Time-Series Analysis and Cyclostratigraphy

Examining stratigraphic records of environmental cycles

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Chapter 1

Introduction

1.1 Cyclostratigraphic data

Increasingly, quantitative records of environmental change covering intervals of between half a day to millions of years are being sought by palaeoceanographers, environmental scientists, palaeoclimatologists, sedimentologists and palaeontologists. The ‘media’ from which these records are obtained range from sediments and sedimentary rocks to living organisms and fossils showing growth bands (especially trees, corals and molluscs), ice cores and cave calcite. This book is concerned with explaining the quantitative methods that can be employed to derive useful information from these records. Much of the discussion is concerned with explaining the problems and limitations of the procedures and with exploring some of the difficulties with interpretation. Most frequently environmental records are obtained from sedimentary sections making up the stratigraphic record and, using a rather broad definition, all the ‘media’ described above are ‘stratigraphic’. The nature of cycles in environmental signals and in stratigraphic records are explored later. However, for now cycles can be thought of as essentially periodic, or regular, oscillations in some variable. The study of stratigraphic records of environmental cycles has been called cyclostratigraphy (Fischer *et al.*, 1990).

By regarding stratigraphic records of environmental change as signals, it is clear that the methods and interpretations reached during analysis must allow for the imperfections inherent in all recording procedures. In cyclostratigraphic data the environmental signal, which is ‘encoded’ during sedimentation, is often corrupted to some extent by interruptions caused by processes that are not part of the normal depositional system. Such processes, for sediments, include non-deposition, erosion, seafloor dissolution or event-bed deposition and they make the later recognition of the normal environmental signal more difficult. Yet the interruptions convey information themselves, and in some cases they result from the extremes of the normal environmental variations. For example, Dunbar *et al.* (1994), in their study of corals, pointed out that growth band thickness was related to sea surface temperature. However, episodes of unusually high sea surface temperatures cause growth band generation to stop completely for several years, thus interrupting the proxy temperature record.

As well as interruptions, the recording processes can introduce distortions that need to be taken into account. For example, accumulation rate variations and diagenesis frequently modify the final shapes of cyclostratigraphic data sets. In a similar manner to the interruptions, the distorting processes often depend on the nature of the environment. Hence, cyclostratigraphic data contain information about normal environmental variability, abnormal environmental variations and the processes that produce the records themselves. In other words, the stratigraphic information that is observed can be regarded as the product of many superimposed environmental and sedimentological, or metabolic, processes.

The methods described in this book are primarily concerned with detecting and describing regular cyclic environmental processes. Hence, the data are treated as though they consist of regular cycles plus irregular oscillations. The irregular components result from both normal and abnormal environmental conditions as well as the effects of sedimentation and diagenesis (or equivalent processes in skeletal growth, etc.). As explained below, there are sound geological reasons for using mathematics to search for regular cycles. Regular components of cyclostratigraphic data are often studied more easily than the irregular components. If methods could be developed to distinguish the various types and origins of the irregular components, much of value could be uncovered. Quantitative studies of the interruption and distortion processes will undoubtedly be useful for understanding ancient environmental and diagenetic mechanisms, but such investigations are relatively rare (e.g. Sadler, 1981; Ricken, 1986; Ricken and Eder, 1991; Ricken, 1993).

The idea that stratigraphic data consist of regular components – the signal, plus irregular components or noise – is based on a linear view of the processes involved. In reality non-linear processes abound in environmental systems (e.g. Le Treut and Ghil, 1983; Imbrie *et al.*, 1993a; Smith, 1994). In non-linear systems, the output does not vary in direct proportion to the input. There are many aspects of cyclostratigraphic data that cannot be easily investigated using the standard linear methods of analysis described in this book. From the perspective of non-linear dynamical systems, part of the irregular components can be considered to be as much a part of the environmental signal as the regular components (Stewart, 1990; Kantz and Schreiber, 1997). Some non-linear methods are described very briefly within Chapter 4 and some non-linear issues in signal distortion are considered in Chapter 5. Despite the view that non-linear approaches might explain more of the data than the linear methods, the latter are currently best understood mathematically and are the most frequently used.

A good demonstration of the success of the standard linear approach to cyclostratigraphic data concerns the time scale developed using late Neogene deep-sea sediments.

Hilgen (Hilgen and Langereis, 1989; Hilgen, 1991) and Shackleton *et al.* (1990) independently derived orbital cycle chronologies based on matching sedimentary cycles and oxygen isotope curves to the calculated history of insolation changes (Section 6.9). The results were at odds with the widely accepted radiometric ages that had been obtained using potassium-argon dating. Subsequently, improved radiometric dating and studies of sea-floor spreading rates confirmed the validity and utility of the so-called astronomical time scale approach (Wilson, 1993; Shackleton *et al.*, 1995a, 1999a). Consequently, a recent geochronometric scale for part of the Neogene has been based directly on orbital-cycle chronology rather than the traditional data derived from radiometrically calibrated rates of sea-floor spreading (Berggren *et al.*, 1995). In this case the standard, linear methods of time-series analysis have yielded results of fundamental importance to many other areas of the Earth Sciences.

