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Quarks and leptons

1.1 Preamble

The subject of elementary particle physics may be said to have begun with the discovery of the electron 100 years ago. In the following 50 years, one new particle after another was discovered, mostly as a result of experiments with cosmic rays, the only source of very high energy particles then available. However, the subject really blossomed after 1950, following the discovery of new elementary particles in cosmic rays; this stimulated the development of high energy accelerators, providing intense and controlled beams of known energy that were finally to reveal the quark substructure of matter and put the subject on a sound quantitative basis.

1.1.1 Why high energies?

Particle physics deals with the study of the elementary constituents of matter. The word 'elementary' is used in the sense that such particles have no known structure, i.e. they are pointlike. How pointlike is pointlike? This depends on the spatial resolution of the 'probe' used to investigate possible structure. The resolution is Δr if two points in an object can just be resolved as separate when they are a distance Δr apart. Assuming the probing beam itself consists of pointlike particles, the resolution is limited by the de Broglie wavelength of these particles, which is $\lambda = h/p$ where p is the beam momentum and h is Planck's constant. Thus beams of high momentum have short wavelengths and can have high resolution. In an optical microscope, the resolution is given by

$$\Delta r \simeq \lambda / \sin\theta$$

where θ is the angular aperture of the light beam used to view the structure of an object. The object scatters light into the eyepiece, and the larger the angle of scatter θ and the smaller the wavelength λ of the incident beam the better is the resolution. For example an ultraviolet microscope has better resolution and reveals more detail than one using visible light. Substituting the de Broglie relation, the resolution becomes

$$\Delta r \simeq \frac{\lambda}{\sin\theta} = \frac{h}{p\sin\theta} \simeq \frac{h}{q}$$

so that Δr is inversely proportional to the momentum q transferred to the photons, or other particles in an incident beam, when these are scattered by the target.† Thus a value of momentum transfer such that $qc = 10\text{ GeV} = 10^{10}$ eV – easily attainable with present accelerator beams – gives a spatial resolution $hc/(qc) \sim 10^{-16}$ m, about 10 times smaller than the known radius of the charge and mass distribution of a proton (see Table 1.1 for the values of the units employed).

In the early decades of the twentieth century, particle-beam energies from accelerators reached only a few MeV (10^6 eV), and their resolution was so poor that protons and neutrons could themselves be regarded as elementary and pointlike. At the present day, with a resolution thousands of times better, the fundamental pointlike constituents of matter appear to be quarks and leptons, which are the main subject of this text. Of course, it is possible that they in turn may have an inner structure, but there is no present evidence for this, and whether they do will be for future experiments to decide.

The second reason for high energies in experimental particle physics is simply that many of the elementary particles are extremely massive and the energy mc^2 required to create them is correspondingly large. The heaviest elementary particle detected so far, the 'top' quark (which has to be created as a pair with its antiparticle) has $mc^2 \simeq 175$ GeV, nearly 200 times the mass–energy of a proton.

At this point it should be mentioned that the total energy in accelerator beams required to create such massive particles in sufficient intensities is quite substantial. For example, an energy per particle of 1 TeV (10^{12} eV) in beams consisting of bunches of 10^{13} accelerated particles every second will correspond to a total kinetic energy in each bunch of 1.6 megajoules, equal to the energy of 30 000 light bulbs, or of a 15 tonne truck travelling at 30 mph.

1.1.2 Units in high energy physics

The basic units in physics are length, mass and time and the SI system expresses these in metres, kilograms and seconds. Such units are not very appropriate in high energy physics, where typical lengths are 10^{-15} m and typical masses are 10^{-27} kg.

Table 1.1 summarises the units commonly used in high energy physics. The unit of length is the *femtometre* or *fermi*, where 1 fm $= 10^{-15}$ m; for example, the root mean square radius of the charge distribution of a proton is 0.8 fm. The

† To be exact, in an elastic collision with a massive target, the momentum transfer will be $q = 2p\sin(\theta/2)$, if θ is the angle of deflection.

