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V. N. Bringi and V. Chandrasekar
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1 Electromagnetic concepts useful for radar applications

The scattering of electromagnetic waves by precipitation particles and their propagation through precipitation media are of fundamental importance in understanding the signal returns from dual-polarized, Doppler weather radars. In this chapter, a number of useful concepts are introduced from first principles for the benefit of those readers who have not had prior exposure to such material. Starting with Maxwell's equations, an integral representation for scattering by a dielectric particle is derived, which leads into Rayleigh scattering by spheres. The Maxwell-Garnet(MG) mixing formula is discussed from an electrostatic perspective (a review of electrostatics is provided in Appendix 1). Faraday's law is used in a simple example to explicitly show the origin of the bistatic Doppler frequency shift. The important concepts of coherent and incoherent addition of waves are illustrated for two- and N-particle cases. The time-correlated bistatic scattering cross section of a single moving sphere is defined, which naturally leads to Doppler spectrum. The transmitting and receiving aspects of a simple Doppler radar system are then explained. This chapter ends with coherent wave propagation through a slab of spherical particles, the concept of an effective propagation constant of precipitation media, and the definition of specific attenuation.

1.1 Review of Maxwell's equations and potentials

Maxwell's time-dependent equations[1] governing the electric ($\vec{E}$) and magnetic ($\vec{B}$) vectors within a material can be written in terms of permittivity of free space (ε_0), permeability of free space (μ_0), and the volume density of polarization ($\vec{P}$). It is assumed here that within the material under consideration, the volume density of free charge is zero, the conductivity is zero (no Ohmic currents), and the volume density of magnetization is zero. Then, $\vec{E}$ and $\vec{B}$ within the interior of the material satisfy,

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}; \quad \varepsilon_0 \nabla \cdot \vec{E} = -\nabla \cdot \vec{P} \tag{1.1a}$$

$$\frac{1}{\mu_0} \nabla \times \vec{B} = \frac{\partial \vec{P}}{\partial t} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}; \quad \nabla \cdot \vec{B} = 0 \tag{1.1b}$$

The term $\partial \vec{P}/\partial t$ is identified as the polarization current, which can be thought of as being maintained by external sources, while $\varepsilon_0 \partial \vec{E}/\partial t$ is the free space displacement

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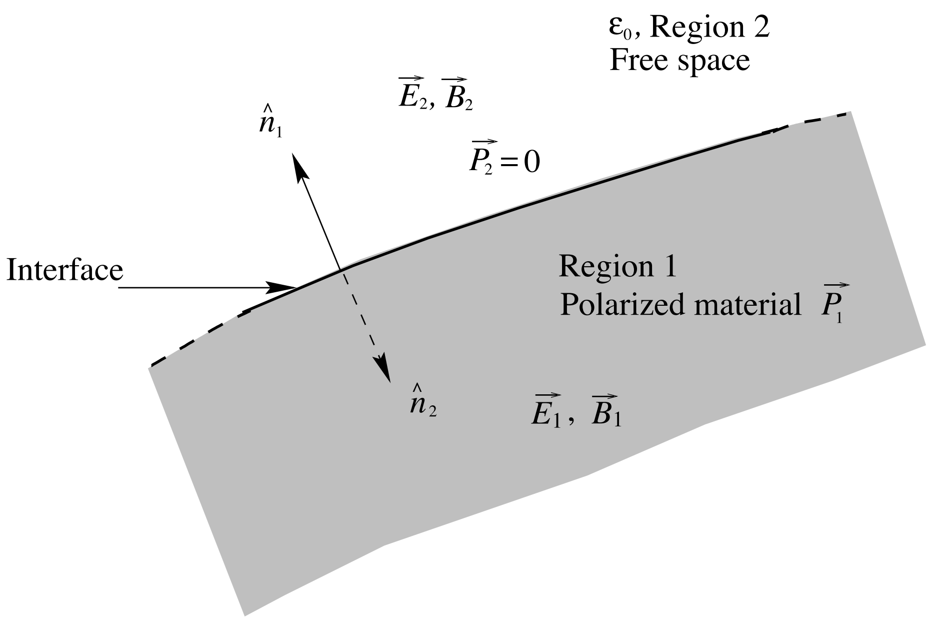

Fig. 1.1. Boundary between a polarized material (Region 1) and free space (Region 2). The unit normals are directed "outward" with respect to the corresponding regions.

