Polarimetric Doppler weather radar: principles and applications

This book provides a detailed introduction to the principles of Doppler and polarimetric radar, focusing in particular on their use in the analysis of precipitation. The design features and operation of practical radar systems are highlighted throughout the book in order to illustrate important theoretical foundations.

The authors begin by discussing background topics such as electromagnetic scattering, polarization, and wave propagation. They then deal in detail with the engineering aspects of pulsed Doppler polarimetric radar, including the relevant signal theory, spectral estimation techniques, and noise considerations. They close by examining a range of key applications in meteorology and remote sensing.

The book will be of great use to graduate students of electrical engineering and atmospheric science as well as to practitioners involved in the meteorological application of radar systems.

V. N. Bringi received his Ph.D. from Ohio State University and is a Professor of Electrical and Computer Engineering at Colorado State University. He is a pioneer in the field of polarimetric radar meteorology and has published over 100 technical articles.

V. Chandrasekar received his Ph.D. from Colorado State University, where he is a Professor of Electrical and Computer Engineering. He has over twenty years' experience working with various aspects of radar systems. He has also been a visiting professor at The National Research Council of Italy.

POLARIMETRIC DOPPLER WEATHER RADAR

Principles and applications

Professor V. N. Bringi

Professor V. Chandrasekar

CAMBRIDGE
UNIVERSITY PRESS

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York, NY 10011-4211, USA
10 Stamford Road, Oakleigh, VIC 3166, Australia
Ruiz de Alarcón 13, 28014, Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa

http://www.cambridge.org

First published 2001

Printed in the United Kingdom at the University Press, Cambridge

Typeface Times 10/13pt. *System* LaTeX 2$_\varepsilon$ [DBD]

A catalogue record of this book is available from the British Library

ISBN 0 521 62384 7 hardback

With humility and reverence, this work is placed at the feet of Ganesha, the remover of all obstacles.

Contents

Preface

Doppler radars are now considered to be an indispensable tool in the measurement and forecasting of atmospheric phenomena. The deployment of WSR–88D radars, the terminal Doppler weather radar (TDWR) at major airports, and wind-profiling radars in the USA can all be considered as major milestones in the operational application of the Doppler principle. Another milestone is the successful deployment of the first precipitation radar in space as part of the Tropical Rainfall Measurement Mission (TRMM). Measurement of the reflectivity and velocity of precipitation particles basically exploits the information contained in the amplitude and phase of the scattered electromagnetic wave. In the last two or three decades, it has become increasingly clear that significant information is also contained in the polarization state of the scattered wave. The physical and experimental basis for the application of radar polarimetry to the study of precipitation is the main subject of this book.

The evaluation of polarimetric measurement options for operational WSR–88D radars is gaining momentum and, if realized, will result in widespread application of polarimetric techniques by a broad segment of the meteorological community. Basic and applied research in polarimetric radar meteorology continues to be strong world-wide, especially in Europe and Japan. We believe that the time has arrived for a detailed treatment of the physical principles underlying coherent polarimetric radar for meteorological applications.

This book is based in part on a graduate class taught by the authors at Colorado State University since the mid-1980s. Our goal has been to present the subject, essentially from first principles, in a self-contained format. Substantial thought has gone into selection and organization of the material in order to provide a good balance between theoretical rigor and practical applications. Examples of radar measurements are frequently given to illustrate the theory and to motivate the reader. Mathematical and physical background normally acquired in an undergraduate program in physics, atmospheric science, or electrical engineering is assumed.

The book is organized into eight chapters and five appendices. Notes are provided at the end of each chapter that provide suggestions for further reading, websites for description of specialized instruments, and examples of data not usually available in the archival journals. The Website for this book is www.engr.colostate.edu/ece/radar_education where specialized software, homework assignments, etc., are available.

Chapters 1–4 provide the foundation for wave scattering and propagation theory as applied to radar polarimetry. Chapter 1 covers a number of important electromagnetic concepts starting with Maxwell's equations and leading to Rayleigh scattering and dielectric mixing formulas. The bistatic Doppler frequency formula is derived by considering an example of a moving loop illuminated by a plane wave. The time-correlated bistatic cross section of a moving particle is defined, which naturally leads to the Doppler spectrum. The elements of a generic pulsed Doppler radar system are then described.

Chapter 2 deals with the amplitude scattering matrix of spheroids in the Rayleigh–Gans limit. In fact, it is quite remarkable that important polarimetric radar observations of precipitation can, to a large degree, be explained using the spheroidal model. For completeness, the Mie scattering solution is formulated as a boundary value problem, with a review of spherical harmonics provided in Appendix 2.

Chapter 3 provides a detailed description of wave, antenna, and radar polarization. The dual-polarized radar range equation is derived in linear and circular polarization bases. The conventional radar cross section concept is generalized to include the concepts of copolar and cross-polar polarization synthesis and optimal polarizations. Scattering from randomly distributed precipitation particles is treated using the Mueller matrix, which leads to the definition of conventional polarimetric observables such as differential reflectivity, linear depolarization ratio, etc. The polarimetric covariance matrix is defined, which represents complete measurements from a dual-polarized radar. Much of this chapter deals with the structure of the covariance matrix in linear and circular bases including simplifications afforded by symmetry arguments.

