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Part I

Plasma physics preliminaries

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Introduction

1.1 Motivation

Under ordinary circumstances, matter on Earth occurs in the three phases of solid, liquid, and gas. Here, 'ordinary' refers to the circumstances relevant for human life on this planet. This state of affairs does not extrapolate beyond earthly scales: astronomers agree that, ignoring the more speculative nature of dark matter, matter in the Universe consists more than 90% of plasma. Hence, *plasma is the ordinary state of matter in the Universe*. The consequences of this fact for our view of nature are not generally recognized yet (see Section 1.3.4). The reason may be that, since plasma is an exceptional state on Earth, the subject of plasma physics is a relative latecomer in physics.

For the time being, the following crude definition of plasma suffices. *Plasma is a completely ionized gas, consisting of freely moving positively charged ions, or nuclei, and negatively charged electrons.*[1] In the laboratory, this state of matter is obtained at high temperatures, in particular in thermonuclear fusion experiments ($T \sim 10^8$ K). In those experiments, the mobility of the plasma particles facilitates the induction of electric currents which, together with the internally or externally created magnetic fields, permits magnetic confinement of the hot plasma. In the Universe, plasmas and the associated large-scale interactions of currents and magnetic fields prevail under much wider conditions.

Hence, we will concentrate our analysis on the two mentioned broad areas of application of plasma physics, viz.

(a) *Magnetic plasma confinement for the purpose of future energy production by controlled thermonuclear reactions* (CTR); this includes the pinch experiments of the 1960s and

[1] In plasma physics, one can hardly avoid mentioning exceptions: in pulsar electron–positron magnetospheres, the role of positively charged particles is taken by positrons. In considerations of fusion reactions with exotic fuels like muonium, the role of negatively charged particles is taken by muons.

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early 1970s, and the tokamaks and alternatives (stellarator, spheromak, etc.) developed in the 1980s and 1990s and, at present, sufficiently matured to start designing prototypes of the fusion reactors themselves;

(b) *The dynamics of magnetized astrophysical plasmas*; this includes the ever growing research field of solar magnetic activity, planetary magnetospheres, stellar winds, interstellar medium, accretion discs of compact objects, pulsar magnetospheres, etc.

The common ground of these two areas is the subject of *plasma interacting with a magnetic field*. To appreciate the power of this viewpoint, we first discuss the conditions for laboratory fusion in Section 1.2, then switch to the emergence of the subject of plasma-astrophysics in Section 1.3, and finally refine our definition(s) of plasma in Section 1.4. In the latter section, we also provisionally formulate the approach to plasmas by means of magnetohydrodynamics.

The theoretical models exploited lead to *nonlinear partial differential equations*, expressing *conservation laws*. The boundary conditions are imposed on an extended spatial domain, associated with the *complex magnetic plasma confinement geometry*, whereas the temporal dependence leads to *intricate nonlinear dynamics*. This gives theoretical plasma physics its particular, mathematical, flavour.

1.2 Thermonuclear fusion and plasma confinement

1.2.1 Fusion reactions

Both fission and fusion energy are due to nuclear processes and, ultimately, described by Einstein's celebrated formula $E = mc^2$. Hence, in nuclear reactions $A + B \rightarrow C + D$, net energy is released if there is a mass defect, i.e. if

$$(m_A + m_B)\, c^2 > (m_C + m_D)\, c^2 \,. \tag{1.1}$$

In laboratory fusion, reactions of hydrogen isotopes are considered, where the deuterium–tritium reaction (Fig. 1.1) is the most promising one for future reactors:

$$\mathrm{D}^2 + \mathrm{T}^3 \;\rightarrow\; \mathrm{He}^4\ (3.5\,\mathrm{MeV}) + \mathrm{n}\ (14.1\,\mathrm{MeV})\,. \tag{1.2}$$

This yields two kinds of products, viz. α-particles (He^4), which are *charged* so that they can be captured by a confining magnetic field, and neutrons, which are *electrically neutral* so that they escape from the magnetic configuration. The former contribute to the heating of the plasma (so-called α-particle heating) and the latter have to be captured in a surrounding $\mathrm{Li}^6/\,\mathrm{Li}^7$ blanket, which recovers the fusion energy and also breeds new T^3.