Table 1.1. *Units in high energy physics*

(a)

Quantity	High energy unit	Value in SI units
length	1 fm	10^{-15} m
energy	1 GeV $= 10^9$ eV	1.602×10^{-10} J
mass, E/c^2	1 GeV/c^2	1.78×10^{-27} kg
$\hbar = h/(2\pi)$	6.588×10^{-25} GeV s	1.055×10^{-34} J s
c	2.998×10^{23} fm s^{-1}	2.998×10^8 m s^{-1}
$\hbar c$	0.1975 GeV fm	3.162×10^{-26} J m

(b)

natural units, $\hbar = c = 1$		
mass, Mc^2/c^2	1 GeV	
length, $\hbar c/(Mc^2)$	1 GeV^{-1} = 0.1975 fm	
time, $\hbar c/(Mc^3)$	1 GeV^{-1} = 6.59×10^{-25} s	
Heaviside–Lorentz units, $\epsilon_0 = \mu_0 = \hbar = c = 1$		
fine structure constant	$\alpha = e^2/(4\pi) = 1/137.06$	
Relations between energy units		
1 MeV = 10^6 eV	1 GeV = 10^3 MeV	1 TeV = 10^3 GeV

commonly used unit of energy is the GeV, convenient because it is typical of the mass–energy mc^2 of strongly interacting particles. For example, a proton has $M_pc^2 = 0.938$ GeV.

In calculations, the quantities $\hbar = h/(2\pi)$ and c occur frequently, sometimes to high powers, and it is advantageous to use units in which we set $\hbar = c = 1$. Having chosen these two units, we are still at liberty to specify one more unit, e.g. the unit of energy, and the common choice, as indicated above, is the GeV. With $c = 1$ this is also the mass unit. As shown in the table, the unit of length will then be 1 GeV^{-1} = 0.197 fermi, while the corresponding unit of time is 1 GeV^{-1} = 6.59×10^{-25} s.

Throughout this text we shall be dealing with interactions between charges – which can be the familiar electric charge of electromagnetic interactions, the strong charge of the strong interaction or the weak charge of the weak interaction. In the SI system the unit electric charge, e, is measured in coulombs and the fine structure constant is given by

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \simeq \frac{1}{137}$$

Here ϵ_0 is the permittivity of free space, while its permeability is defined as μ_0, such that $\epsilon_0\mu_0 = 1/c^2$. For interactions in general, such units are not useful and we can define e in Heaviside–Lorentz units, which require $\epsilon_0 = \mu_0 = \hbar = c = 1$, so that

$$\alpha = \frac{e^2}{4\pi} \simeq \frac{1}{137}$$

with similar definitions that relate charges and coupling constants analogous to α in the other interactions.

1.1.3 Relativistic transformations

In most of the processes to be considered in high energy physics, the individual particles have relativistic or near relativistic velocities, $v \sim c$. This means that the result of a measurement, e.g. the lifetime of an unstable particle, will depend on the reference frame in which it is made. It follows that one requirement of any theory of elementary particles is that it should obey a fundamental symmetry, namely invariance under a relativistic transformation, so that the equations will have the same form in all reference frames. This can be achieved by formulating the equations in terms of 4-vectors, which we now discuss briefly, together with the notation employed in this text.

The relativistic relation between total energy E, the vector 3-momentum $\mathbf{p}$ (with Cartesian components p_x, p_y, p_z) and the rest mass m for a free particle is

$$E^2 = \mathbf{p}^2c^2 + m^2c^4$$

or, in units with $c = 1$

$$E^2 = \mathbf{p}^2 + m^2$$

The components p_x, p_y, p_z, E can be written as components of an energy–momentum 4-vector p_μ, where $\mu = 1, 2, 3, 4$. In the Minkowski convention used in this text, the three momentum (or space) components are taken to be real and the energy (or time) component to be imaginary, as follows:

$$p_1 = p_x, \quad p_2 = p_y, \quad p_3 = p_z, \quad p_4 = iE$$

so that

$$p^2 = \sum_\mu p_\mu^2 = p_1^2 + p_2^2 + p_3^2 + p_4^2 = \mathbf{p}^2 - E^2 = -m^2 \tag{1.1}$$

Thus p^2 is a relativistic invariant. Its value is $-m^2$, where m is the rest mass, and clearly has the same value in all reference frames. If E, $\mathbf{p}$ refer to the values measured in the lab frame $\sum$ then those in another frame, say $\sum'$, moving along

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the x-axis with velocity βc are found from the Lorentz transformation, given in matrix form by

$$p'_\mu = \sum_{\nu=1}^{4} \alpha_{\mu\nu} p_\nu$$

where

$$\alpha_{\mu\nu} = \begin{vmatrix} \gamma & 0 & 0 & i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & \gamma \end{vmatrix}$$

and $\gamma = 1/\sqrt{1-\beta^2}$. Thus

$$\begin{aligned} p'_1 &= \gamma p_1 + i\beta\gamma p_4 \\ p'_2 &= p_2 \\ p'_3 &= p_3 \\ p'_4 &= -i\beta\gamma p_1 + \gamma p_4 \end{aligned}$$

In terms of energy and momentum

$$\begin{aligned} p'_x &= \gamma(p_x - \beta E) \\ p'_y &= p_y \\ p'_z &= p_z \\ E' &= \gamma(E - \beta p_x) \end{aligned}$$

with, of course, $\mathbf{p}'^2 - E'^2 = -m^2$. The above transformations apply equally to the space–time coordinates, making the replacements $p_1 \to x_1(=x)$, $p_2 \to x_2(=y)$, $p_3 \to x_3(=z)$ and $p_4 \to x_4(=it)$.