Chapter 4 deals with dual-polarized wave propagation in precipitation media and how differential attenuation and differential phase between the two "characteristic" waves can be measured. The structure of the propagation-modified covariance matrix in linear and circular bases is treated in detail. Radar measurements in linear and circular bases are used to illustrate the theory. The hybrid measurement mode, which is currently being explored for the WSR–88D radar system, is explained.

Doppler radar theory, signal statistics, and signal processing form the subject matter of Chapter 5 and are dealt with in a fairly rigorous manner. Although a very brief review of signals and systems theory is provided, the level of treatment assumes prior exposure and familiarity with linear systems theory. The application of Doppler radar to wind retrieval or to the study of storm dynamics is not covered as these topics are dealt with in detail in existing books (Sauvageot 1992; Doviak and Zrnić 1993).

Chapters 6–8, which form nearly 50% of the book, deal with dual-polarized radar systems and applications to meteorology, and would be of most relevance to professionals in the field. Chapter 6 describes how polarization diversity and agility are configured in different radar systems. The topics of antenna performance and system polarization errors are treated in detail. A significant portion of this chapter is devoted to estimation of covariance matrix elements from signal samples under three different pulsing schemes. The practical topic of deriving specific differential phase from range profiles of differential propagation phase is also covered in detail.

Chapter 7 deals with the polarimetric basis for characterizing precipitation and describes methods used to infer hydrometeor types and amounts. Commensurate with the importance of this topic, this chapter occupies nearly one-third of the book. A large number of examples are used to illustrate different methods and approaches. Correction of measured reflectivity and differential reflectivity for attenuation, and differential attenuation due to rain along the propagation path are treated in detail. This chapter ends with a description of fuzzy logic methods applied to the problem of hydrometeor classification, and several examples are provided. With real-time implementation of such methods already in place on some research radars, this topic is both timely and relevant for applications.

Chapter 8 treats the radar rainfall measurement problem from the viewpoint of both physically based and statistical/engineering-based approaches. The error structure of polarimetric-based rain rate algorithms is explained via simulations. The theoretical basis of the area–time integral (ATI) and probability-matched methods (PMM) is covered as well as their polarimetric extensions. A separate section on neural networks and their application to rainfall estimation is included together with examples of performance.

Five appendices are provided, three of which are reviews (electrostatics, spherical harmonics, and the T-matrix method). Appendix 4 derives the transmission matrix, while Appendix 5 details procedures for calculating the variance of the magnitude and phase of correlation functions. It is beyond the scope of this book to give a historical account of polarimetric Doppler radar, for which we defer to *Radar in Meteorology* (American Meteorological Society 1990, Ed. D. Atlas). Since emphasis in this book is primarily on pulsed, dual-polarized Doppler radar, other radar techniques, such as frequency modulated–continuous waveform (FM–CW), pulse compression, or synthetic aperture radar are not covered, nor are radars based on airborne or spaceborne platforms. However, the principles covered in this book will be helpful to the reader wishing to pursue these topics.

Finally, the opportunity to pursue exciting interdisciplinary research, to teach classes in our research area, and to interact with graduate students both in electrical engineering and atmospheric science, have all played a significant role in the writing of this book. Indeed, it is our hope that students will enjoy learning from this book. At the same time, we hope that researchers in the field will find the topics both timely and helpful for their own work.

V. N. Bringi
V. Chandrasekar

Acknowledgments

It is a great pleasure for us to acknowledge the assistance of our colleagues and students. To our students and post-doctoral fellows (Ji Ran, Yoong-Goog Cho, Gwo-Jong Huang, Hongping Liu, Li Liu, John Hubbert, John Beaver, Steve Bolen, Gang Xu, Konrad Gojara, Max Seminario, A. Al-Zaben, and L. Ramaswamy), we are indeed grateful for their assistance given in many forms. A number of our colleagues carefully read draft versions of many of the chapters and provided valuable feedback for which we are grateful (Archibald Hendry, Richard Doviak, Eugene Mueller, Anthony Holt, John Hubbert, Richard Strauch, Eugenio Gorgucci, J. Vivekanandan, and David Brunkow). Many colleagues provided us with original figures which are included in the book, and these have been acknowledged with each figure. The Bureau of Meteorology Research Center, Melbourne, Australia provided C-POL radar data and disdrometer data described in Chapter 7. The 2D-video disdrometer data were provided by Joanneum Research, Graz, Austria. A significant impetus which motivated us in large part to the writing of this book was the relocation of the CHILL radar to Colorado State University in 1990, and now referred to as the CSU–CHILL radar facility operated via a cooperative agreement with the National Science Foundation. The close association that the authors have had with this facility (Eugene Mueller and David Brunkow) and with the Department of Atmospheric Science (Steve Rutledge) has without doubt lead to a better exposition of the subject matter of this book. Another significant factor has been the continued research support from the National Science Foundation (Ron Taylor, Program Manager for Physical Meteorology, now retired) and from the US Army Research Office (Walter "Bud" Flood, retired, and Walter Bach) for our polarimetric radar research, particularly from the early 1980s to the mid-1990s. We are grateful for this continuous period of support during which much of the earlier exciting research results using the NCAR CP-2 radar were obtained (in collaboration with Thomas Seliga and Kultegin Aydin). One of the authors (V.C.) also acknowledges close collaboration with Eugenio Gorgucci and Gianfranco Scarchilli, particularly with reference to polarimetric rain rate algorithms in Chapter 8. Finally, both authors are especially grateful for the support of their respective spouses (Sreedevi Bringi and Nirmala Chandra) during the writing of this book, and to their parents for being a continuous source of inspiration.