1.2 Past studies of cyclic sediments

Examination of cyclic sediments intensified in the 1960s as modern depositional environments were better understood and conceptual models became more sophisticated. Historically sedimentologists were looking for explanations for cyclic stratigraphic sequences that did not simply require **random** (i.e. unconnected, meaning uncorrelated or 'independent') events. Perhaps if the underlying controls could be uncovered, more could be learnt about the environment of deposition. Cycle-generating processes were described as autocyclic if they originated inside the basin of deposition. Alternatively, allocyclic processes originated outside the basin (Beerbower, 1964). Coal measure cyclothems were a particular target for investigation since they had a wide range of interbedded lithologies, and resulted from a range of suspected autocyclic and allocyclic mechanisms. The definition of a cyclothem (Wanless and Weller, 1932) soon became contentious once the variety of lithological successions and inferred origins was appreciated (Duff *et al.*, 1967; Riegel, 1991). Simpler cyclic sections involving two alternating lithologies, often described as rhythmic, were often mentioned in reviews of cyclic sedimentation but, aside from sequences that were inferred to contain varves, they were little studied (e.g. Anderson and Koopmans, 1963; Schwarzacher, 1964).

In many early investigations, pattern recognition was centred on the analysis of the observed sequences of lithologies. This made sedimentological sense as the predictions of qualitative models could be compared with the observations. Of course no reasonably long stratigraphic section actually corresponded exactly to the pattern predicted by the models. Unfortunately, since it was easy to imagine situations where the expected or 'ideal cycle' (Pearn, 1964) was not encoded in the sedimentary rocks, it proved impossible to falsify the models. Duff and Walton (1962) argued that sedimentary cycles can be recognized as having a particular order of lithologies that frequently occur in a particular sequence. They called the most frequently occurring sequence a modal cycle. However, their definition of cyclicity was criticized as being so vague that it could include sequences that are indistinguishable from the result of random fluctuations – which would also exhibit modal cycles (Schwarzacher, 1975).

Markov chain analysis was used to test sequences for the presence of a **Markov property** or the dependence of successive observations (lithologies or numbers) on previous observations. This captured some of the concept of a 'pattern' in a cyclic sequence since it implied a certain preferred order to the observed lithologies. However, stratigraphic data as structured for Markov analysis apparently always have preferred lithological transitions, and thus never correspond to a truly independent random sequence (Schwarzacher, 1975). This is because environmental systems include a degree of 'inertia'. Even instantaneous changes in the 'boundary conditions' (e.g. sea level, rainfall, etc.) do not cause instantaneous changes in the environment. For example, it can be as much as a few years before the release, over a few weeks or months, of a large volume of sulphate aerosols into the atmosphere by a volcanic eruption causes a drop in global atmospheric temperatures (Stuiver *et al.*, 1995). Therefore, the ubiquitous detection of a Markov property in cyclic sections merely indicated that there is a degree of 'smoothness' in the transitions between successive observations. Since virtually all physical systems exhibit inertia, the detection of a Markov property proved to be of little use for characterizing sedimentary cyclicity. Nevertheless, Markov analysis is useful when, for example, the particular order of lithologies helps in the description of sedimentological processes (e.g. Wilkinson *et al.*, 1997).

Schwarzacher's (1975) book represented a landmark in the examination of sedimentary cyclicity. Instead of just examining the transitions between lithologies at bed boundaries in Markov chain analysis, he reasoned that the thickness of successive beds provided information of fundamental importance in the assessment of sedimentary cycles. This meant that the stratigraphic data should be collected as **time series**. Time series include any sequence of measurements or observations collected in a particular order. Usually the measurements are made at constant intervals of some scale of measurement such as cumulative rock thickness, geographic distance, time, growth band number, etc. Some authors have referred to data collected relative to a depth or thickness scale as 'depth series', but time series is actually the correct mathematical term for historical reasons (Schwarzacher, 1975; Priestley, 1981; Schwarzacher, 1993). The variable that is recorded need not be restricted to lithology of course, and this significantly widens the scope of potential investigations of sedimentary cyclicity. The quantitative techniques used for the study of such data are described as methods of **time-series analysis**.

Schwarzacher argued that to be meaningful the term 'sedimentary cycles' must refer to oscillations having perfectly or nearly perfectly constant **wavelength**. Only if the wavelength can be measured in time does one refer to the cycle's **period**. However, whether a time or thickness scale is being used, oscillations of constant wavelength are described by mathematicians as **periodic**, and those of nearly constant wavelength as **quasi-periodic**. Periodic or quasi-periodic cyclostratigraphic sections have repetitions of a particular observation (such as a particular rock type) at essentially constant stratigraphic intervals. To many mathematicians stratigraphic sections that do not exhibit this type of regularity should not be termed cyclic at all (Schwarzacher, 1975). Yet sedimentary cyclicity is a perfectly useful field term for sections with interbedded rock types where event deposition is not involved (Einsele *et al.*, 1991). The mathematician's approach would require mathematical investigations before the term sedimentary cyclicity could be applied. I argue here that 'cyclicity' and 'sedimentary cycles' are liable to be used by sedimentologists, however vaguely, for the foreseeable future. Instead I have used the terms **regular cycles** and **regular cyclicity** to denote oscillations in stratigraphic records that can be shown, using time-series analysis, to have near-constant wavelengths (i.e. rock thickness) or periods. The issue of nomenclature of cyclic sediments is currently being assessed by the Working Group on Cyclostratigraphy appointed by the International Subcommission on Stratigraphic Classification (Hilgen *et al.*, 2001).