current. Fields $\vec{E}$ and $\vec{B}$ are the macroscopic fields in the material, not the "microscopic" or "local" fields. The boundary conditions on the interface between the material and the free space [see Fig. 1.1 and note the directions of the unit normal vectors $(\hat{n}_{1,2})$] are given as,

$$\hat{n}_1 \times \vec{E}_1 + \hat{n}_2 \times \vec{E}_2 = 0; \quad \varepsilon_0(\hat{n}_1 \cdot \vec{E}_1 + \hat{n}_2 \cdot \vec{E}_2) = -\hat{n}_1 \cdot \vec{P}_1 \tag{1.2a}$$

$$\frac{1}{\mu_0}(\hat{n}_1 \times \vec{B}_1 + \hat{n}_2 \times \vec{B}_2) = 0; \quad \hat{n}_1 \cdot \vec{B}_1 + \hat{n}_2 \cdot \vec{B}_2 = 0 \tag{1.2b}$$

The tangential components of the electric and magnetic fields are continuous across the interface; the normal component of the magnetic vector is continuous; and the normal component of the electric vector is discontinuous by an amount equal to $-\hat{n}_1 \cdot \vec{P}_1/\varepsilon_0$ or $-\eta_b/\varepsilon_0$, where η_b is the surface density of the bound charge. If the interface is between two materials with different $\vec{P}_1$ and $\vec{P}_2$ on either side, then $\varepsilon_0(\hat{n}_1 \cdot \vec{E}_1 + \hat{n}_2 \cdot \vec{E}_2) = -(\hat{n}_1 \cdot \vec{P}_1 + \hat{n}_2 \cdot \vec{P}_2)$. Note that in Fig. 1.1, $\vec{P}_2$ is taken as zero in free space.

When the time variation of the external sources (e.g. radiating antenna) that maintain the polarization current within the material (or particle) is in sinusoidal steady state at a fixed angular frequency (ω), then Maxwell's equations (1.1) can be transformed by defining,

$$\vec{E}(\vec{r}, t) = \mathrm{Re}\left[\vec{E}^c(\vec{r})e^{j\omega t}\right] \tag{1.3a}$$

$$\vec{B}(\vec{r}, t) = \mathrm{Re}\left[\vec{B}^c(\vec{r})e^{j\omega t}\right] \tag{1.3b}$$

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$$\vec{P}(\vec{r},t) = \text{Re}\left[\vec{P}^c(\vec{r})e^{j\omega t}\right] \tag{1.3c}$$

where $\vec{E}^c(\vec{r})$, $\vec{B}^c(\vec{r})$, and $\vec{P}^c(\vec{r})$ are vector-phasors or complex vectors. Substituting (1.3) into (1.1) yields,

$$\nabla \times \vec{E}^c = -j\omega\vec{B}^c; \quad \varepsilon_0\nabla\cdot\vec{E}^c = -\nabla\cdot\vec{P}^c \tag{1.4a}$$

$$\frac{1}{\mu_0}\nabla \times \vec{B}^c = j\omega\vec{P}^c + j\omega\varepsilon_0\vec{E}^c; \quad \nabla\cdot\vec{B}^c = 0 \tag{1.4b}$$

The boundary conditions in (1.2) are valid without any change by replacing the real, instantaneous vectors by complex vectors. Note that,

$$\vec{E}^c(\vec{r}) = \vec{E}^c_{\text{real}} + j\vec{E}^c_{\text{im}} \tag{1.5a}$$

$$\vec{E}(\vec{r},t) = \text{Re}\left[\vec{E}^c(\vec{r})e^{j\omega t}\right] \tag{1.5b}$$

Henceforth, the superscript c will be dropped as only the sinusoidal steady state will be considered. For linear materials, a complex relative permittivity is defined ($\varepsilon_r = \varepsilon_r' - j\varepsilon_r''$) as discussed in Appendix 1, where $\vec{P} = \varepsilon_0(\varepsilon_r - 1)\vec{E}$ within the material.

1.1.1 Vector Helmholtz equation

The vector Helmholtz equation for $\vec{E}$ is derived by taking the curl of $\vec{E}$ in (1.4a),

$$\nabla \times \nabla \times \vec{E} = -j\omega\nabla \times \vec{B} \tag{1.6a}$$

$$= -j\omega\mu_0\left(j\omega\vec{P} + j\omega\varepsilon_0\vec{E}\right) \tag{1.6b}$$

$$= \omega^2\mu_0\vec{P} + \omega^2\mu_0\varepsilon_0\vec{E} \tag{1.6c}$$

$$= \omega^2\mu_0\vec{P} + k_0^2\vec{E} \tag{1.6d}$$

Thus, the inhomogeneous vector Helmholtz equation for $\vec{E}$ is,

$$\nabla \times \nabla \times \vec{E} - k_0^2\vec{E} = \omega^2\mu_0\vec{P} \tag{1.7a}$$

