

# *Tides*

A SCIENTIFIC HISTORY

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# I

## *Introduction – the overall pattern of enquiry*

Excepting Mediterranean and Baltic communities, the daily pulse of the tide has always been familiar to those who live by a sea exposed to the oceans. Both the rise and fall at the shoreline and the swing of the currents offshore are obvious. The close alliteration of the Anglo-Saxon words *Zeit*, *Gezeiten*, *Time* and *Tide*, (which ‘wait for no man’), testifies to early awareness of the tide’s regularity by North Sea coastal dwellers, fishermen, navigators and pirates. ‘Rules-of-thumb’ expressing a relationship between times of High Water and lunar phases, based on careful observation, were embodied in medieval tide-tables and clocks (Figures 3.1, 3.2) and in archaic terminology such as ‘What moon maketh a Full Sea?’ (Chapter 3). For many centuries, then, or at least throughout the Christian era, the tide has been a commonplace phenomenon, apparently as predictable as sunrise and Full Moon. What, some may ask, has made it a subject for serious research up to the present day?

Leaving aside the fact that, to a professional astronomer the accurate prediction of the times of sunrise and Full Moon demands sophisticated computations, research on the oceanic tides (and later, tides of air and earth) has been driven by practical needs and by the variety of fundamental questions which have been posed. Practical needs originally stemmed from harbor management, protection from coastal and estuarine flooding, coastal navigation and surveying, but more recently have been additionally concerned with amphibious military operations, harnessing tidal power, and precise corrections to measurements to and from artificial satellites. In all such cases, easy solutions which suffice for rough purposes have proved inadequate, while every improvement in accuracy of measurement and prediction has led to further fundamental research into previously hidden details.

From the dawn of scientific enquiry, basic questions about the *mechanism* of how the moon and sun drive the tides and how the ocean responds to the driving forces have inspired distinguished philosophers and earth-scientists. Descartes

and Galileo (Chapter 4) proposed theories which were later proved untenable by Isaac Newton. Pierre Simon, Marquis de Laplace, the pioneer of tidal dynamics, declared the subject to be ‘... *ce problème, le plus épineux de toute la mécanique céleste* ...’ (*the spiniest problem in all celestial mechanics*.) Solution of Laplace’s tidal equations, even in seas of idealised shape, taxed mathematicians for well over a century until the advent of modern computers. Even then, some decades were to elapse before computers were large enough to represent the global ocean in sufficient detail, and techniques had improved sufficiently to give realistic results.

From a more empirical viewpoint, ever since primitive measurements from the western shores of the Atlantic became known, natural philosophers from Sir Francis Bacon onwards became curious to know how the tides behave in mid-ocean. Observing, for example, that High Water on the Atlantic coast of Florida occurs at the same time as at the Canary Islands, what happens in between? Does the tide progress northwards everywhere as observed along the coast of western Europe, or does it behave differently in the east and west Atlantic with a region of low amplitude in between? This important question was not seriously tackled until the 19th century, and then only after much speculation, – see Chapter 9 and later Chapters.

The 19th century polymath, William Whewell of Trinity College, Cambridge, became obsessed with this very question, and while unable to solve it, continually stressed the need for worldwide coastal measurements and berated the astronomers for having abandoned this fundamental science for more celestial pursuits.<sup>(1)</sup> Similar accusations have been leveled at oceanographers in the 20th century, and for analogous reasons. The fact is, that when a scientific problem does not yield to currently available tools, scientists tend to turn to other subjects which, if no easier, at least have the attraction of novelty. The tides have been an ‘old subject’ for a long time.

From time to time a new idea has arisen to cast fresh light on the subject. While such events have spurred some to follow up the new ideas and their implications, they have also had a negative effect by appearing superficially to solve all the outstanding problems. Newton’s gravitational theory of tides (Chapter 5) explained so many previously misunderstood phenomena that British scientists in the 18th century saw little point in pursuing the subject further. The initiative passed to the French Académie Royale, culminating in the work of Laplace (Chapter 7) who took over where Newton stopped, at the dynamic response of the ocean to Newton’s correctly defined force field. Similarly, William Thomson’s idea of harmonic analysis, which stemmed from Laplace’s theory, was so successful (after development by George Darwin) in providing for accurate predictions at any site where the tide had been measured for a long enough period of time, that one of the mainstays of research, namely from the commercial and naval producers of tide-tables, was transferred to routine computing activity. It was left to altruistic bodies like the British Association for the Advancement of Science (who had in fact promoted the



development of the ‘harmonic method’ of prediction) to encourage further research into the *spatial* and *global* properties of tides and their currents.