In the late 1970s and 1980s two revolutions in sedimentological thinking profoundly influenced the study of cyclic sediments. Firstly, following extensive deep-sea drilling, improvements in the measurement of remnant magnetization and in radiometric dating, it became clear that the orbital or Milankovitch Theory of climatic change (Section 6.9) should be taken seriously as an explanation for the Pleistocene climate changes (Hays *et al.*, 1976; Imbrie and Imbrie, 1979; Imbrie *et al.*, 1984). This promoted intense interest in evidence for pre-Pleistocene orbital-climatic cycles (Sections 6.9.3 and 6.9.4). In the absence of accurate time scales, the most convincing demonstrations of ancient orbital-climatic cycles came from the time-series analysis methods advocated by Schwarzacher (1975) and used extensively by the palaeoceanographers examining Pleistocene sediments (Weedon, 1993). Pioneering time-series analyses of cyclic sequences (Preston and Henderson, 1964; Schwarzacher, 1964; Carrs and Neidell, 1966; Dunn, 1974) seem to have lacked the long data sets and time control needed to make sufficiently convincing cases for Milankovitch cyclicity to the wider community. Concurrent with the increased interest in Milankovitch cyclicity, the attempt to detect regular climatic and weather cycles possessing much shorter periods met with increasing success (Burroughs, 1992).

Meanwhile Vail *et al.* (1977, 1991) changed the way sedimentologists interpreted lithostratigraphic successions. By employing sequence stratigraphic methods, sedimentary sections can be divided into genetically related stratigraphic units. Stacks of sequences were explained in terms of changing base level, especially relative sea level. However, because a large variety of processes were believed to be ultimately responsible for sequence generation, a classification scheme based on the duration of sea level cycles was adopted (e.g. Vail *et al.*, 1991). This ranged from 'first order' sequences lasting more than 50 million years to 'sixth order' sequences formed in 10,000 to 30,000 years. Although the duration or 'order' of sequences was believed to provide a clue to their likely origin, regularity was not implied by their use of the term 'sea level cycle'. Nevertheless, the higher order sea level cycles were explained in terms of Milankovitch cycles, especially acting through glacio-eustasy (Goldhammer *et al.*, 1990; Naish and Kamp, 1997). The resulting sequences were termed parasequences if relatively complete, or simple sequences if bounded by stratigraphic gaps.

By the 1990s a more descriptive approach to cyclic sequences was being advocated (Einsele *et al.*, 1991). Studies of the links between ancient climatic changes and cyclic sedimentation increased, utilizing several Pleistocene models that include and exclude ice sheets and glacio-eustasy. The description and study of cyclic sedimentary sections became known as cyclostratigraphy (Fischer *et al.*, 1990). As discussed earlier, it is likely that future studies of irregular processes, particularly utilizing non-linear dynamic systems methods (Sections 4.7 and 4.8), will be fruitful. Studies of Milankovitch cyclicity are currently particularly concerned with the development of time scales based on counts of Milankovitch cycles for pre-Cenozoic sequences and matches with orbital 'templates' for the younger part of the Cenozoic (Section 6.9.3, Shackleton *et al.*, 1999a). However, a great deal of work on cyclostratigraphic signals is now being undertaken by palaeoceanographers, climatologists and environmental scientists concerned with climatic oscillations that have periods shorter than the orbital cycles (i.e. <20,000 years) as discussed in Chapter 6.

1.3 Time-series analysis – an introduction

As shown in Fig. 1.1 a simple oscillation can be described in terms of its **amplitude** and wavelength. Additionally, the position within the oscillation or its **phase angle** or **phase** (ranging from 0 to 360° or from 0 to 2π radians) can be measured from some sort of origin along the time or cumulative thickness/depth axis. Geologically the position of the origin is determined arbitrarily by wherever the data collection started. However, mathematically this type of simple oscillation is usually described using a sinusoid; if it starts at the mid-point of an oscillation it is a sine wave and if it starts at a maximum it is a cosine wave (Fig. 1.1). Sine and cosine waves are convenient for describing oscillations mathematically. To produce a sinusoid that starts at a phase angle of 45° it is only necessary to add together a sine and cosine wave of the same wavelength and the same amplitude (Fig. 1.2a). Any other starting angle can be generated by controlling the relative amplitudes of the sine and cosine waves used (Fig. 1.2b). Observational time series rarely have oscillations of such a simple shape, but more complicated shapes, such as cuspate waves with narrow troughs and long peaks, can be represented by adding sine and cosine waves with particular wavelengths (Section 5.2.4).