Cambridge University Press
0521623847 - Polarimetric Doppler Weather Radar: Principles and Applications
V. N. Bringi and V. Chandrasekar
Frontmatter
More information

Notation

List of symbols

a — radius of a sphere or radius of the principal equatorial circle of a spheroid (Fig. 1.8*a*)

a_n^s — expansion coefficients of the scattered electric field (van de Hulst definition), Section 2.4.1

A — specific attenuation, eq. (1.133)

A_{dp} — specific differential attenuation between the two characteristic waves, eq. (4.7)

$A_{h,v}$ — specific attenuation at horizontal (h) or vertical (v) polarization states, eq. (4.76)

A_{sm} — asymmetry ratio, eq. (2.99)

ATI — area–time integral

$\vec{A}$ — magnetic vector potential, eq. (1.13a)

b — semi-major axis length of a prolate spheroid or semi-minor axis length of an oblate spheroid (Fig. 1.8*a*)

b_n^s — expansion coefficients of the scattered electric field (van de Hulst definition), Section 2.4.1

B — bandwidth of a radar receiver

$\vec{B}^{i,s}$ — magnetic field; superscripts i, s correspond to incident and scattered waves

c — speed of light in vacuum

c — fractional volume concentration, eq. (1.69)

D — diameter of a sphere or characteristic dimension of a spheroid (equi-volume spherical diameter)

$D_{\max}$ — maximum diameter in a measured size distribution

D_m — mass-weighted mean diameter, eq. (7.13)

D_0 — median volume diameter, eq. (7.11)

D_z — reflectivity-weighted mean diameter, eq. (7.25)

$\hat{e}_{i,r,s}$ — unit polarization vector of the incident (i), reflected (r), or scattered (s) waves

$\hat{e}_{h,v}$ — unit polarization vector for horizontal (h) and vertical (v) states

$\hat{e}_{R,L}$ — unit polarization vector for right-hand (R) and left-hand (L) circular states, eqs. (3.6, 3.8)

$\hat{e}_t$ — unit polarization vector of the transmitting antenna

E_0 — electric field amplitude

$E_h^{i,r,s}$; $E_v^{i,r,s}$ — components of the electric field in the (h, v)-basis, superscript (i, r, s) refers to incident, reflected, or scattered waves, respectively

E_R; E_L — components of the electric field in circular basis (R = right-hand component, L = left-hand component)

$E_l^{i,s}$; $E_r^{i,s}$ — components of the electric field in the van de Hulst convention (Section 2.4.1); i, s refer to incident

	and scattered fields, l, r refer to parallel (l) and perpendicular (r) components
$\vec{E}^i$	incident electric field vector
$\vec{E}_T^{\text{in}}$	total electric field inside the scatterer, eq. (1.18b)
$\vec{E}^r$	reflected electric field vector
$\vec{E}^s$	scattered electric field vector
$\mathbf{E}^i$; $\mathbf{E}^r$; $\mathbf{E}^s$	2×1 column, or Jones vector, of the incident, reflected, or scattered electric field, eqs. (2.7, 2.12)
f	frequency
f	ice fraction, eq. (7.87c)
f_c	carrier frequency
f_D	Doppler frequency
f_{IF}	intermediate frequency
f_h; f_v	normalized antenna power pattern functions, eq. (6.20)
f_h; f_v	forward scattering amplitude at horizontal and vertical polarizations, eq. (8.5)
$f_{\varepsilon h}$; $f_{\varepsilon v}$	cross-polar pattern functions, eq. (6.21)
$f(\theta, \phi)$	normalized antenna pattern function, eq. (5.38)
$f(R_T)$	rain volume fraction above a threshold rain rate, eq. (8.39b)
f_η	normalized range–time profile of reflectivity, eq. (5.40)
$f(x)$	probability density function, where x can be voltage, phase, power, etc.
$f_D(D)$	probability density function of drop diameter, eq. (8.2)
$\vec{f}$	vector scattering amplitude, eq. (1.24a)
F	dielectric factor, eq. (1.69)
F	volume flux of rain, eq. (8.35)
F	noise figure, eq. (5.144)
F_e	equivalent noise figure, eq. (5.151)
$\vec{F}$	vector radiation amplitude, eq. (3.30)
$g(t)$	impulse response of a receiver filter, eq. (5.51)
G_0	peak boresight gain of an antenna, eq. (5.38)
G_r	receiver power gain
$G(\theta, \phi)$	antenna power gain function, eq. (5.38)
$G_{h,v}$	antenna power gain function associated with the h- or v-port of a dual-polarized antenna, eq. (6.19)
G_{0h}; G_{0v}	peak boresight gains associated with the h- or v-port of a dual-polarized antenna
G_R; G_L	antenna power gain functions associated with the right-hand (R)-port or left-hand (L)-port of a dual-circularly polarized antenna, eq. (3.102)
$G(f)$	frequency-response function of a receiver, eq. (5.50)
$G_n(f)$	normalized frequency response function of a receiver
$h_n^{(2)}$	spherical Hankel function of the second kind, eq. (A2.6)
$h(\tau; t)$	impulse response of the medium, eq. (5.101)
$\hat{h}_{i,r,s}$	unit horizontal polarization vector of incident, reflected, or scattered electric field
$\vec{h}$	effective vector "length" of an antenna, eq. (3.37)
$H(f; t)$	frequency response function of the medium, eq. (5.100)
H_{dr}	hail signature function, eq. (7.88)
i	$\sqrt{-1}$
$\hat{i}$	unit vector along the direction of the incident wave (Fig. 1.7)