Deuterium abounds in the oceans: out of 6500 molecules of water one molecule is $\mathrm{D_2O}$. Thus, in principle, 1 litre of sea water contains 10^{10} J of deuterium fusion

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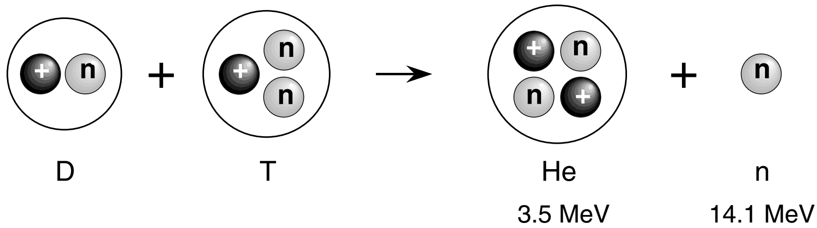

Fig. 1.1. Deuterium–tritium reactions.

energy. This is a factor of about 300 more than the combustion energy of 1 litre of gasoline, which yields 3×10^7 J.

A number of other reactions also occur, in particular reactions producing T^3 and He^3 which may be burned again. Complete burn of all available D^2 would involve the following reactions:

$$\begin{aligned} D^2 + D^2 &\to He^3\ (0.8\,\text{MeV}) + n\ (2.5\,\text{MeV})\,, \\ D^2 + D^2 &\to T^3\ (1.0\,\text{MeV}) + p\ (3.0\,\text{MeV})\,, \\ D^2 + T^3 &\to He^4\ (3.5\,\text{MeV}) + n\ (14.1\,\text{MeV})\,, \\ D^2 + He^3 &\to He^4\ (3.7\,\text{MeV}) + p\ (14.6\,\text{MeV})\,, \end{aligned} \tag{1.3}$$

so that in effect

$$6D^2 \;\to\; 2\,He^4 + 2\,p + 2\,n + 43.2\,\text{MeV}\,. \tag{1.4}$$

In the liquid Li blanket, fast neutrons are moderated, so that their kinetic energy is converted into heat, and the following reactions occur:

$$\begin{aligned} n + Li^6 &\to T^3\ (2.1\,\text{MeV}) + He^4\ (2.8\,\text{MeV})\,, \\ n\ (2.5\,\text{MeV}) + Li^7 &\to T^3\ + He^4 + n\,. \end{aligned} \tag{1.5}$$

This provides the necessary tritium fuel for the main fusion reaction (1.3)(c) [156].

Typical numbers associated with thermonuclear fusion reactors, as presently envisaged, are:

$$\begin{array}{llll} \text{temperature} & T \sim 10^8\,\text{K}\ (10\,\text{keV})\,, & \text{power density} & \sim 10\,\text{MW}\,\text{m}^{-3}\,, \\ \text{particle density} & n \sim 10^{21}\,\text{m}^{-3}\,, & \text{time scale} & \tau \sim 100\,\text{s}\,. \end{array} \tag{1.6}$$

It is often said that controlled thermonuclear fusion in the laboratory is an attempt to harness the power of the stars. This is actually a quite misleading statement since the fusion reactions which take place in, e.g., *the core of the Sun* are

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different reactions of hydrogen isotopes, viz.

$$\begin{aligned} p + p &\rightarrow D^2 \ + e^+ + \nu_e \ + 1.45\,\text{MeV} \quad (2\times)\,, \\ p + D^2 &\rightarrow He^3 \ + \gamma + 5.5\,\text{MeV} \qquad\qquad (2\times)\,, \\ He^3 + He^3 &\rightarrow He^4 + 2\,p + 12.8\,\text{MeV}\,, \end{aligned} \tag{1.7}$$

so that complete burn of all available hydrogen amounts to

$$4\,p \ \rightarrow \ He^4 + 2\,e^+ + 2\,\nu_e\ (0.5\,\text{MeV}) + 2\,\gamma\ (26.2\,\text{MeV})\,. \tag{1.8}$$

The positrons annihilate with electrons, the neutrinos escape, and the gammas (carrying the bulk of the thermonuclear energy) start on a long journey to the solar surface, where they arrive millions of years later (the mean free path of a photon in the interior of the Sun is only a few centimetres) [190]. In the many processes of absorption and re-emission the wavelength of the photons gradually shifts from that of gamma radiation to that of the visible and UV light escaping from the photosphere of the Sun, and producing one of the basic conditions for life on a planet situated at the safe distance of one astronomical unit (1.5×10^{11} m) from the Sun.

