Molecular and Cellular Biophysics

This book provides advanced undergraduate and beginning graduate students with a foundation in the basic concepts of molecular and cellular biophysics. Students who have taken physical chemistry and calculus courses will find this book an accessible and valuable aid in learning how these concepts can be used in biological research. The text provides a rigorous treatment of the fundamental theories in biophysics and illustrates their application with examples. Conformational transitions of proteins are studied first using thermodynamics, and subsequently with kinetics. Allosteric theory is developed as the synthesis of conformational transitions and association reactions. Basic ideas of thermodynamics and kinetics are applied to topics such as protein folding, enzyme catalysis and ion channel permeation. These concepts are then used as the building blocks in a treatment of membrane excitability. Through these examples, students will gain an understanding of the general importance and broad applicability of biophysical principles to biological problems.

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Molecular and Cellular Biophysics

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I have tried to present the subject of biophysics from a conceptual perspective. This needs to be stated because biophysics is too often defined as a collection of physical methods that can be used to study molecular and cellular biology. This technical emphasis often fosters narrowness, and in the worst cases leads to shallowness, where sophisticated measurements are interpreted with little consideration for the physical principles that govern the special complexities of the macromolecular world of biology.

The conceptual emphasis of this book has lead to a heavy dose of theory. Theoretical analysis is essential in a conceptual approach, but I must admit that the theoretical emphasis of this book also reflects my own personal fascination with the insights that can be gained by applying physical theory to biological questions. In developing theoretical topics I have tried to be practical. I have steered toward more basic forms of mathematics wherever possible. Much of the analysis is at the level of an introductory calculus course. Where more sophisticated mathematics is involved I have tried to teach the mathematics in parallel with the development of the subject at hand. Six mathematical appendices have been added to help the reader. These may be useful guides, but are certainly not rigorous or thorough. Readers who desire a better background in mathematics will have to find appropriate texts that treat subjects such as matrices and partial differential equations. The relevant chapters in a book on mathematical methods for physics or chemistry will probably fill the gap adequately.

The level of the mathematics is not the critical issue. The most essential prerequisite here is physical chemistry. Everything has been written with the assumption that the reader has taken an undergraduate course that introduces thermodynamics, kinetics, and statistical mechanics. Some of the essentials are reviewed but my summaries cannot substitute for some intensive study focused on these topics. I also assume that the reader has had some exposure to biochemistry.

The concepts developed here are often quite general, and illustrations with specific examples are vital. Finding suitable examples has been a challenge. I have tried to avoid excessive reliance on examples from areas closer to my own research such as membranes and ion channels, but this has been hard to avoid. The concept teaches the example as often as the example teaches the concept. In order to make this book useful to an audience beyond those who share my particular research interests, I have attempted to cast a wide net and roam far and wide to present examples from the many different fields that biophysicists study.

Much of this book presents subjects that are fundamental but have not yet found their way into textbooks. Distilling such work
and rendering it in an accessible form requires difficult decisions to be made about organization and topic selection. I can only hope that this has been successful. I am painfully aware of the many interesting and important aspects of biophysics that I have not written about. However, there is already more than enough here for a one semester course for advanced undergraduates and beginning graduate students. I can only hope that studying this book will bring the many omitted topics within reach of the initiated students.

The material covered in this book varies in difficulty. Sections that are more difficult and not essential for continuity are designated with a star (•).
I owe a very special thanks to two graduate students who worked in my laboratory while I was in the final stage of writing this book. I originally asked Payne Chang and Xue Han to read a few chapters, but in the end they read every page. They have done a remarkable job of finding errors and requesting greater clarity. They both followed the Chinese adage “I respect my professor but I respect the truth more,” to the enormous benefit of this book.

I am also indebted to the following friends and colleagues for critical comments on one or more chapters: Ed Chapman, Claudio Grossmann, Enfu Hui, Matt Jones, Peter Jordan, Stuart Licht, Andrew Lokuta, Cathy Morris, Bob Pearce, Steve Redman, Kimberly Taylor, Jeff Walker, and Jim Weisshaar. A final thanks to Adam Van Wynsberghe for help with the cover picture.