Observational time series are of course usually composed of many different wavelength oscillations. According to **Fourier's theorem**, any time series, no matter what shape it is provided it has some oscillations and no infinite values, can be recreated by adding together regular sine and cosine waves having the correct wavelengths and amplitudes. Sine and cosine waves form a set of so-called **orthogonal functions**. Orthogonal functions are simply groups of waves that can be added together to describe any time series, but none of the individual component waves can be constructed from combinations of other waves in the group. There are other sets of orthogonal functions, which can be used in place of sines and cosines (e.g. **Walsh functions**, Section 3.4.6,

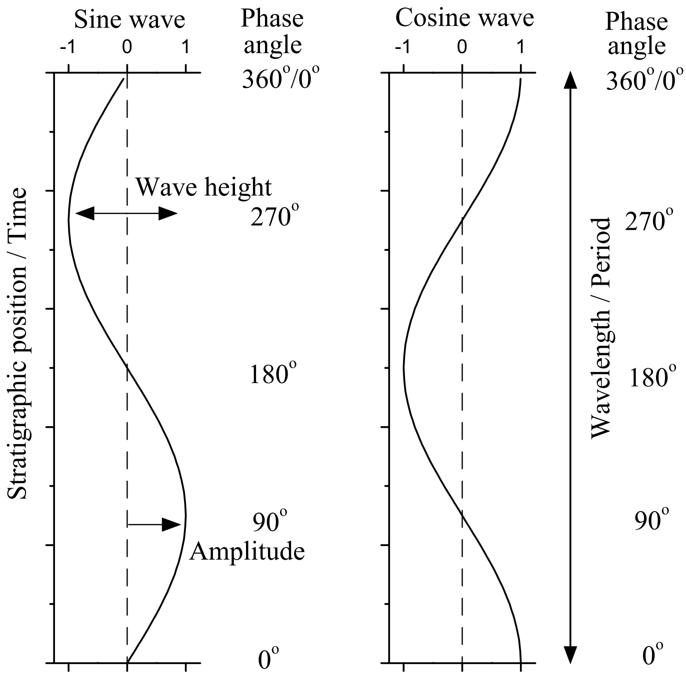

Fig. 1.1 The principal parameters needed to describe sinusoidal waves. Amplitude is measured as the maximum deviation from the zero line. Period (time interval) or wavelength (thickness interval) is defined as the interval from peak to peak or trough to trough, etc. The phase angle indicates the relative position within the complete cycle and is measured from the base of the data set. The phase angle, or more simply phase, ranges from 0 to 360 degrees (or 0 to 2π radians). Sine and cosine waves of the same wavelength are identical except that the phase differs by 90°. The sine and cosine waves shown have a wavelength or period equal to the length of the whole time series.

Many cyclostratigraphic records from cores are labelled using depth or age from the top of the data downwards. However, it is standard lithostratigraphic practice, when studying sections exposed on land, to denote stratigraphic position by height or time increasing from the base upwards. Throughout this book the measurements or observations from the youngest strata are always located at the top of the time series plots (i.e. the top measurements relate to minimum depth or maximum height).

Beauchamp, 1984). However, most stratigraphic time series consist of approximately sinusoidal oscillations, so usually sine and cosine waves are the most naturally employed. Examination of time series using sines and cosines is often referred to as **Fourier analysis**.

Clearly it would be convenient to be able to take a time series and quickly assess how many regular component oscillations are present. This is most readily achieved by using **power-spectral analysis** (Chapter 3). Put simply the **power spectrum** shows the relative amplitudes (strictly squared amplitudes) and wavelengths or periods of all the regular components in the time series. By convention the horizontal axis of a power spectrum is plotted as **frequency** (frequency = 1/period) with highest frequencies (shortest oscillations) appearing on the right. Zero frequency refers to oscillations that