I	in-phase component of a signal
I	element of the 4×1 Stokes' vector, eq. (3.25a)
$\mathbf{I}$	4×1 Stokes' vector, eq. (3.25)
$\bar{\bar{\mathbf{I}}}$	identity tensor, or unit diagonal matrix, eq. (2.28b)
j	$\sqrt{-1}$
j_n	spherical Bessel function, eq. (A2.6)
$\mathbf{J}$	coherency matrix, eq. (3.146). Additional subscripts cp and up define completely polarized and completely unpolarized waves, respectively
$\mathbf{J}_{10}$; $\mathbf{J}_{01}$	coherency matrix when the transmitted wave is horizontally polarized (subscript "10") or vertically polarized (subscript "01"), eq. (3.181)
$\vec{J}_p$	polarization current density, eq. (1.54)
k	Boltzman constant, 1.38×10^{-23} J K^{-1}, eq. (5.140)
k	wave number in a dielectric, eq. (1.8)
k_{eff}	effective wave number (or propagation constant) in a composite material, eq. (1.70)
$k_{\text{eff}}^{\text{re}}$; $k_{\text{eff}}^{\text{im}}$	real (re) or imaginary (im) component of the effective propagation constant, eq. (1.129)
k_{eff}^{h}; k_{eff}^{v}	effective propagation constant when the two characteristic waves are polarized horizontally (h) and vertically (v), eq. (4.67)
$k_{\text{re}}^{h,v}$; $k_{\text{im}}^{h,v}$	real (re) and imaginary (im) components of k_{eff}^{h} or k_{eff}^{v}, eq. (4.68a,b)
$\hat{k}_{i,r,s}$	unit vector along direction of the incident (i), reflected (r), or scattered (s) waves
$\lvert K_{p,w}\rvert^2$	dielectric factor of a particle (p) or of water (w), eqs. (3.167, 5.49a)
K_{dp}	specific differential phase, eq. (4.8)
$\mathbf{K}$	covariance matrix of signal samples, eq. (6.73)
l_r	finite bandwidth loss factor, eq. (5.56)
l_{sw}	insertion loss factor of a polarization switch, eq. (6.61)
l_{wg}	waveguide loss factor, eq. (6.53)
L	loss factor of a lossy device, eq. (5.152)
L	linear depolarization ratio; $L = 10^{0.1(\text{LDR})}$, eq. (3.232)
LDR$_{vh}$; LDR$_{hv}$	linear depolarization ratio for an ensemble of particles, subscript "vh" stands for horizontal transmit/vertical receive, while "hv" stands for vertical transmit/horizontal receive, eq. (3.169)
m	refractive index, eq. (1.126)
$\vec{M}$	vector spherical harmonic, eq. (A2.9a)
$M_{h,v}$	transmitter wave amplitudes exciting the horizontal or vertical, h- or v-, ports of a dual-polarized antenna. $M_h = 1$, $M_v = 0$, represents transmission of a h-polarized wave, while $M_h = 0$, $M_v = 1$, represents a v-polarized wave, eq. (3.59)
$M_{R,L}$	as above except for the right-hand (R) or left-hand (L) ports of a dual-polarized antenna, eq. (3.102)
$\mathbf{M}$	4×4 Mueller matrix, eq. (3.28)
n; n_c; N	number of particles per unit volume, eqs. (4.3), (7.26), (1.64)
N	number of signal samples

