Fast analytical techniques for electrical and electronic circuits

Today, the only method of circuit analysis known to most engineers and students is nodal or loop analysis. Although this works well for obtaining numerical solutions, it is almost useless for obtaining analytical solutions in all but the simplest cases.

In this unique book, Vorpérian describes remarkable alternative techniques to solve, almost by inspection, complicated linear circuits in symbolic form and obtain meaningful analytical answers for any transfer function or impedance.

Although not intended to replace traditional computer-based methods, these techniques provide engineers with a powerful set of tools for tackling circuit design problems. They also have great value in enhancing students' understanding of circuit operation. The numerous problems and worked examples in this book make it an ideal textbook for senior/graduate courses or a reference book.

This book will show you how to:

- use less algebra and do most of it directly on the circuit diagram,
- obtain meaningful analytical solutions to complex circuits with reactive elements and dependent sources by reducing them to a set of simple and purely resistive circuits which can be analyzed by inspection,
- analyze feedback amplifiers easily using the simplest and most natural formulation,
- analyze PWM converters easily using the model of the PWM switch.

Originally developed and taught at institutions and companies around the world by Professor David Middlebrook at Caltech, the extended and new techniques described in this book are an indispensable set of tools for linear electronic circuit analysis and design.

Vatché Vorpérian received his PhD in Electrical Engineering in 1984 from the California Institute of Technology and joined the faculty of Electrical Engineering at Virginia Tech in the same year. In 1991 he joined the Jet Propulsion Laboratory where he is currently a senior member of the technical staff. He has published over 35 conference and journal papers in the field of power electronics and has taught many professional advancement courses to industry.

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To my parents Edward and Azadouhi Vorpérian

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Preface

The title of this book could easily have been called *Variations on a Theme by Middlebrook*, or *Applications of The Extra Element Theorem and its Extensions*. Neither title, however, would have captured the unique message of this book that one can solve very complicated linear circuits in symbolic form almost by inspection and obtain more than one meaningful analytical answer for any transfer function or impedance. The well-known and universally practiced method of nodal or loop analysis not only becomes intractable when applied to a complicated linear circuit in symbolic form, but also yields unintelligible answers consisting of a massive collection of symbols. In a meaningful analytical answer, the symbols must be grouped together in *low-entropy* form – a term coined by R. D. Middlebrook – clearly indicating series and parallel combination of circuit elements, and sums and products of time constants. The illustrative examples in Chapter 1 serve as a quick and informal introduction to the basic concepts behind the radically different approach to network analysis presented in this book.

Today, the only method of circuit analysis known to most engineers, students and professors is the method of nodal or loop analysis. Although this method is an excellent general tool for obtaining *numerical solutions*, it is almost useless for obtaining *analytical solutions* in all but the simplest cases. Anyone who has attempted inverting a matrix with symbolic entries – sometimes as low as second-order – knows how tedious the algebra can get and how ridiculous the resulting high-entropy expressions can look. The purpose of this book is not to eliminate the linear algebra approach to network analysis, but instead to provide additional new and efficient tools for obtaining analytical solutions with great ease and without letting the algebra run into a brick wall.

Among the most important techniques discussed in this book are the extra element theorem (EET) and its extension the *N*-extra element theorem (NEET). These two theorems are discussed in Chapters 3 and 4 after a brief and essential review of transfer functions given in Chapter 2. The EET and its proof were given by R. D. Middlebrook. The NEET was given without proof by Sarabjit Sabharwal, an undergraduate at Caltech in 1979. In Chapter 4, a completely original treatment of the NEET is given, where it is stated in its most general form using a new compact notation and, for the first time, proven directly using matrix analysis.

The subject of electronic feedback is treated in Chapter 5 using the EET for dependent sources, and another theorem by R. D. Middlebrook called simply "the feedback theorem". Both methods lead to a much more *natural* formulation of electronic feedback than the well-known block diagram approach found in most textbooks. Block diagrams are useful tools in linear system theory to help visualize abstract concepts, but they tend to be very awkward tools in network analysis. For instance, in an electronic feedback circuit neither the impedance loading nor the bi-directional transmission of the feedback network are easily captured by the single-loop feedback block diagram unless the feedback network and the amplifier circuit are both manipulated and *forced* to fit the block diagram. The fact is block diagrams bear little resemblance to circuits and their use in network analysis mainly results in loss of time and insight.

The examples presented in Chapters 6 and 7 are a *tour de force* in analysis of complicated circuits which demonstrate the efficacy of the fast analytical techniques developed in the previous chapters. Among the examples discussed in these chapters are higher-order passive filters and a MESFET amplifier. Some infinite networks, including fractal networks, are discussed in Chapter 7 where an interesting, and possibly new, result is presented. It is shown that a resistor, an inductor and a capacitor are all special cases of a single, two-terminal, linear element whose voltage and current are related by a fractional derivative or its inverse, the Riemann–Liouville fractional integral.

Pulse-width-modulated (PWM) switching dc-to-dc power converters are introduced in Chapter 8 to illustrate further the applications of the fast analytical techniques presented in this book. The analysis of PWM converters has been one of the hot topics of nearly every conference in power electronics since the early 1970s, and many specialized analytical techniques have been developed since. The simplest and fastest of these techniques is based on the equivalent circuit model of the PWM switch, which is introduced after a discussion of basic PWM converters. The PWM switch is a three-terminal nonlinear device which is solely responsible for the dc-to-dc conversion function inside a PWM converter. Hence, the PWM switch and its equivalent circuit model are to a PWM converter what the transistor and its equivalent circuit are to an amplifier. To analyze the dynamics of a PWM converter, one simply replaces the PWM switch with its equivalent circuit model and proceeds in exactly the same way as in an amplifier circuit analysis.

This book is based on my experience in electronic circuit analysis as a student, design engineer, teacher and researcher. The limitations of the "standard" circuit analysis I studied as an undergraduate soon became apparent on my first job as a power supply design engineer at Digital Equipment Corporation, Maynard, MA. I spent inordinate amounts of time deriving various small-signal transfer functions of switching converters in order to understand and improve their stability and dynamic behavior. Most of the senior engineers around me had acquired excellent design skills mostly by experience and did not rely too much on analysis. When I returned to graduate school at Caltech, I took Middlebrook's course which engendered a complete turn around: I learned how to handle complicated linear networks and obtain transfer functions, in low-entropy form, using very simple and elegant techniques. I gradually adopted these techniques in my seven years of teaching at Virginia Polytechnic Institute and State University confirming the adage, "the best way to learn something is to teach it."

Logically, Middlebrook's book, which is still in preparation, should have preceded mine. I began writing this book in the summer of 1996 with the intention of completing it by the winter of 1997. Clearly, I did not realize that writing a book at nights and on weekends would be considerably more difficult and time consuming than I had ever imagined. Fortunately, I had the constant support and encouragement of family, friends and colleagues. I would especially like to thank Gene Wester and Dave Rogers, both at the Jet Propulsion Laboratory, for their careful review and corrections of some of the chapters of this book. I would also like to thank my former supervisor Robert Detwiler; my current supervisor Mark Underwood; my colleagues Chris Stell, Tony Tang, Roman Gutierrez, Avo Demirjian, Dan Karmon, Mario Matal, Joseph Toczylowsky, Karl Yee, James Gittens, Mike Newell, David Hykes, Chuck Derksen and Tien Nguyen for making JPL an enjoyable place to work. Although this book is dedicated to my parents for their countless sacrifices, I would not have been able to write it without the enduring support, love and care of my favorite mezzo-soprano, best friend and wife Shoghig.

Vatché Vorpérian
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