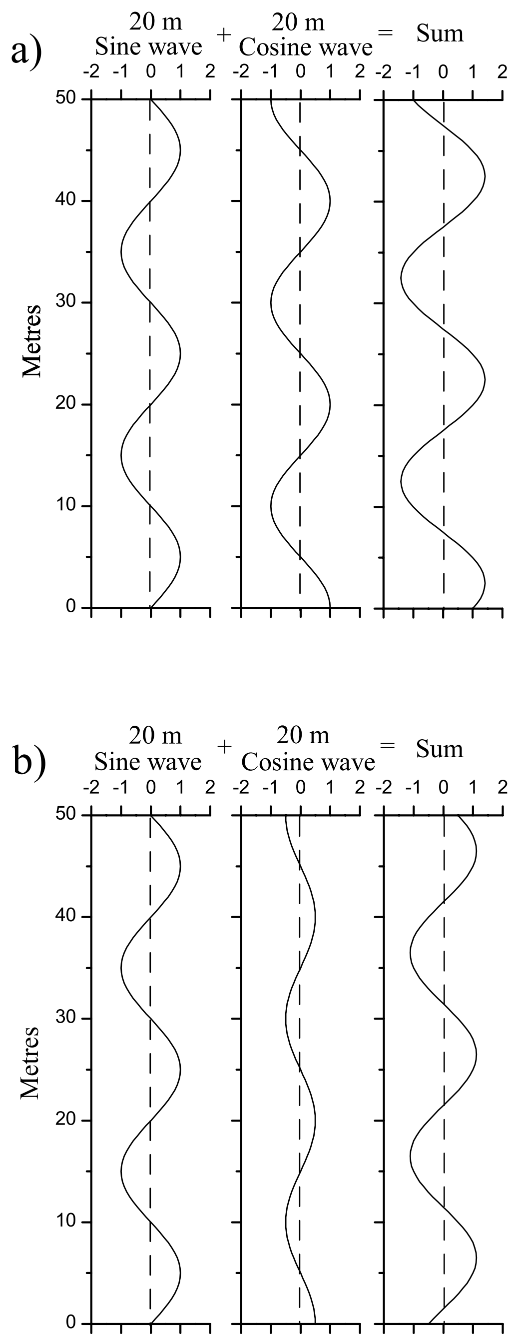

Fig. 1.2 (a) When sine and cosine waves with the same wavelength and equal amplitude are added together, the resulting sinusoid has a phase which is intermediate between that of the components (i.e. it differs by 45°). (b) Adding a sine wave with an amplitude of one unit to a cosine wave with an amplitude of half a unit produces a sinusoid with a phase of 67.5°. This means that any sinusoid can be considered to represent the sum of one sine and one cosine wave having the same wavelength and the correct relative amplitudes.

have wavelengths or periods exceeding the length of the whole data set. If the data are collected as a function of time, frequency is measured in ‘numbers of cycles per time unit’ which is usually shortened to ‘cycles per time unit’ (e.g. cycles per thousand years). If a thickness or depth scale is used then instead of frequency some authors refer to the **wave number** (i.e. wave number = 1/wavelength in thickness). However, for clarity in cyclostratigraphic studies it is usual for one to speak of frequency even though the units do not include a time element (e.g. cycles per metre).

Spectral analysis requires amplitude measurements determined as positive or negative deviations from some zero line. However, although the zero line is sometimes defined using the average or mean of the data, usually a more complicated definition is involved as discussed later (Section 3.2). Consequently, it is often confusing, when inspecting a time series plot, if the zero line is indicated, so this has only been illustrated for the time series in Figs. 1.1 and 1.2. Geologists often prefer to plot stratigraphic position or time running vertically up the page. However, frequently palaeoceanographers and environmental scientists plot data relative to time so that the time axis runs horizontally, with younger data on the left of the page. To simplify the layout of the figures I have plotted all the time series the same way so that either stratigraphic position or time runs up the page, hence the youngest data are found at the top.

The vertical axis of the spectrum is usually plotted as squared amplitude and by analogy with physics it is described as ‘power’ (energy per time interval), hence the name power spectrum. Since amplitude refers to deviation from the zero line, squared amplitude can be thought of as squared deviation and so sometimes one speaks of the **variance spectrum** (variance equals squared standard deviation). Occasionally amplitude, rather than squared amplitude, is plotted against frequency, so creating an **amplitude spectrum** (also known as a **magnitude spectrum**). If small spectral peaks need to be studied together with large peaks then the log of power is plotted against frequency. In electronic signal processing, for comparing power values the **decibel scale** is used (i.e. $10 \times \log_{10}$power) so that a power value of 0.01 equals −20 dB. (Note that for comparing voltages, analogous to amplitude in time-series analysis, decibels are calculated as $20 \times \log_{10}$voltage/amplitude.)

It is sometimes useful to be able to think of spectral analysis using physical analogies. Thus the rainbow effect produced by a glass prism acting on a beam of white light is a classical example of a spectrum. The brightness of different parts of the rainbow corresponds to the power and the various colours the frequency. The ear and brain similarly apparently analyse sound (fluctuating air pressure) as though it is a time series made up of components with different amplitude/power (loudness) and frequency (pitch, Taylor, 1965, 1976). Thus different parts of the brain are activated by different frequencies, though the size of the response depends on musical training/skill and the type of sound (e.g. Pantev *et al.*, 1998).

Figure 1.3 illustrates an example where a 10 m sine wave has been added to a 2.78 m sine wave. The resulting time series, shown as ‘Sum’ on the right, would have looked different if the relative phase of the two components and/or the relative amplitudes