At higher temperatures another chain of reactions is effective, where carbon acts as a kind of catalyst. This so-called CNO cycle involves a chain of fusion reactions where C^{12} is successively converted into N^{13}, C^{13}, N^{14}, O^{15}, N^{15}, and back into C^{12} again. However, the net result of incoming and outgoing products is the same as that of the proton–proton chain, viz. Eq. (1.8).

Typical numbers associated with thermonuclear reactions in the stars, in particular the core of the Sun, are the following ones:

$$\begin{array}{llll} \text{temperature} & T \sim 1.5\times 10^7\,\text{K}\,, & \text{power density} & \sim 3.5\,\text{W}\,\text{m}^{-3}\,, \\ \text{particle density} & n \sim 10^{32}\,\text{m}^{-3}\,, & \text{time scale} & \tau \sim 10^7\ \text{years}\,. \end{array} \tag{1.9}$$

Very different from the numbers (1.6) for a prospective fusion reactor on Earth!

1.2.2 Conditions for fusion

Thermonuclear fusion happens when a gas of, e.g., deuterium and tritium atoms is sufficiently heated for the thermal motion of the nuclei to become so fast that they may overcome the repulsive Coulomb barrier (Fig. 1.2) and come close enough for the attractive nuclear forces to bring about the fusion reactions discussed above. This requires particle energies of ~ 10 keV, i.e. temperatures of about 10^8 K. At these temperatures the electrons are completely stripped from the atoms (the ionization energy of hydrogen is ~ 14 eV) so that a plasma rather than a gas is obtained (cf. our crude definition of Section 1.1).

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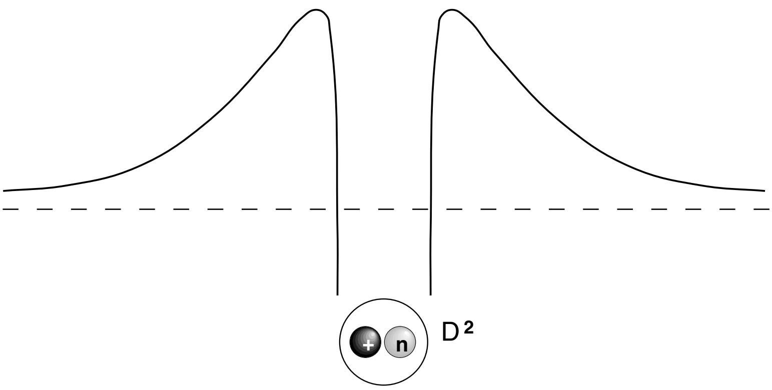

Fig. 1.2. Nuclear attraction and Coulomb barrier of a deuteron.

Because the charged particles (occurring in about equal numbers of opposite charge) are freely moving and rarely collide at these high temperatures, a plasma may be considered as a *perfectly conducting fluid* for many purposes. In such fluids, electric currents are easily induced and the associated magnetic fields in turn interact with the plasma to confine or to accelerate it. The appropriate theoretical description of this state of matter is called *magnetohydrodynamics* (MHD), i.e. the dynamics of magneto-fluids (Section 1.4.2).

Why are magnetic fields necessary? To understand this, we need to discuss the power requirements for fusion reactors (following Miyamoto [156] and Wesson [244]). This involves three contributions, viz.

