and Xue Han to read a few chapters, but in the end they read every page. They have done a remarkable job of finding errors and requesting greater clarity. They both followed the Chinese adage “I respect my professor but I respect the truth more,” to the enormous benefit of this book.

I am also indebted to the following friends and colleagues for critical comments on one or more chapters: Ed Chapman, Claudio Grossmann, Enfu Hui, Matt Jones, Peter Jordan, Stuart Licht, Andrew Lokuta, Cathy Morris, Bob Pearce, Steve Redman, Kimberly Taylor, Jeff Walker, and Jim Weisshaar. A final thanks to Adam Van Wynsberghe for help with the cover picture.