N_0	intercept parameter of an exponential or gamma drop size distribution, eq. (7.12)
$N_0/2$	power spectral density of "white" noise, eq. (5.141)
N_w	"intercept" parameter of a normalized gamma drop size distribution, eq. (7.61)
$N(D)$	drop size distribution, eq. (1.128)
$\vec{N}$	vector spherical harmonic, eq. (A2.9b)
p, χ	parameters representing differential attenuation and differential phase between the two characteristic waves, eq. (4.13b)
$p_{x,y,z}$	Cartesian components of the dipole moment vector, eq. (1.36a)
$p(t)$	impulse train sampling function, eq. (5.111)
$p(\theta)$; $p(\phi)$	probability density function (pdf) of the spherical coordinate angles θ or ϕ
$p(\Omega)$	pdf of the symmetry axis in the solid angle interval
$\vec{p}$	vector dipole moment, eq. (1.32c)
P_t	transmitter pulse power, eq. (3.33)
$P_t^{h,v}$	transmitter pulse power coupled to the horizontal or vertical, h- or v-ports of a dual-polarized antenna, eqs. (3.57, 3.58)
$P_t^{R,L}$	as above except for a dual-circularly polarized antenna
P_{rad}	power radiated by an antenna, eq. (3.31)
P_{rec}	received power, eq. (3.130)
$P_{h,v}$	power received by the h-receiver or v-receiver in the hybrid mode, eq. (6.106)
P_{co}^h	power received at the copolar (co) h-port of a dual-polarized antenna when horizontal (h) polarization is transmitted, eq. (3.177a)
P_{co}^v	power received at the v-port when the transmitted state is vertical (v), eq. (3.177b)
P_{cx}	cross-polar (cx) received power, eq. (3.178)
$\bar{P}_{\text{o}}$	averaged received (or signal) power at the receiver output, eq. (6.52a)
$\bar{P}_{\text{ref}}$	averaged received (or signal) power at the reference plane, eq. (6.53)
$\bar{P}_{S+N}$	sum of averaged signal power and noise power
P_N	noise power, eq. (5.140)
P_s	total power scattered by a particle when illuminated by an ideal, linearly polarized plane wave, eq. (1.44)
P_d	time-averaged power dissipated within a particle, eq. (1.53)
$\mathbf{P}$	2×2 ensemble-averaged forward scattering matrix, eq. (4.39)
P_{hh}; P_{hv}; P_{vh}; P_{vv}	elements of the 2×2 ensemble-averaged forward scattering matrix, eq. (4.40)
$\vec{P}$	volume density of polarization within a dielectric particle
Q	quadrature phase component of a signal
Q	element of the 4×1 Stokes' vector, eq. (3.25b)
Q_{ext}	normalized extinction cross section of a spherical particle, eq. (1.129c)
r	spherical coordinate of a point (r, θ, ϕ) or radar range
r	axis ratio of a spheroid ($r = b/a$), Fig. 1.8
$\vec{r}$; $\vec{r}'$	position vectors

$\vec{r}_i$	position vector to a point on the plane of constant phase of a plane wave, Fig. 1.6*b*
$\hat{r}$	unit vector ($\hat{r} = \vec{r}/r$)
r	magnitude of $\vec{r}$
$\bar{r}_m$	mass-weighted mean axis ratio, eq. (7.33)
$\bar{r}_z$	reflectivity-weighted mean axis ratio, eq. (7.35)
$\overline{r^2}$	mean-square value of the axis ratio, eq. (3.220a)
$\bar{r}$	mean axis ratio, eq. (3.219a)
$\vec{R}$	vector connecting source and observation points, $\vec{R} = \vec{r} - \vec{r}'$
R	magnitude of $\vec{R}$
$\hat{R}$	unit vector, along the direction of $\vec{R}$
R	rain rate, eq. (7.64)
$\bar{R}$	areal average rain rate, eq. (8.38a)
R; R_v	autocorrelation function of the received voltage, eqs. (5.80, 5.128)
R_N	autocorrelation function of "white" noise, eq. (5.158)
R_{co}	correlation between the copolar received voltages (hh and vv returns), eq. (3.179c)
R^h_{cx}	correlation between copolar and cross-polar signal returns (hh and vh) when the transmitted polarization is horizontal (h), eq. (3.179a)
R^v_{cx}	correlation between coplar and cross-polar signal returns (vv and hv) when transmitted state is vertical (v), eq. (3.179b)
$R_{hh,hh}[l]$	autocorrelation of the hh signal return at lag l, eq. (6.76a)
$R_{hh,vh}[l]$	correlation between hh and vh returns at lag l, eq. (6.76b)
$R_{hh,vv}[l]$	correlation between hh and vv returns at lag l, eq. (6.76c)
$R_{vh,vh}[l]$	autocorrelation of the vh signal at lag l, eq. (6.76d)
$R_{vv,hv}[l]$	correlation between vv and hv returns at lag l, eq. (6.76e)
$R_{vv,vv}[l]$	autocorrelation of the vv signal at lag l, eq. (6.76f)
$R_{v,h}[0]$	correlation between the h and v signal returns at lag 0 in the hybrid mode, eq. (6.107)
$\mathbf{R}$	2×2 rotation matrix, eq. (2.93a)
$\mathbf{R}$	4×4 covariance matrix of the complex signal vector, eq. (5.168d)
$\hat{s}$	unit vector along the direction of scattering, Fig. 1.6
s	distance along the direction of scattering
$s(t)$	complex signal representation, eq. (1.92)
$s_{tr}(t)$	transmitted signal, eq. (5.5a)
S_σ	bistatic Doppler power spectrum, eq. (1.113a)
$S(\omega)$; $S(f)$	Doppler frequency spectrum or power spectral density, eqs. (5.124, 5.125)
$S(v)$	Doppler velocity spectrum, eq. (5.127)
$S_N(f)$	power spectral density of "white" noise, eq. (5.141)
$S_{11,22}$	principal plane elements of the scattering matrix, eq. (2.89)
$S_{1,2}$	functions related to the Mie solution, eq. (2.108)
S_{hh}; S_{vh}	first column of the 2×2 amplitude scattering matrix, eq. (2.6a)
S_{hv}; S_{vv}	second column of the 2×2 amplitude scattering matrix, eq. (2.6a)
S_{RR}; S_{LR}	first column of the 2×2 amplitude scattering matrix in the circular basis, eq. (3.88a)