The 4-momentum squared in (1.1) is an example of a Lorentz scalar, i.e. the invariant scalar product of two 4-vectors, $\sum p_\mu p_\mu$. Another example is the phase of a plane wave, which determines whether it is at a crest or a trough and which must be the same for all observers. With $\mathbf{k}$ and ω as the propagation vector and the angular frequency, and in units $\hbar = c = 1$,

$$\text{phase} = \mathbf{k}\cdot\mathbf{x} - \omega t = \mathbf{p}\cdot\mathbf{x} - Et = \sum p_\mu x_\mu$$

The Minkowski notation used here for 4-vectors defines the *metric*, namely the square of the 4-vector momentum $p = (\mathbf{p}, iE)$ so that

$$\text{metric} = (\text{4-momentum})^2 = (\text{3-momentum})^2 - (\text{energy})^2$$

In analogy with the space–time components, the components $p_{x,y,z}$ of 3-momentum are said to be *spacelike* and the energy component E, *timelike*. Thus, if q denotes the 4-momentum transfer in a reaction, i.e. is $q = p - p'$ where p, p' are the initial and final 4-momenta, then

$$\begin{aligned} q^2 &> 0 \quad \text{is spacelike,} \quad \text{e.g. in a scattering process} \\ q^2 &< 0 \quad \text{is timelike,} \quad \text{e.g. the squared mass of a free particle} \end{aligned} \tag{1.2}$$

A different notation is used in texts on field theory. These avoid the use of the imaginary fourth component ($p_4 = iE$) and introduce the negative sign via the metric tensor $g_{\mu\nu}$. The scalar product of 4-vectors A and B is then defined as

$$AB = g_{\mu\nu} A_\mu B_\nu = A_0 B_0 - \mathbf{A} \cdot \mathbf{B} \tag{1.3}$$

where all the components are real. Here $\mu, \nu = 0$ stand for the energy (or time) component and $\mu, \nu = 1, 2, 3$ for the momentum (or space) components, and

$$g_{00} = +1, \quad g_{11} = g_{22} = g_{33} = -1, \quad g_{\mu\nu} = 0 \quad \text{for} \quad \mu \neq \nu \tag{1.4}$$

This metric results in Lorentz scalars with sign opposite to those using the Minkowski convention in (1.2), so that a spacelike (or timelike) 4-momentum has $q^2 < 0$ (or $q^2 > 0$) respectively. Sometimes, to avoid writing negative quantities, re-definitions have to be made. In deep inelastic electron scattering, q^2 is spacelike and negative, as defined in (1.3), and in discussing such processes it has become common to define the positive quantity $Q^2 = -q^2$. This simply illustrates the fact that the sign of the metric is just a matter of convention and does not in any way affect the physical results.

1.1.4 Fixed-target and colliding beam accelerators

As an example of the application of 4-vector notation, we consider the energy available for particle creation in fixed-target and in colliding-beam accelerators (see also Chapter 11).

Suppose an incident particle of mass m_A, total energy E_A and momentum $\mathbf{p}_A$ hits a target particle of mass m_B, energy E_B, momentum $\mathbf{p}_B$. The total 4-momentum, squared, of the system is

$$p^2 = (\mathbf{p}_A + \mathbf{p}_B)^2 - (E_A + E_B)^2 = -m_A^2 - m_B^2 + 2\mathbf{p}_A \cdot \mathbf{p}_B - 2E_A E_B \tag{1.5}$$

The centre-of-momentum system (cms) is defined as the reference frame in which the total 3-momentum is zero. If the total energy in the cms is denoted E^*, then we also have $p^2 = -E^{*2}$.