and a similar equation for $\vec{B}$ is,

$$\nabla \times \nabla \times \vec{B} - k_0^2\vec{B} = j\omega\mu_0\nabla \times \vec{P} \tag{1.7b}$$

where $k_0 = \omega\sqrt{\varepsilon_0\mu_0}$ is the wave number of free space. Here, $\vec{E}$ and $\vec{B}$ are the total fields within the material, and $\vec{P}$ is maintained by external sources. The volume density of polarization or the polarization current, in this context, can be thought of as induced "sources" in (1.7a, b). If the linear relation $\vec{P} = \varepsilon_0(\varepsilon_r - 1)\vec{E}$ is used within the material, then $\vec{E}$ and $\vec{B}$ satisfy the more familiar homogeneous vector Helmholtz equations,

$$\nabla \times \nabla \times \vec{E} - k^2\vec{E} = 0 \tag{1.8a}$$

$$\nabla \times \nabla \times \vec{B} - k^2\vec{B} = 0 \tag{1.8b}$$

where $k = \omega\sqrt{\varepsilon_0\varepsilon_r\mu_0} = k_0\sqrt{\varepsilon_r}$ is the complex wave number of the material.

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1.1.2 Scalar, vector and electric Hertz potentials

The $(\vec{E}, \vec{B})$ fields can be expressed in terms of simpler potential functions $(\phi, \vec{A})$, where $\vec{B} = \nabla \times \vec{A}$ and $\vec{E} = -\nabla\phi - j\omega\vec{A}$. These follow from the two Maxwell equations, $\nabla\cdot\vec{B} = 0$ and $\nabla \times \vec{E} + j\omega\vec{B} = 0$. The function ϕ is termed the electric scalar potential, and reduces to the electrostatic potential when $\omega = 0$. The function $\vec{A}$ is termed the magnetic vector potential. These two potentials must also satisfy Maxwell's remaining two equations,

$$\varepsilon_0 \nabla\cdot\vec{E} = -\nabla\cdot\vec{P} \tag{1.9a}$$

$$\frac{1}{\mu_0}\nabla \times \vec{B} = j\omega\vec{P} + j\omega\varepsilon_0\vec{E} \tag{1.9b}$$

Substituting for $\vec{B}$ and $\vec{E}$ in terms of the potentials ϕ and $\vec{A}$ gives,

$$\nabla^2\phi + j\omega\nabla\cdot\vec{A} = \frac{1}{\varepsilon_0}\nabla\cdot\vec{P} \tag{1.10a}$$

$$\nabla \times \nabla \times \vec{A} = j\omega\mu_0\vec{P} - j\omega\mu_0\varepsilon_0\nabla\phi + k_0^2\vec{A} \tag{1.10b}$$

Since $\nabla \times \nabla \times \vec{A} = \nabla(\nabla\cdot\vec{A}) - \nabla^2\vec{A}$,

$$\nabla(\nabla\cdot\vec{A}) - \nabla^2\vec{A} = j\omega\mu_0\vec{P} - j\omega\varepsilon_0\mu_0\nabla\phi + k_0^2\vec{A} \tag{1.11}$$

The magnetic vector potential is not unique as yet, since only its curl is defined as equal to $\vec{B}$. Its divergence can be assigned any convenient value and, in particular, if the Lorentz gauge is used, i.e. $\nabla\cdot\vec{A} = -j\omega\varepsilon_0\mu_0\phi$, then (1.10a, b) become,

$$(\nabla^2 + k_0^2)\phi = \frac{1}{\varepsilon_0}\nabla\cdot\vec{P} \tag{1.12a}$$

$$(\nabla^2 + k_0^2)\vec{A} = -j\omega\mu_0\vec{P} \tag{1.12b}$$

The above are the (inhomogeneous) Helmholtz equations for the potentials $(\phi, \vec{A})$ from which $\vec{E}$ and $\vec{B}$ are derived. In practice, $\vec{E}$ and $\vec{B}$ can be derived from $\vec{A}$ alone using the Lorentz gauge relation,

$$\vec{B} = \nabla \times \vec{A} \tag{1.13a}$$

$$\vec{E} = -\nabla\phi - j\omega\vec{A} \tag{1.13b}$$

$$= \frac{-j}{\omega\varepsilon_0\mu_0}\nabla(\nabla\cdot\vec{A}) - j\omega\vec{A} \tag{1.13c}$$