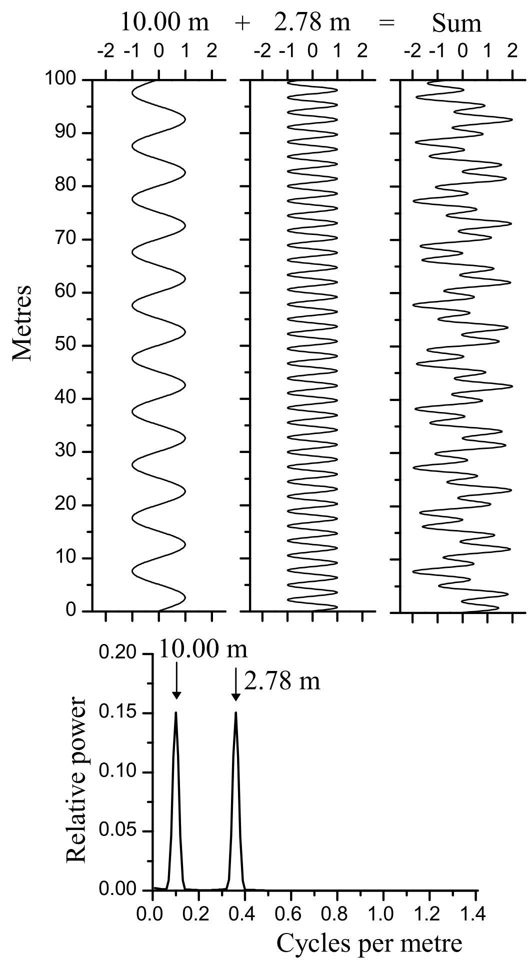

Fig. 1.3 Adding sinusoids with different wavelengths produces a time series with multiple frequency components. Power spectra are used to: (a) identify which frequency components are present (frequency = 1/wavelength); and (b) determine their relative amplitudes. In this case two sine waves of equal amplitude, but different wavelengths, have been added to produce the time series labelled "Sum". The corresponding power spectrum has peaks that occur at frequencies corresponding to the component wavelengths. The peaks are equal in height because the components have the same amplitude. Note that it is impossible to tell from the spectrum whether the components are sine or cosine waves – in other words the spectrum is independent of the phase of the components.

differed. The power spectrum in Fig. 1.3 shows that the time series consists of just two frequency components. When power spectra are generated all the phase information is discarded. As a result, changing the relative positions or phases of the 2.78 m oscillations relative to the 10.0 m oscillations, and hence the shape of the time series, would not influence the shape of the spectrum. The heights of the two spectral peaks in Fig. 1.3 are identical because the amplitudes of the component oscillations are identical. The larger the spectral peak, the greater the amplitude of the corresponding wavelength of oscillation and the greater its 'importance' in controlling the overall shape of the time series. The frequency of the spectral peaks can be read from the horizontal axis and indicates, of course, that oscillations with wavelengths of 10.00 m and 2.78 m are present in the time series.

Sinusoids with varying amplitude are said to exhibit **amplitude modulation** or **AM**. There are two types of amplitude modulation. In **heterodyne AM** the addition of two sinusoids with similar wavelengths creates a new single oscillation (Olsen, 1977). The new oscillation has a frequency that is the average of the frequencies of

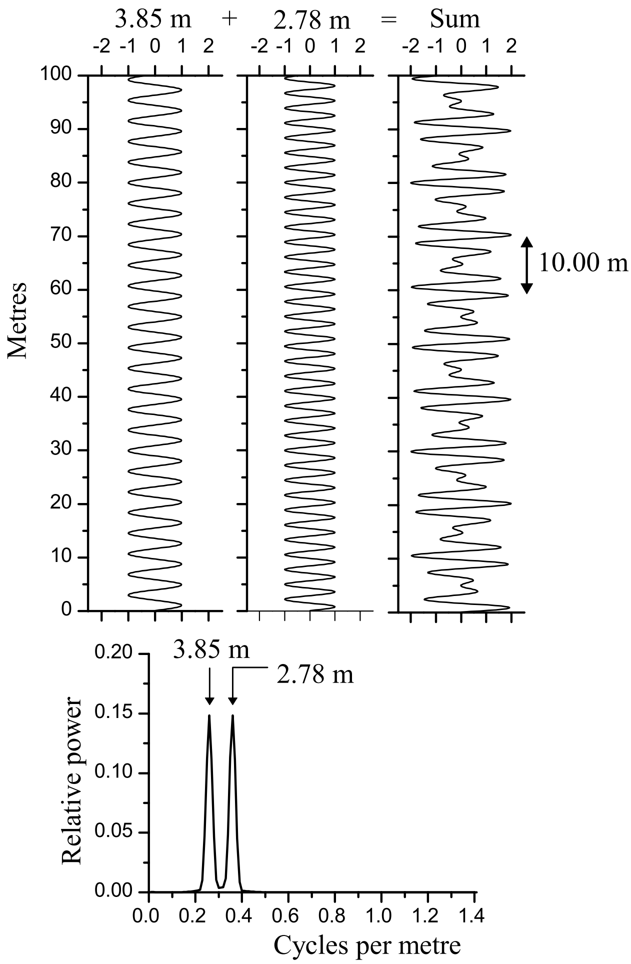

Fig. 1.4 Heterodyne amplitude modulation occurs when two sinusoids with similar frequencies are added together. Here sine waves with wavelengths of 3.85 m and 2.78 m are summed. The result is an oscillation with a wavelength of 3.29 m and a beat wavelength of 10 m. Note that the spectrum does not have a spectral peak corresponding to the beat frequency (0.1 cycles per metre) because there are no 10 m oscillations, just 10 m variations in amplitude. If the spectrum of this type of time series had a lower frequency resolution than illustrated here, the two spectral peaks would appear as one broad peak (Section 3.3.2).