(a) the thermonuclear output power per unit volume:

$$P_{\rm T} = n^2 f(\tilde{T}) \,, \qquad f(\tilde{T}) \equiv \tfrac{1}{4}\langle\sigma v\rangle E_{\rm T} \,, \quad E_{\rm T} \approx 22.4 \ \text{MeV} \,, \tag{1.10}$$

where n is the particle density, σ is the cross-section of the D-T fusion reactions, v is the relative speed of the nuclei, $\langle\sigma v\rangle$ is the average nuclear reaction rate, which is a well-known function of temperature, and $E_{\rm T}$ is the average energy released in the fusion reactions (i.e. more than the 17.6 MeV of the D-T reaction (1.3)(c) but, of course, less than the 43.2 MeV released for the complete burn (1.4));

(b) the power loss by Bremsstrahlung, i.e. the radiation due to electron–ion collisions:

$$P_{\rm B} = \alpha n^2 \tilde{T}^{1/2} \,, \quad \alpha \approx 3.8 \times 10^{-29} \ \text{J}^{1/2}\,\text{m}^3\,\text{s}^{-1} \,; \tag{1.11}$$

(c) the losses by heat transport through the plasma:

$$P_{\rm L} = \frac{3n\tilde{T}}{\tau_{\rm E}} \,, \tag{1.12}$$

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where $3n\tilde{T}$ is the total plasma kinetic energy density (ions + electrons), and $\tau_{\rm E}$ is the energy confinement time (an empirical quantity). The latter estimates the usually anomalous (i.e. deviating from classical transport by Coulomb collisions between the charged particles) heat transport processes.

Here, we have put a tilde on the temperature to indicate that energy units of keV are exploited:

$$\tilde{T}\,(\mathrm{keV}) = 8.62 \times 10^{-8}\, T\,(\mathrm{K})\,,$$

since $\tilde{T} = 1\,\mathrm{keV} = 1.60 \times 10^{-16}\,\mathrm{J}$ corresponds with $T = 1.16 \times 10^{7}\,\mathrm{K}$ (using Boltzmann's constant, see Appendix Table B.1).

If the three power contributions are considered to become externally available for conversion into electricity and back again into plasma heating, with efficiency η, *the Lawson criterion* [140],

$$P_{\rm B} + P_{\rm L} = \eta\,(P_{\rm T} + P_{\rm B} + P_{\rm L})\,, \tag{1.13}$$

tells us that there should be power balance between the losses from the plasma (LHS) and what is obtained from plasma heating (RHS). Typically, $\eta \approx 1/3$. Inserting the explicit expressions (1.10), (1.11), and (1.12) into Eq. (1.13) leads to a condition to be imposed on the product of the plasma density and the energy confinement time:

$$n\tau_{\rm E} = \frac{3\tilde{T}}{\dfrac{\eta}{1-\eta}\,f(\tilde{T}) - \alpha\tilde{T}^{1/2}}\,. \tag{1.14}$$

This relationship is represented by the lower curve in Fig. 1.3. Since Bremsstrahlung losses dominate at low temperatures and transport losses dominate at high temperatures, there is a minimum in the curve at about

$$n\tau_{\rm E} = 0.6 \times 10^{20}\,\mathrm{m}^{-3}\,\mathrm{s}\,, \quad \text{for } \tilde{T} = 25\,\mathrm{keV}\,. \tag{1.15}$$

This should be considered to be the threshold for a fusion reactor under the given conditions.

By a rather different, more recent, approach of fusion conditions, *ignition* occurs when the total amount of power losses is balanced by the total amount of heating power. The latter consists of α-particle heating P_α and additional heating power $P_{\rm H}$, e.g. by radio-frequency waves or neutral beam injection. The latter heating sources are only required to bring the plasma to the ignition point, when α-particle heating may take over. Hence, at ignition we may put $P_{\rm H} = 0$ and the power balance becomes

$$P_{\rm B} + P_{\rm L} = P_\alpha = \tfrac{1}{4}\langle\sigma v\rangle n^2 E_\alpha\,, \quad E_\alpha \approx 3.5\,\mathrm{MeV}\,. \tag{1.16}$$

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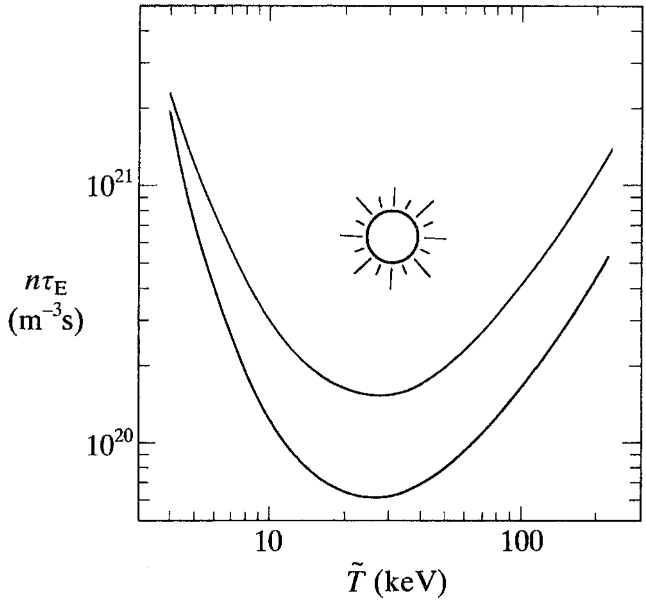