It is convenient to define the electric Hertz vector $\vec{\Pi} = \vec{A}/j\omega\mu_0\varepsilon_0$, so that $\vec{\Pi}$ satisfies,

$$(\nabla^2 + k_0^2)\vec{\Pi} = \frac{-\vec{P}}{\varepsilon_0} \tag{1.14}$$

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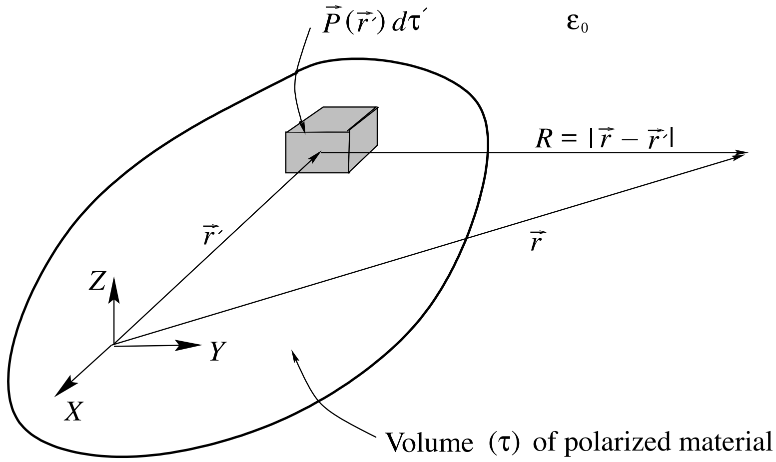

Fig. 1.2. Continuous volume density of polarization $\vec{P}(\vec{r}')$ is defined within the volume τ of polarized material.

and $(\vec{E}, \vec{B})$ are derived from $\vec{\Pi}$ as,

$$\vec{E} = \nabla(\nabla\cdot\vec{\Pi}) + k_0^2\vec{\Pi} \tag{1.15a}$$

$$= \nabla\times\nabla\times\vec{\Pi} + (\nabla^2 + k_0^2)\vec{\Pi} \tag{1.15b}$$

$$= \nabla\times\nabla\times\vec{\Pi} - \frac{\vec{P}}{\varepsilon_0} \tag{1.15c}$$

$$\vec{B} = j\omega\mu_0\varepsilon_0\nabla\times\vec{\Pi} \tag{1.15d}$$

Referring to Fig. 1.2, the particular integral (PI) solution to (1.14) is,

$$\vec{\Pi}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_\tau \vec{P}(\vec{r}')\frac{e^{-jk_0|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}\,d\tau' \tag{1.16}$$

where,

$$G_0(\vec{r}/\vec{r}') = \frac{1}{4\pi\varepsilon_0}\frac{e^{-jk_0|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \tag{1.17}$$

is the free space Green function for the scalar Helmholtz equation. Note that $\vec{\Pi}$ satisfies no boundary condition (because it is a PI solution) except the radiation condition on a spherical surface at infinity. This radiation condition implies that $\vec{\Pi}$ at large distances (R) must be of the form of outward propagating spherical waves, $\exp(-jk_0R)/R$ (for $\exp(j\omega t)$ time dependence), where $R = |\vec{r} - \vec{r}'|$. One can consider (1.16) as analogous to the electrostatic PI solution given in (A1.5) for the electrostatic potential, where in both cases $\vec{P}$ is maintained by external sources as yet unspecified. When the observation point $\vec{r}$ is outside the volume (τ) of polarized material and in empty space (see Fig. 1.2), then the electric field $\vec{E}(\vec{r})$ from (1.15c) is given as $\nabla\times\nabla\times\vec{\Pi}(\vec{r})$, since $\vec{P}$ is zero in free space.

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1.2 Integral representation for scattering by a dielectric particle

Now consider the case of a dielectric particle placed in a known incident field ($\vec{E}^i$) as illustrated in Fig. 1.3a. As discussed in Appendix 1, $\vec{E}^i$ is perturbed by the dielectric particle. What was termed the perturbation field ($\vec{E}^p$), in Appendix 1, in an electrostatic context, is now conventionally termed the scattered field ($\vec{E}^s$). Figure 1.3b illustrates that the "source" for the scattered electric field is the unknown volume density of polarization ($\vec{P}$) within the particle. Inside the particle, $\vec{E}^s$ satisfies $\nabla\times\nabla\times\vec{E}^s - k_0^2\vec{E}^s = \omega^2\mu_0\vec{P}$, and thus $\vec{E}^s$ can be derived from the electric Hertz vector $\vec{\Pi}_s$. Since $\vec{\Pi}_s$ satisfies $(\nabla^2 + k_0^2)\vec{\Pi}_s = -\vec{P}/\varepsilon_0$ inside the particle, the principal integral solution is given by (1.16), and $\vec{E}^s$ outside the particle is given as,

$$\vec{E}^s(\vec{r}) = \nabla\times\nabla\times\frac{1}{4\pi\varepsilon_0}\int_\tau \frac{\vec{P}(\vec{r}\,')e^{-jk_0|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}\,d\tau' \tag{1.18a}$$