the two added sinusoids. The variation in amplitude of the new oscillation is called the **beat** and this has a frequency that equals the difference in the frequencies of the added sinusoids (Taylor, 1965). For example, in Fig. 1.4 the addition of oscillations with frequencies of 1/3.85 m and 1/2.78 m generates an oscillation with a frequency of 1/3.22 m (i.e. = (1/2.78 m + 1/3.85 m)/2) and a beat with a wavelength of 10.00 m (i.e. 1/10.00 m = 1/2.78 m – 1/3.85 m). Note that the spectrum reveals the presence of the two original cycles, but no peak at the frequency of the beat frequency. This is because there are no oscillations with a 10 m wavelength in the record, just 10 m variations in amplitude.

Imposed AM occurs when a large period/wavelength signal (the beat) is used to vary the amplitude of another oscillation (the primary cycle, Olsen, 1977). In such cases, a peak at the primary cycle frequency dominates the spectrum, but small **combination tone** peaks are generated on either side due to the imposed beat frequency (discussed further in Section 5.2.4). The frequencies of the combination tone peaks

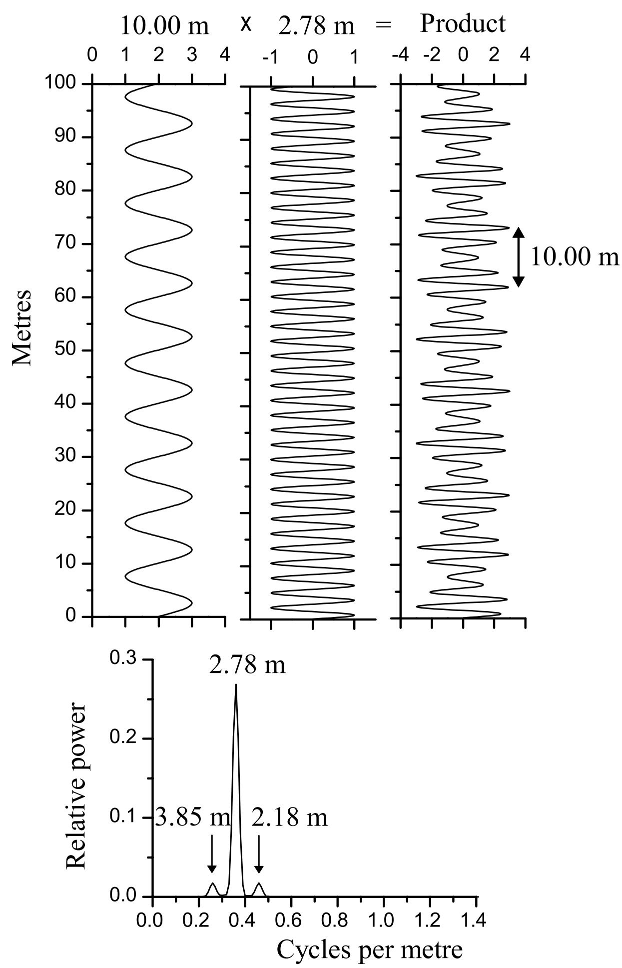

Fig. 1.5 Imposed amplitude modulation (AM) occurs when a separate signal is used to vary the amplitude of a regular sinusoid. In this case the amplitude of a 2.78 m cycle is made to vary between 1.0 and 1.5 by multiplication by a cycle with a wavelength of 10 m. The resulting time series looks very similar to that in Fig. 1.4, but the spectrum bears the hallmark of imposed AM with combination tone peaks on either side of the primary frequency. As for heterodyne AM there are no 10 m oscillations, so the spectrum does not contain a spectral peak at 0.1 cycles per metre.