Fig. 1.3. Conditions for net fusion energy production according to the Lawson criterion (lower curve) and according to the view that power losses should be completely balanced by α-particle heating (upper curve). Adapted from Wesson [244].

Formally, this may be described by the same equation (1.14) taking now $\eta \approx 0.135$ so that a 2.5 times higher threshold for fusion is obtained:

$$n\tau_E = 1.5 \times 10^{20}\,\mathrm{m}^{-3}\,\mathrm{s}\,, \quad \text{for } \tilde{T} = 30\,\mathrm{keV}\,. \tag{1.17}$$

This relationship is represented by the upper curve of Fig. 1.3.

Roughly speaking then, products of density and energy confinement time $n\tau_E \sim 10^{20}\,\mathrm{m}^{-3}\,\mathrm{s}$ and temperatures $\tilde{T} \sim 25\,\mathrm{keV}$, or $T \sim 3 \times 10^8$ K, are required for controlled fusion reactions. As a figure of merit for fusion experiments one frequently constructs the product of these two quantities, which should approach

$$n\tau_E\tilde{T} \sim 3 \times 10^{21}\,\mathrm{m}^{-3}\,\mathrm{s\,keV} \tag{1.18}$$

for a fusion reactor. To get rid of the radioactive tritium component, one might consider pure D-D reactions in a more distant future. This would require yet another increase of the temperature by a factor of 10. Considering the kind of progress obtained over the past 40 years, though (see Fig. 1.1.1 of Wesson [244]: a steady increase of the product $n\tau_E T$ with a factor of 100 every decade!), one may hope that this difficulty eventually will turn out to be surmountable.

Returning to our question on the magnetic fields: no material containers can hold plasmas with densities of $10^{20}\,\mathrm{m}^{-3}$ and temperatures of 100–300 million K

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during times in the order of minutes, or at least seconds, without immediately extinguishing the 'fire'. One way to solve this problem is to make use of the *confining properties of magnetic fields*, which may be viewed from quite different angles:

(a) the charged particles of the plasma rapidly and tightly gyrate around the magnetic field lines (they 'stick' to the field lines, see Section 2.2);
(b) fluid and magnetic field move together ('the magnetic field is frozen into the plasma', see Section 2.4), so that engineering of the geometry of the magnetic field configuration also establishes the geometry of the plasma;
(c) the thermal conductivity of plasmas is highly anisotropic with respect to the magnetic field, $\kappa_\perp \ll \kappa_\parallel$ (see Sections 2.3.1 and 3.3.2), so that heat is easily conducted along the field lines and the magnetic surfaces they map out, but impeded across.

Consequently, what one needs foremost is a *closed magnetic geometry* facilitating stable, static plasma equilibrium with roughly bell-shaped pressure and density profiles and nested magnetic surfaces. This is the subject of the next section.