Using the fact that $\vec{P}(\vec{r}\,') = \varepsilon_0(\varepsilon_r - 1)\vec{E}_T^{\text{in}}(\vec{r}\,')$, where $\vec{E}_T^{\text{in}}$ is the total electric field inside the particle, an integral representation for the scattered electric field outside the particle, $\vec{E}^s(\vec{r})$, can be obtained as a volume integral,

$$\vec{E}^s(\vec{r}) = \nabla\times\nabla\times\frac{1}{4\pi}\int_\tau (\varepsilon_r - 1)\vec{E}_T^{\text{in}}(\vec{r}\,')\frac{e^{-jk_0R}}{R}\,d\tau' \tag{1.18b}$$

where $R = |\vec{r}-\vec{r}\,'|$, and may be compared with the electrostatic representation for $\vec{E}^p$ in (A1.14b). For homogeneous particles, the term $(\varepsilon_r - 1)$ can be brought outside the integral.

When $r \gg r'$, and recalling that $\vec{r}\,'$ locates the differential volume element ($d\tau'$) inside the particle, the far-field integral representation for $\vec{E}^s(\vec{r})$ can be derived as follows. First, note that the dual curl operator $[\nabla\times\nabla\times(\cdots)]$ operates on $\vec{r} \equiv (x, y, z)$ only and can be taken inside the integral in (1.18b), Thus,

$$\nabla\times\left[\vec{E}_T^{\text{in}}(\vec{r}\,')\frac{e^{-jk_0R}}{R}\right] = \nabla\left(\frac{-e^{jk_0R}}{R}\right)\times\vec{E}_T^{\text{in}}(\vec{r}\,') \tag{1.19a}$$

$$= \left[\frac{1}{R}\nabla(e^{-jk_0R}) + e^{-jk_0R}\nabla\left(\frac{1}{R}\right)\right]\times\vec{E}_T^{\text{in}}(\vec{r}\,') \tag{1.19b}$$

$$= \left[\frac{-jk_0e^{-jk_0R}\nabla(R)}{R} + \frac{e^{-jk_0R}(-\hat{R})}{R^2}\right]\times\vec{E}_T^{\text{in}}(\vec{r}\,') \tag{1.19c}$$

$$= \frac{-jk_0(e^{-jk_0R})\hat{R}}{R}\left(1+\frac{1}{jk_0R}\right)\times\vec{E}_T^{\text{in}}(\vec{r}\,') \tag{1.19d}$$

Since $r \gg r'$ and $R = |\vec{r}-\vec{r}\,'|$, the term $|1/jk_0R|$ may be neglected, in comparison to unity in (1.19d). Also, the term $(1/R)$ can be approximated as $(1/r)$, while $\hat{R}$ is parallel to $\hat{r}$ (see Fig. 1.4). Hence, (1.19d) reduces to,

$$\nabla\times\left[\vec{E}_T^{\text{in}}(\vec{r}\,')\frac{e^{-jk_0R}}{R}\right] \approx \frac{(-jk_0)}{r}e^{-jk_0R}\hat{r}\times\vec{E}_T^{\text{in}}(\vec{r}\,') \tag{1.20}$$

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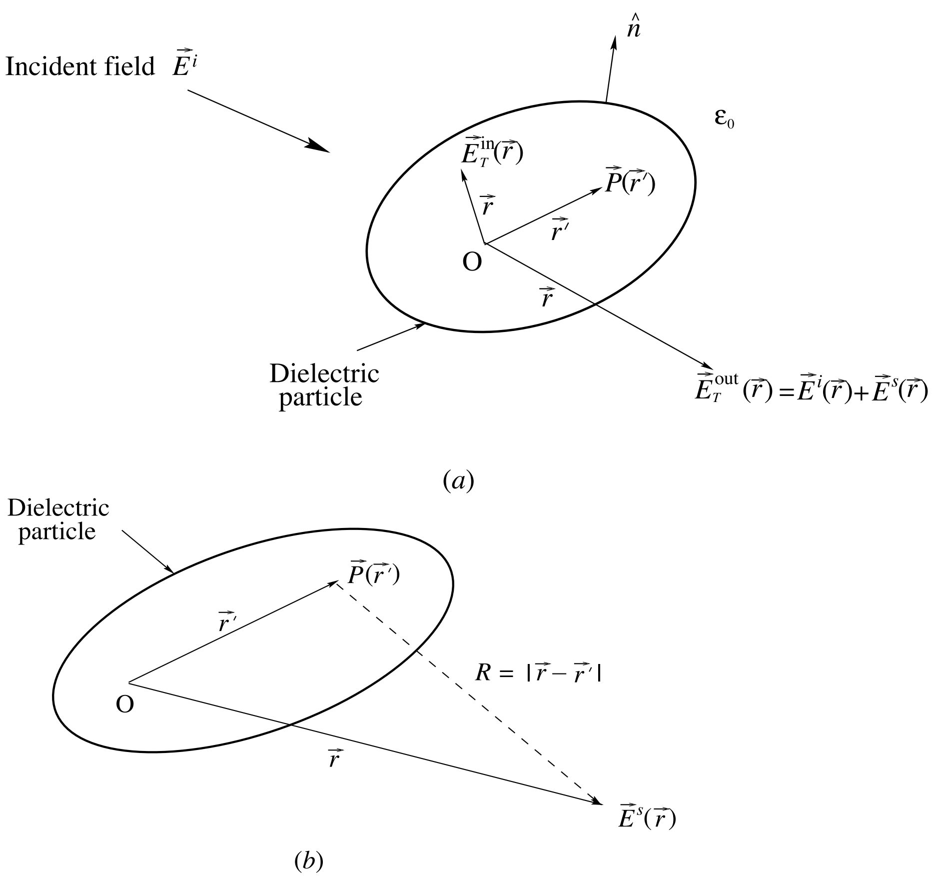