When the new subject of *geophysics* began to develop towards the end of the 19th century, its investigators soon found that many of its problems involved the large-scale properties of the tides of ocean, atmosphere and the elastic earth.<sup>(2)</sup> Increasingly, these problems were seen to be global in character and solutions to them few or lacking. Of central and lasting interest was the total rate of dissipation of energy by the oceanic tides, and its implications for the apparent acceleration of the moon’s longitude and (later) the rate of increase in the length of the day. Progress was now made, not by the tide-table experts, the naval hydrographers and academic mathematicians, who had hitherto kept tidal lore to themselves, but by geophysicists and by certain oceanographers inclined to mathematical physics.

The central problem of the 20th century, essential to the understanding of global energy dissipation as well as to a host of other geophysical problems, has remained that of determining the behavior of tides in the deep ocean. This is essentially the same problem as had bothered Whewell in the previous century, but at an altogether more refined level of precision than Whewell ever imagined. Persistently this problem defied formal mathematical analysis and measurement technique, and final or nearly final solutions have had to await modern technology.

Research on oceanic tides in the modern sense, then, has spanned at least four centuries. It has involved scientists from disciplines ranging from astronomy and satellite geodesy to ocean instrument technology, and activity from mathematical analysis and computing to sea-going expeditions. Relatively few people have been involved at any one time, but the subject seems to have had a peculiar fascination for ‘lone workers’. As one worker has ‘shot his bolt’ or retired, another has taken up the challenge from a different viewpoint or discipline. Schools of expertise in different countries have led certain aspects of the field at different times, chiefly in Britain, France, Germany, Russia and USA. Only in the last decades of the 20th century, with the enormously increased power of computation and space geodesy, have the major goals been achieved. This book is therefore unusual in being concerned with the history of a science which has both a recognisable beginning and an ‘end’, or at least a temporary plateau, taking the reader from the earliest writings to the most recent research. The present is particularly timely for such a history.

Previous historians of tidal science have concentrated on the minutiae of limited, rather distant epochs of research, chiefly in the 16th to 18th centuries. The papers of the late E.J. Aiton in *Annals of Science*<sup>(4)</sup> (see Chapters 4 and 5) are particularly well studied in depth and have made useful guides to their epochs. Other historians from whose writings I have learnt much are Margaret Deacon on the debates about tides led by the early Royal Society of London (Chapters 4 and 6) and David Kushner on Sir George Darwin and the controversies over lunar acceleration among 19th century astronomers (Chapter 10).

Some textbooks have included a Chapter or part-Chapter on early history. Most comprehensive of these is in Book I, Part I of Rollin Harris's 5-volume monograph,<sup>(3)</sup> written very nearly a century ago and now long out-of-print. Harris's history has been widely acknowledged as the most thorough guide to early ideas on tides from antiquity to near the end of the 19th century. However, it was written as an introduction to Harris's own painstaking construction of a world map of tidal times. Ironically, he too probably thought the science was reaching the end of an era, but the dynamical theory on which his constructions were based was unsound, and was soon criticised on rigorous standards by George Darwin, though later accepted by Henri Poincaré as a reasonable compromise (Chapter 9). As one who has played a modest part in the international activity in tidal science during most of the last third of the 20th century, I feel confident that the year 1996 has a stronger claim to have reached the end of a long epoch of research, with the achievement of centuries-old objectives.

The above statements should not, however, be taken to imply that research on the tides is likely to come to a halt. Some details of the physical mechanism of energy dissipation are still unclear, and at the time of writing, *acoustic tomography* and *satellite altimetry* are revealing unexpected features of *internal waves* of tidal periodicity.<sup>(7)</sup> Tidal motion at diurnal and semidiurnal periods has also been observed for the first time in the rotation of the earth, (see Chapter 15). The small-scale dynamics of tidal motion on continental shelves, not to mention coastal engineering problems, will no doubt continue to demand attention.<sup>(5,6)</sup> This history is more concerned with the *global* aspects of tidal science, which do indeed seem to have reached a state of near-culmination.

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