1.2.3 Magnetic confinement and tokamaks

Controlled thermonuclear fusion research started in the 1950s in the weapons laboratories after the 'successful' development of the hydrogen bomb: fusion energy had been unleashed on our planet! The development of the peaceful, controlled, counterpart appeared to be a matter of a few years, as may become clear by considering the simplicity of early pinch experiments. The history of the subject is schematically illustrated in Fig. 1.4. In the upper part the two early attempts with the simple schemes of θ- and z-pinch are shown. Here, θ and z refer to the direction of the plasma current in terms of a cylindrical r, θ, z coordinate system. Since it is relatively straightforward to produce plasma by ionizing hydrogen gas in a tube, a very conductive fluid is obtained in which a strong current may be induced by discharging a capacitor bank over an external coil surrounding the gas tube. In a *z-pinch experiment*, this current is induced in the z-direction and it creates a transverse magnetic field B_θ, so that the resulting Lorentz force $(\mathbf{j} \times \mathbf{B})_r = -j_z B_\theta$ is pointing radially inward. In this manner, the confining force as well as near thermonuclear temperatures ($\sim 10^7$ K) are easily produced. There is only one problem: the curvature of the magnetic field B_θ causes the plasma to be extremely unstable, with growth rates in the order of μseconds. To avoid these instabilities, the orthogonal counterpart, the *θ-pinch experiment*, suggested itself. Here, current is induced in the θ-direction, it causes a radial decrease of the externally applied magnetic field B_z, so that the net Lorentz force $j_\theta\, \Delta B_z$ is again directed inward. In the θ-pinch, thermonuclear temperatures are also obtained, and the plasma is now macroscopically stable. However, pinching of the plasma column produces

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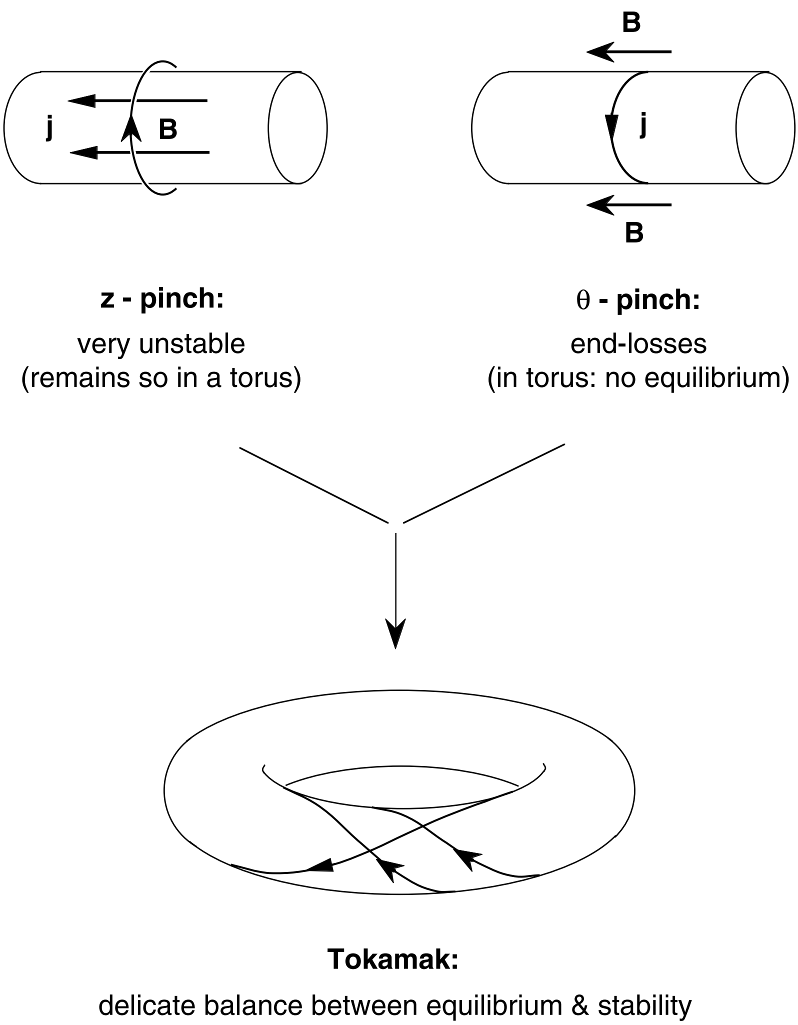

Fig. 1.4. Interaction of currents and magnetic fields: a schematic history of plasma confinement experiments.

unbalanced longitudinal forces so that the plasma is squirted out of the ends, again terminating plasma confinement on the μs time scale. In conclusion, in pinch experiments the densities and temperatures needed for thermonuclear ignition are easily produced but the confinement times fall short by a factor of a million to a billion.

With these obstacles ahead, the nations involved with thermonuclear research decided it to be opportune to declassify the subject. This fortunate decision was landmarked by the Second International UN Conference on Peaceful Uses of Atomic Energy in Geneva in 1958, where all scientific results obtained so far were

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