Suppose first of all that the target particle (m_B) is at rest in the laboratory (lab) system, so that $\mathbf{p}_B = 0$ and $E_B = m_B$, while E_A is the energy of the incident particle in the lab system. Then

$$E^{*2} = -p^2 = m_A^2 + m_B^2 + 2m_B E_A \tag{1.6}$$

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Secondly, suppose that the incident and target particles travel in opposite directions, as would be the case in an e^+e^- or a $p\bar{p}$ collider. Then, with p_A and p_B denoting the absolute values of the 3-momenta, the above equation gives

$$\begin{aligned} E^{*2} &= -p^2 = 2(E_A E_B + p_A p_B) + (m_A^2 + m_B^2) \\ &\simeq 4E_A E_B \end{aligned} \tag{1.7}$$

if $m_A, m_B \ll E_A, E_B$. This result is for a head-on collision. For two beams crossing at an angle θ, the result would be $E^{*2} = 2E_A E_B(1 + \cos\theta)$. We note that the cms energy available for new particle creation in a collider with equal energies E in the two beams rises linearly with E, i.e. $E^* \simeq 2E$, while for a fixed-target machine the cms energy rises as the square root of the incident energy, $E^* \simeq \sqrt{2m_B E_A}$. Obviously, therefore, the highest possible energies for creating new particles are to be found at colliding-beam accelerators. As an example, the cms energy of the Tevatron $p\bar{p}$ collider at Fermilab is $E^* = 2$ TeV $= 2000$ GeV. To obtain the same cms energy with a fixed-target accelerator, the energy of the proton beam, in collision with a target nucleon, would have to be $E_A = E^{*2}/(2m_B) \simeq 2 \times 10^6$ GeV $= 2000$ TeV.

1.2 The Standard Model of particle physics

1.2.1 The fundamental fermions

Practically all experimental data from high energy experiments can be accounted for by the so-called *Standard Model* of particles and their interactions, formulated in the 1970s. According to this model, all matter is built from a small number of fundamental spin $\frac{1}{2}$ particles, or *fermions*: six *quarks* and six *leptons*. For each of the various fundamental constituents, its symbol and the ratio of its electric charge Q to the elementary charge e of the electron are given in Table 1.2.

The *leptons* carry integral electric charge. The electron e with unit negative charge is familiar to everyone, and the other charged leptons are the muon μ and the tauon τ. These are heavy versions of the electron. The neutral leptons are called *neutrinos*, denoted by the generic symbol ν. A different ‘flavour’ of neutrino is paired with each ‘flavour’ of charged lepton, as indicated by the subscript. For example, in nuclear β-decay, an electron e is emitted together with an electron-type neutrino, ν_e. The charged muon and tauon are both unstable, and decay spontaneously to electrons, neutrinos and other particles. The mean lifetime of the muon is 2.2×10^{-6} s, that of the tauon only 2.9×10^{-13} s.

Neutrinos were postulated by Pauli in 1930 in order to account for the energy and momentum missing in the process of nuclear β-decay (see Figure 1.1). The actual existence of neutrinos as independent particles, detected by their interactions, was

Table 1.2. *The fundamental fermions*

Particle	Flavour			$Q/\|e\|$
leptons	e	μ	τ	-1
	ν_e	ν_μ	(ν_τ)	0
quarks	u	c	t	$+\frac{2}{3}$
	d	s	b	$-\frac{1}{3}$

first demonstrated in 1956. The tau neutrino is shown in parentheses because its interactions have not so far (1999) been observed.

The *quarks* carry fractional charges, of $+\frac{2}{3}|e|$ or $-\frac{1}{3}|e|$. In the table, the quark masses increase from left to right, just as they do for the leptons (see Tables 1.4 and 1.5). And, just as for the leptons, the quarks are grouped into pairs differing by one unit of electric charge. The quark type or 'flavour' is denoted by a symbol: u for 'up', d for 'down', s for 'strange', c for 'charmed', b for 'bottom' and t for 'top'. How did such odd names get chosen? The 's for strange' quark terminology came about because these quarks turned out to be constituents of the so-called 'strange particles' discovered in cosmic rays (long before quarks were postulated). Their behaviour was strange in the sense that they were produced prolifically in strong interactions, and therefore would be expected to decay on a strong interaction timescale (10^{-23} s); instead they decayed extremely slowly, by weak interactions. The solution to this puzzle was that these particles carried a new quantum number, S for strangeness, conserved in strong interactions – so that they were always produced in pairs with $S = +1$ and $S = -1$ but they decayed singly and weakly, with a change in strangeness, $\Delta S = \pm 1$, into non-strange particles (see Figure 1.10). The choice of the name 'c for charm' was perhaps a reaction to strangeness, while 'top' and 'bottom' are logical names for the partners of up and down quarks. In turn, the up and down quarks were so named because of isospin symmetry (see Section 3.12), according to which each possesses one of the two components $\pm\frac{1}{2}$ of an isospin vector of value $I = \frac{1}{2}$, which, like a spin vector, can point 'up' or 'down'.