Fig. 1.3. (*a*) Dielectric particle in the presence of an incident electric field $\vec{E}^i$. The particle gets polarized $\vec{P}$ and causes a scattered field component $\vec{E}^s$ outside the particle. The total field is $\vec{E}_T^{\text{in}}$ inside the particle. (*b*) Illustration showing that the "source" of the scattered field $\vec{E}^s$ is the volume density of polarization $\vec{P}$ within the particle.

In a similar manner,

$$\nabla \times \nabla \times \left[\vec{E}_T^{\text{in}}(\vec{r}\,') \frac{e^{-jk_0R}}{R} \right] \approx \frac{(-jk_0)(-jk_0)}{r} e^{-jk_0R} \hat{r} \times \hat{r} \times \vec{E}_T^{\text{in}}(\vec{r}\,') \tag{1.21a}$$

$$= \frac{-k_0^2}{r} e^{-jk_0R} \hat{r} \times \hat{r} \times \vec{E}_T^{\text{in}}(\vec{r}\,') \tag{1.21b}$$

Substituting (1.21b) into (1.18b),

$$\vec{E}^s(\vec{r}) = \frac{-k_0^2(\varepsilon_r - 1)}{4\pi} \left(\frac{1}{r} \right) \int_\tau e^{-jk_0R} \hat{r} \times \hat{r} \times \vec{E}_t^{\text{in}}(\vec{r}\,')\, d\tau' \tag{1.22}$$

The final far-field approximation is in the exponential term $\exp(-jk_0R) = \cos(k_0R) - j\sin(k_0R)$, where $R \approx r - \vec{r}\,' \cdot \hat{r}$ (see Fig. 1.4). It is not permissible to approximate

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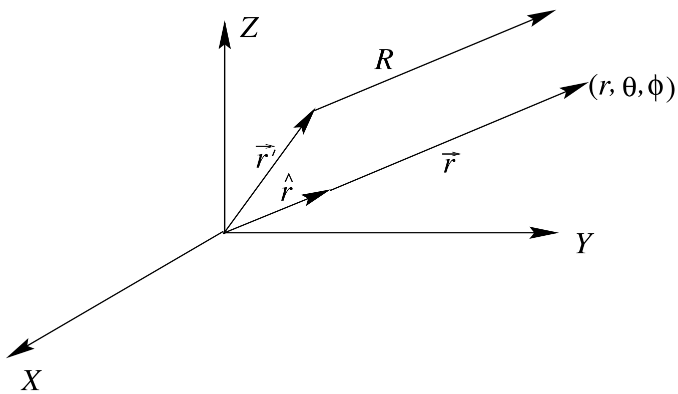

Fig. 1.4. Approximation for R in the far-field, $R \approx r - \vec{r}'\cdot\hat{r}$.