S_{RL}; S_{LL}	second column of the 2 × 2 amplitude scattering matrix in the circular basis, eq. (3.88)
$\mathbf{S}^c_{\text{BSA}}$	2 × 2 scattering matrix in circular basis, eq. (3.88a)
$\mathbf{S}_{\text{BSA}}$; $\mathbf{S}_{\text{FSA}}$	2 × 2 scattering matrix in back scatter alignment (BSA) or forward scatter alignment (FSA) conventions, eqs. (2.6b, 2.12)
t	time
t'	retarded time, eq. (1.108b)
T_0	pulsewidth of a transmitted pulse
T_0	ambient temperature, eq. (5.145)
T_s	pulse repetition time
T_e	equivalent noise temperature, eq. (5.149)
T_D	coherence time of a medium, eq. (5.96)
T_D	device noise temperature, eq. (5.143)
T_N	noise temperature, eq. (5.140)
$T_{11,22}$	principal plane elements of the transmission matrix, eq. (4.6b)
T_{hh}; T_{vh}; T_{hv}; T_{vv}	elements of the 2 × 2 transmission matrix, eqs. (4.19c, 4.42)
$\mathbf{T}$	transmission matrix in (h, v)-basis, eq. (4.62)
$\mathbf{T}^c$	transmission matrix in circular basis
U	element of the 4 × 1 Stokes' vector, eq. (3.25c)
U_R	correlation between the RHC and LHC received votage components when the transmitted state is RHC, eq. (4.24), or LHC, eq. (4.25)
$U_{\text{tr}}(t)$	transmitted waveform, eq. (5.5a)
$\mathbf{U}$	2 × 2 polarization basis transformation matrix, eq. (3.73a)
v; v_0	particle velocity or Doppler velocity
$\vec{v}$	particle velocity vector, eq. (1.108a)
$\bar{v}$	mean Doppler velocity, eq. (5.127)
$v(D)$	terminal velocity of a drop of diameter D, eq. (7.65a)
V	volume of sphere or spheroid
V	element of the 4 × 1 Stokes' vector, eq. (3.25d)
V; V_r	voltage or received voltage
$V_{h,v}$	received voltage at the h- or v-port of a dual-polarized antenna; also, received voltages in the hybrid mode, eqs. (3.60, 6.106)
$V_{R,L}$	received voltage at the RHC- or LHC-port of a dual-circularly polarized antenna, eqs. (3.105, 3.108a)
V_h^{10}	received voltage at the h-port of an antenna when the transmitted state is horizontal (denoted by superscript "10").
V_v^{10}	received voltage at the v-port of an antenna when the transmitted state is horizontal
V_v^{01}	received voltage at the v-port when the transmitted state is vertical (denoted by "01")
V_h^{01}	received voltage at the h-port of an antenna when the transmitted state is vertical
V_h^{11}; V_v^{11}	received voltage at the h- or v-port when transmitted state (denoted by "11") is slant 45° (or hybrid mode), eq. (4.141)
$V_h^{1,-1}$; $V_v^{1,-1}$	received voltage at the h- or v-port when the transmitted state ("1, −1") is slant −45°, eq. (4.143)
V_{hh}; V_{vh}; V_{hv}; V_{vv}	time samples of the received signal vector, eq. (6.72)
W	rainwater content, eq. (7.11)
W; W'	correlation between the RHC and LHC voltage components; prime refers to transmission-modified version, eq. (4.122c)

W	window function, eq. (5.132)
$W(t)$	range–time weighting function, eq. (5.52)
$W_n(t)$	normalized range–time weighting function, eq. (5.52c)
$W(r, \theta, \phi)$	three-dimensional weighting function, eq. (5.60a)
W_1; W_1'	copolar received power (“weak” channel) in a circularly polarized radar; prime refers to transmission-modified version, eq. (4.122a)
W_2; W_2'	cross-polar received power (“strong” channel) in a circularly polarized radar; prime refers to transmission-modified version, eq. (4.122b)
$(W/W_2)_+$	complex correlation between the RHC and LHC voltage components when the transmitted polarization state is RHC (subscript “+”), eq. (4.92a)
$(W/W_2)_-$	as above complex correlation between the RHC and LHC voltage components when the transmitted polarization state is LHC (subscript “−”), eq. (4.92b)
$\mathbf{X}$	polarization error matrix, eq. (6.33)
$\mathfrak{z}_{\text{dr}}$	differential reflectivity ratio ($\mathfrak{z}_{\text{dr}} = 10^{0.1(Z_{\text{dr}})}$), eq. (3.231c)
Z	reflectivity factor, eq. (3.166)
Z_e	equivalent reflectivity factor, eq. (5.49a)
$Z_{h,v}$	equivalent reflectivity factor at horizontal or vertical polarization, eq. (6.61c,d)
Z_0	intrinsic impedance of empty space, eq. (1.29)
Z_{dr}	differential reflectivity, eq. (3.168)
Z_{dp}	difference reflectivity, eq. (7.86a)
Z_h'	transmission-matrix modified Z_h
Z_{dr}'	transmission-matrix modified Z_{dr}
α	polarizability of a sphere, eq. (A1.27a)
α	slope of a linear relation between specific attenuation and specific differential phase, Table 7.1
$\alpha_{x,y,z}$	elements of the polarizability matrix, eqs. (A1.28, A1.29)
α_{z_b}	polarizability of a spheroid along its symmetry axis, eq. (2.45)
$\boldsymbol{\alpha}$	polarizability matrix, eq. (2.28a)
$\bar{\bar{\boldsymbol{\alpha}}}$	polarizability tensor, eq. (2.28b)
α_{o1n}	expansion coefficients related to the Mie solution, eq. (2.104a)
$\alpha_{hh}, \alpha_{vh}, \alpha_{hv}, \alpha_{vv}$	phase of the elements of the “instantaneous” scattering matrix, eq. (3.174)
β	canting angle in the plane of polarization, Fig. 2.10*a*
β	angle between the dipole moment and the scattering direction, Fig. 1.9
β	slope of a linear relation between specific differential attenuation and specific differential phase, Table 7.1
β_0	mean canting angle
β_{e1n}	expansion coefficients related to the Mie solution, eq. (2.104b)
β_a^2	gain inequality between the two ports of a dual-polarized antenna, eq. (6.22a)
γ	slope of a linear relation between drop axis ratio and diameter, eq. (7.29a)
$\gamma_{hh}; \gamma_{vh}; \gamma_{hv}; \gamma_{vv}$	magnitude of the elements of an “instantaneous” scattering matrix, eq. (3.174)
$\Gamma(n)$	gamma function