While leptons exist as free particles, quarks seem not to do so. It is a peculiarity of the strong forces between the quarks that they can be found only in combinations such as uud, not singly. This phenomenon of quark confinement is, even today, not properly understood.

Protons and neutrons consist of the lightest u and d quarks, three at a time: a proton consists of uud, a neutron consists of ddu. The common material of the present universe is the stable particles, i.e. the electrons e and the u and

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d quarks. The heavier quarks s, c, b, t also combine to form particles akin to, but much heavier than, the proton and neutron, but these are unstable and decay rapidly (in typically 10^{-13} s) to u, d combinations, just as the heavy leptons decay to electrons. Only in very high energy collisions at man-made accelerators, or naturally in cosmic rays, are the heavy, unstable varieties observed.

Table 1.2 shows that the three lepton pairs are exactly matched by the three quark pairs. As we shall see later, it is necessary to introduce a further degree of freedom for the quarks: each flavour of quark comes in three different *colours* (the word ‘colour’ is simply a name to distinguish the three types). If we allow for three colours, the total charge of the u, c, t quarks is $3 \times 3 \times \frac{2}{3} = 6|e|$, that of the d, s, b quarks is $-3 \times 3 \times \frac{1}{3} = -3|e|$ and that of the leptons is $-3 \times 1|e| = -3|e|$. The total charge of all the fermions is then zero. This is the actual condition that the Standard Model should be free of so-called ‘anomalies’ and is a renormalisable field theory. It is also, it turns out, a property of the grand unified theories that unify the strong, electromagnetic and weak interactions at very high energies, as described in Chapter 9.

1.2.2 The interactions

We have looked at the particles; the Standard Model also comprises their interactions. As we discuss in the next chapter, the different interactions are described in quantum language in terms of the exchange of characteristic *bosons* (particles of integral spin) between the fermion constituents. These boson mediators are listed in Table 1.3.

There are four types of fundamental interaction or field, as follows.

Strong interactions are responsible for binding the quarks in the neutron and proton, and the neutrons and protons within nuclei. The interquark force is mediated by a massless particle, the *gluon*.

Electromagnetic interactions are responsible for virtually all the phenomena in extra-nuclear physics, in particular for the bound states of electrons with nuclei, i.e. atoms and molecules, and for the intermolecular forces in liquids and solids. These interactions are mediated by *photon* exchange.

Weak interactions are typified by the slow process of nuclear β-decay, involving the emission by a radioactive nucleus of an electron and neutrino. The mediators of the weak interactions are the $W^{\pm}$ and Z^0 bosons, with masses of order 100 times the proton mass.

Gravitational interactions act between all types of particle. On the scale of experiments in particle physics, gravity is by far the weakest of all the fundamental interactions, although of course it is dominant on the scale of the universe. It is supposedly mediated by exchange of a spin 2 boson, the *graviton*. Very refined

Fig. 1.1. Cloud chamber photograph of the birth of an antineutrino. It depicts the β-decay of the radioactive nucleus $^6\text{He} \to {}^6\text{Li} + e^- + \bar{\nu}_e + 3.5$ MeV. The long track is that of the electron, the short thick track that of the recoiling ^{6}Li nucleus. Some momentum is missing, and has to be ascribed to an uncharged particle (an antineutrino) travelling upwards in the picture (after Csikay and Szalay 1957). The cloud chamber consists essentially of a glass-fronted cylindrical tank of gas saturated with water vapour. Upon applying a sudden expansion by means of a piston at the rear of the chamber, the gas cools adiabatically and becomes supersaturated. Water vapour therefore condenses as droplets, preferentially upon charged ions created, for example, by the passage of a charged particle through the gas. The cloud chamber was invented by C.T.R. Wilson for a quite different purpose: to try to reproduce, in the laboratory, the 'glory' phenomenon he had observed on a Scottish mountain top. Wilson failed in this endeavour but by 1912 had given the world a valuable new technique for nuclear research.

Table 1.3. *The boson mediators*

Interaction	Mediator	Spin/parity
strong	gluon, G	1^-
electromagnetic	photon, γ	1^-
weak	$W^\pm$, Z^0	1^-, 1^+
gravity	graviton, g	2^+

experiments to detect gravitons (en masse, as gravitational waves) are currently under way.

To have four independent and apparently unrelated interaction fields is rather unsatisfactory, and physicists from Einstein onwards have speculated that the