$R \approx r$ in the exponential term since $\theta = k_0 R \approx k_0 r - k_0\vec{r}'\cdot\hat{r}$, which lies between 0 and 2π. Since k_0 is the wave number of free space ($k_0 = \omega/c = 2\pi/\lambda$, where λ is the wavelength and c the velocity of light in free space, respectively), the correction term $k_0\vec{r}'\cdot\hat{r} = \vec{r}'\cdot\hat{r}(2\pi/\lambda)$ may become significant if the maximum extent of $\vec{r}'$ is even a small fraction of λ. Since $\vec{r}'$ is a variable of integration and covers all differential volume elements within the particle, the maximum extent of $\vec{r}'$ is of the same order as the maximum dimension of the particle. With this final approximation, (1.22) reduces to,

$$\vec{E}^s(\vec{r}) = \frac{k_0^2}{4\pi}(\varepsilon_r - 1)\frac{e^{-jk_0 r}}{r}\int_\tau \left[\vec{E}_T^{\text{in}}(\vec{r}') - \hat{r}(\hat{r}\cdot\vec{E}_T^{\text{in}})\right] e^{jk_0\vec{r}'\cdot\hat{r}}\,d\tau' \tag{1.23}$$

where the vector identity $\vec{a}\times\vec{b}\times\vec{c} = \vec{b}(\vec{a}\cdot\vec{c}) - \vec{c}(\vec{a}\cdot\vec{b})$, with $\vec{a}\equiv\hat{r}$, $\vec{b}\equiv\hat{r}$, and $\vec{c}\equiv\vec{E}_T^{\text{in}}$, is used. The far-field vector scattering amplitude ($\vec{f}$) is defined as,

$$\vec{E}^s = \vec{f}\,\frac{e^{-jk_0 r}}{r} \tag{1.24a}$$

$$\vec{f} = \frac{k_0^2(\varepsilon_r - 1)}{4\pi}\int_\tau \left[\vec{E}_T^{\text{in}}(\vec{r}') - \hat{r}(\hat{r}\cdot\vec{E}_T^{\text{in}})\right] e^{jk_0\vec{r}'\cdot\hat{r}}\,d\tau' \tag{1.24b}$$

where $\vec{f}\equiv\vec{f}(\theta,\phi)$, the spherical coordinates (θ,ϕ) referring to the direction of $\vec{r}$ (see Fig. 1.4), i.e. to the scattering direction. From (1.15d) the scattered magnetic vector $\vec{B}^s$ is,

$$\vec{B}^s = j\omega\varepsilon_0\mu_0\nabla\times\vec{\Pi} \tag{1.25a}$$

$$= j\omega\varepsilon_0\mu_0\nabla\times\frac{1}{4\pi}\int_\tau\frac{(\varepsilon_r - 1)\vec{E}_T^{\text{in}}(\vec{r}')e^{-jk_0 R}}{R}\,d\tau' \tag{1.25b}$$

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Using the far-field approximation,

$$\vec{H}^s = \frac{\vec{B}^s}{\mu_0} = j\omega\varepsilon_0(-jk_0)(\varepsilon_r - 1)\frac{e^{-jk_0 r}}{4\pi r}\int_\tau \hat{r} \times \vec{E}_T^{\text{in}}(\vec{r}\,')e^{jk_0\vec{r}\,'\cdot\hat{r}}\,d\tau' \tag{1.26}$$

where $\vec{H}^s$ is the far-field magnetic intensity vector. Using $k_0 = \omega/c$, $c^2 = 1/\varepsilon_0\mu_0$, and defining the impedance of free space as $Z_0 = \sqrt{\mu_0/\varepsilon_0}$, the far-field relation between $\vec{E}^s$ and $\vec{H}^s$ reduces to the simple form,

$$\vec{E}^s = Z_0(\vec{H}^s \times \hat{r}); \quad \vec{H}^s = Z_0^{-1}(\hat{r} \times \vec{E}^s) \tag{1.27}$$

From both (1.23) and (1.26) it is easy to see that $\hat{r}\cdot\vec{E}^s = \hat{r}\cdot\vec{H}^s = 0$; thus the radial component of the far-field electric and magnetic vectors is zero. Since $\vec{E}^s = \vec{f}(\theta, \phi)\exp(-jk_0 r)/r$, it follows that,

$$\vec{H}^s = \frac{1}{Z_0}\left[\hat{r} \times \vec{f}(\theta, \phi)\right]\frac{e^{-jk_0 r}}{r} \tag{1.28}$$

$$\left|\frac{\vec{E}^s}{\vec{H}^s}\right| = Z_0 \tag{1.29}$$

Recall that $\vec{E}^s$ and $\vec{H}^s$ are complex vectors or vector-phasors, and so is the vector scattering amplitude $\vec{f}(\theta, \phi)$. The function $rF = |\vec{f}(\theta, \phi)|\exp(-jk_0 r)$ satisfies the spherical wave equation,

$$\frac{\partial^2}{\partial r^2}(rF) + k_0^2(rF) = 0 \tag{1.30}$$

as can be verified by direct substitution. The function $F(r, \theta)$, thus describes spherical waves that expand radially with constant velocity of light. The equiphase surfaces are given by $k_0 r \equiv$ constant; these are spherical and they are separated by the constant distance $\lambda = 2\pi/k_0$, where λ is the wavelength.