$\Gamma(\Delta f, \Delta t)$	spaced-time/spaced-frequency coherency function, eq. (5.106a)
δ	Dirac delta function
δ_{co}	scattering differential phase, eq. (7.49)
δ_c	scattering differential phase at circular polarization, eq. (3.98)
$\delta_{hh}; \delta_{vh}; \delta_{hv}; \delta_{vv}$	phase angle of the scattering matrix elements, eq. (4.69)
ΔZ_e	enhancement in equivalent reflectivity factor in the bright-band relative to the rain below
ε_0	permittivity of empty space
ε_r	relative permittivity of a dielectric
$\varepsilon_r'; \varepsilon_r''$	real and imaginary parts of ε_r
ε_{eff}	effective permittivity
ε_m	radar measurement error, eq. (8.23)
ε_p	error due to parameterization of the rain rate, eq. (8.23)
ε_T	sum of ε_m and ε_p, eq. (8.23)
$\varepsilon_{h,v}$	complex error terms of an antenna polarization error matrix, eq. (3.63)
$\varepsilon_{R,L}$	complex error terms for circular polarization, eqs. (3.113, 3.114)
η	radar reflectivity, eq. (3.167a)
η_c	radar reflectivity at circular polarization, eq. (3.202a)
$\eta_{hh,vv}$	copolar radar reflectivities at horizontal and vertical polarizations, eq. (3.164)
η_{vh}	cross-polar radar reflectivity; transmit polarization is h and receive polarization is v (subscript "vh"), eq. (3.164c)
η_{hv}	cross-polar radar reflectivity; transmit state is v and receive is h
$\bar{\eta}$	resolution-volume averaged radar reflectivity, eq. (5.62)
$\theta_{i,s}$	spherical coordinate angle related to incidence (i) and scattered (s) directions
$\theta_{R,L}$	phase angle of RHC and LHC components
θ_b	spherical coordinate angle of the particle symmetry axis, Fig. 2.6*a*
λ	wavelength
$\lambda_{1,2}$	eigenvalues of the two characteristic waves, eq. (4.6a)
$\lambda_{x,y,z}$	depolarization factors of a spheroid, Table A1.1
λ_{z_b}	depolarizing factor along the spheroid symmetry axis
Λ	slope of an exponential drop size distribution or slope parameter of a gamma dsd, eq. (7.12)
$\Lambda_{x,y,z}$	related to $\lambda_{x,y,z}$, Table A1.1
μ	parameter of a gamma drop size distribution, eq. (7.12)
μ_0	permeability of empty space
ν	parameter of a gamma dsd, note that $\mu = \nu - 1$, eq. (7.27)
ν	back scatter amplitude ratio at circular polarization, eq. (3.97a)
$\nu^2; \overline{\nu^2}$	circular depolarization ratio; ensemble-average, eq. (3.202c)
$\vec{\Pi}$	electric Hertz vector, eq. (1.14)
ρ	polarization match factor, eq. (3.44)
ρ_0	size parameter of a sphere
ρ_p	particle density
ρ_w	water density
$\rho_{2,4}$	orientation parameters, eq. (3.201)
ρ_{co}	copolar correlation coefficient between the hh and vv return signals, eq. (3.170a)

$\rho[l]$ correlation coefficient between signal samples at lag l, eq. (5.190)

$\rho_p[l]$ correlation coefficient for samples of the signal power, eq. (5.190)

$\rho(t_s)$ normalized autocorrelation function of the received signal, eq. (5.95)

ρ_{cx}^h, ρ_{cx}^v correlation coefficient between the copolar and cross-polar return signals when the transmitted polarization is horizontal (superscript "h"), eq. (4.78a) or vertical (superscript "v"), eq. (4.78b)

$\rho_{hh,hh}[l]$ correlation coefficient of the hh return signal at lag l; see in relation to eq. (6.76a)

$\rho_{hh,vh}[l]$ as above except between hh and vh returns; see eq. (6.76b)

$\rho_{hh,vv}[l]$ as above except between hh and vv returns; see eq. (6.76c)

σ; σ^2 standard deviation; variance

σ_m standard deviation of the mass-weighted mean diameter, eq. (7.50a)

σ_v standard deviation of velocity distribution, eq. (5.93b)

σ_v standard deviation of the Doppler velocity spectrum, eq. (5.127)

$\sigma_{a,s,b}$ absorption (a), scattering (s) and back scatter (b) cross section of a sphere, eqs. (1.56, 1.45, and 1.51), respectively

σ_{bi} bistatic radar cross section, eq. (1.46b)

σ_{co} copolar radar cross section, eq. (3.137b)

σ_{cx} cross-polar radar cross section, eq. (3.137c)

σ_{rt} generalized bistatic radar cross section, eq. (3.134)

σ_{hh}; σ_{vv} copolar radar cross sections at horizontal and vertical polarizations, eq. (8.4)

σ_{ext} extinction cross section of a sphere, eq. (1.57)

$\mathbf{\Sigma}$ polarimetric covariance matrix in (h, v)-basis, eq. (3.183)

$\mathbf{\Sigma}^c$ polarimetric covariance matrix in circular basis, eq. (3.195)

τ volume of a non-spherical scatterer

τ range–time

τ time difference

τ ellipticity angle of a polarization ellipse, eq. (3.13)

τ mean canting angle in McCormick and Hendry's definition, eq. (4.33)

ϕ canting angle in the polarization plane, Oguchi's definition, eq. (4.44)

ϕ scalar potential function, eq. (1.10a)

$\phi_{i,s}$ spherical coordinate angle related to directions of incidence (i) and scattering (s)

ϕ_b spherical coordinate angle related to the symmetry axis of the spheroid, Fig. 2.6

$\Phi_{h,v}$ phase angles associated with the h- or v-ports of a dual-polarized antenna, eq. (6.19)

Φ_{dp} differential propagation phase, eq. (4.65b)

χ characteristic function of a random variable, eq. (5.93a)

$\vec{\chi}$ vector associated with Rayleigh scattering, eq. (1.48)

$\chi_{i,s,r}$ complex polarization ratio of an incident (i), scattered (s), or reflected (r) wave, eq. (3.3–3.5)

ψ	angle between the direction of the incident wave and the particle symmetry axis, Fig. 2.10*a*
ψ	orientation angle of a polarization ellipse, Fig 3.3*b*
ψ_m	magnetic flux, eq. (1.72)
Ψ_{dp}	differential phase between copolar (hh and vv) received signals, eq. (4.65a)
ω	angular frequency
ω_D	angular Doppler frequency
Ω	solid angle, eq. (2.68)
Ω	frequency axis for discrete time signals
$\mathbf{\Omega}$	feature vector, eq. (3.175)

Abbreviations

AGL	above ground level
ATI	area–time integral
BMRC	Bureau of Meteorology Research Center
BPN	back propagation network
BSA	back scatter alignment
CaPE	convective and precipitation/electrification experiment
CDF	cumulative distribution function
CDR	circular depolarization ratio
COHO	coherent oscillator
CW	continuous waves
CSU–CHILL	Colorado State University–University of Chicago/Illinois State Water Survey
DDA	discrete dipole approximation
DFT	discrete Fourier transform
DLR	German Aerospace Research Establishment
dsd	drop size distribution
DSP	digital signal processing
DTFT	discrete time Fourier transform
EDR	elliptical depolarization ratio
EMF	electromotive force
ETL	Environmental Technology Laboratory
FDTD	finite difference–time domain
FFT	fast Fourier transform
FHC	fuzzy logic hydrometeor classifier
FIR	finite impulse response
FSA	forward scatter alignment
HVPS	high volume particle spectrometer
IEEE	Institute of Electrical and Electronic Engineers
IF	intermediate frequency
iid	independent and identically distributed
IIR	infinite impulse response
IWC	ice–water content
LHC	left-hand circular
LNA	low noise amplifier
MBF	membership function
M_EMF	motional EMF
MG	Maxwell-Garnet
NCAR	National Center for Atmospheric Research
NOAA	National Oceanic and Atmospheric Administration
NRC	National Research Council of Canada
OMT	orthomode transducer
ORTT	apparent degree of orientation measured by a circularly polarized radar
pdf	probability density function
PI	particular integral

PMM	probability-matching method
PPI	plan position indicator
PPMM	polarimetric probability matching method
PRF	pulse repetition frequency
PROM	programmable read only memory
PRT	pulse repetition time
RAL	Rutherford Appleton Laboratory
RBF	radial basis function
RHC	right-hand circular
RHI	range–height indicator
RRV	radar resolution volume
SD	standard deviation
SDSM&T	South Dakota School of Mines and Technology
SLDR	slant linear depolarization ratio
SNR	signal-to-noise ratio
STALO	stable local oscillator
T_EMF	transformer EMF
TDWR	terminal Doppler weather radar
TRMM	Tropical Rain Measurement Mission
TWT	traveling wave tube
UTC	universal time coordinates
VDH	van de Hulst
VSWR	voltage standing wave ratio
WPMM	window probability-matching method
WSR–88D	Weather Service Radar–1988 Doppler
XPI	cross-polarization isolation