A further simplification of the far-field structure of the scattered wave follows if interest is confined to a relatively small region of space surrounding the point P_0 in Fig. 1.5, whose radial extent is much smaller than r_0 and whose solid angle subtended at the origin is also small. Within this small volume of space, the function $|\vec{f}(\theta, \phi)|r^{-1}$ can be considered constant and equal to $|\vec{f}(\theta_0, \phi_0)|r_0^{-1}$. Consider another point P within this small volume, as also shown in Fig. 1.5. It is clear that the direction to P as described by $\hat{s}$ will be the same as $\hat{r}_0$. However, the phase of the spherical wave at P is approximated as $\exp(-jk_0 s)$, while its amplitude is approximated as a constant equal to $K \equiv |\vec{f}(\theta_0, \phi_0)|r_0^{-1}$. Thus within the small volume, the scattered wave characteristics can be written as $F(s) \approx K\exp(-jk_0 s)$, which is the so-called "local" plane wave approximation, since the surfaces of the constant phase are given by $k_0 s$ = constant, which are plane surfaces orthogonal to $\hat{s}$. The function $F(s)$ satisfies the plane wave equation,

$$\frac{d^2 F}{ds^2} + k_0^2 F = 0 \tag{1.31}$$

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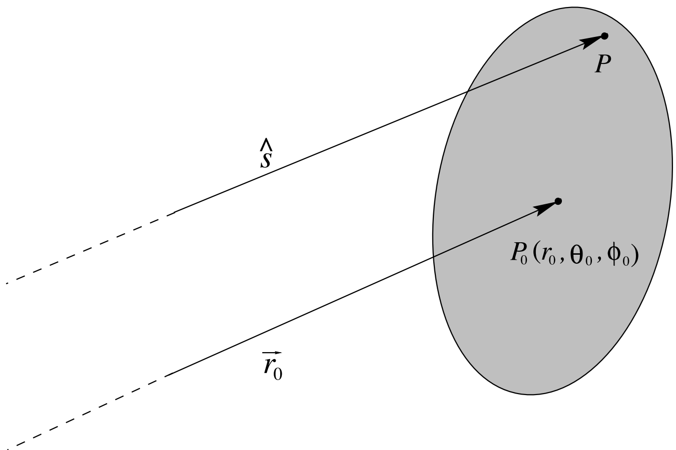

Fig. 1.5. Local plane wave approximation within a small volume centered at P_0.

as can be verified by direct substitution. Thus (1.30) which is the spherical wave equation, is approximated by (1.31), which is the local plane wave equation (King and Prasad 1986).

1.3 Rayleigh scattering by a dielectric sphere

The integral representation for the far-field vector scattering amplitude, $\vec{f}(\theta, \phi)$, given in (1.24b) is fundamental, with the basic unknown being the total electric field in the interior of the dielectric particle. When this internal field is approximated by the corresponding electrostatic solution it is called Rayleigh scattering. In Appendix 1, the case of a dielectric sphere in an uniform incident field is described and extensions are made to the case of prolate/oblate spheroids. For arbitrary-shaped dielectric particles, numerical methods must be used: a surface integral equation technique is described in Van Bladel (1985; Section 3.9) with numerical results given, for example, in Herrick and Senior (1977).

Referring to Fig. 1.6*a*, the uniform incident field $\vec{E}^i = \hat{z}E_0$ can be used as an electrostatic approximation for an ideal plane wave propagating along the positive Y-axis with amplitude E_0 and linearly polarized along the $\hat{z}$-direction, of the form $\vec{E}^i = \hat{z}E_0 \exp(-jk_0 y)$ (wave polarization will be treated in detail in Chapter 3). As discussed under the electroquasistatic approximation in Appendix 1, this is valid when the sphere diameter is very small compared with the wavelength λ. Substituting from Appendix 1 (A1.24), the electrostatic solution $\vec{E}_T^{\text{in}} = \hat{z}3E_0/(\varepsilon_r + 2) = \vec{E}^i(3/\varepsilon_r + 2)$ in (1.24b), and noting that $\vec{E}_T^{\text{in}}$ is constant inside the sphere and that $\exp(jk_0\vec{r}'\cdot\hat{r}) \approx 1$, since the maximum extent of $\vec{r}'$ is the sphere radius, results in,

$$\vec{f}(\theta, \phi) = \frac{k_0^2}{4\pi}\frac{(\varepsilon_r - 1)}{(\varepsilon_r + 2)}3V\left[\vec{E}^i - \hat{r}(\hat{r}\cdot\vec{E}^i)\right] \tag{1.32a}$$

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