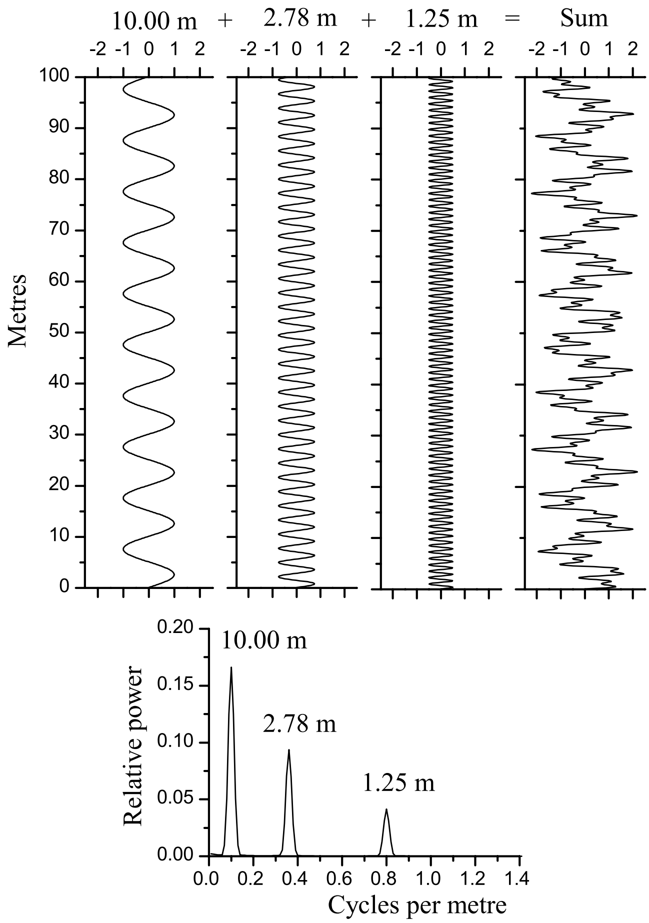

Fig. 1.6 When three sine waves of different wavelengths are added together, the resulting time series can look quite complicated. Nevertheless, as power spectra are unaffected by phase, the three component wavelengths and their relative amplitudes are readily determined.

correspond to the primary oscillation frequency minus the beat frequency and to the primary oscillation frequency plus the beat frequency (Taylor, 1965). In the example shown in Fig. 1.5, 10 m imposed AM of the 2.78 m oscillations generates sidebands at frequencies of 1/3.85 m (i.e. = 1/2.78 m – 1/10.0 m) and 1/2.18 m (i.e. = 1/2.78 m + 1/10.0 m). As for heterodyne AM, the spectrum of an imposed AM signal does not have a peak at the beat frequency because there are no 10 m oscillations present (just 10 m variations in amplitude). A crucial point is that power spectra reveal average power. Thus, except in rare clear-cut cases (e.g. Figs. 1.4 and 1.5), power spectra cannot be used to infer how the amplitude of an oscillation varies along the length of a time series.

Time series can look exceedingly complicated when only a few regular cycles are added together. In Fig. 1.6 the addition of three oscillations with different amplitudes results in a moderately complicated looking data set. The spectrum contains three

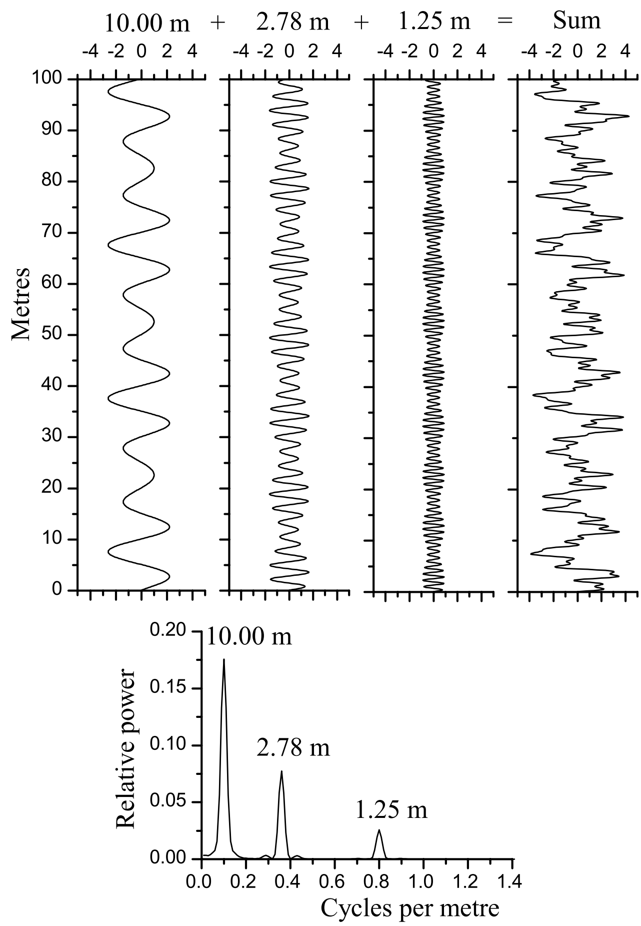

Fig. 1.7 Adding three sine waves with different wavelengths and varying amplitudes produces a time series that looks extremely complicated. It is unlikely that mere visual inspection of the summed time series illustrated would allow one to recognize that just three frequency components are present, or to determine the wavelengths involved. However, as the peak height depends only on (squared) *average* amplitude, the spectrum is very similar to the spectrum in Fig. 1.6.

spectral peaks, the relative peak heights indicating the average relative squared amplitudes of the component cycles. If the same three regular cycles have varying amplitudes along the series, the result looks considerably more complex (Fig. 1.7), but the corresponding spectrum is dominated by the same three peaks. Most people would be hard-pressed to recognize the presence of just three regular components in the time series of Fig. 1.7 merely by visual inspection. It would also be very difficult to establish the wavelengths involved. Therefore, except with very simple or especially characteristic data sets, it is unwise to claim the detection of regular cyclicity in a time series by visual inspection alone or by using simple analysis of the wavelength distribution of the oscillations (e.g. histograms of bed thickness).

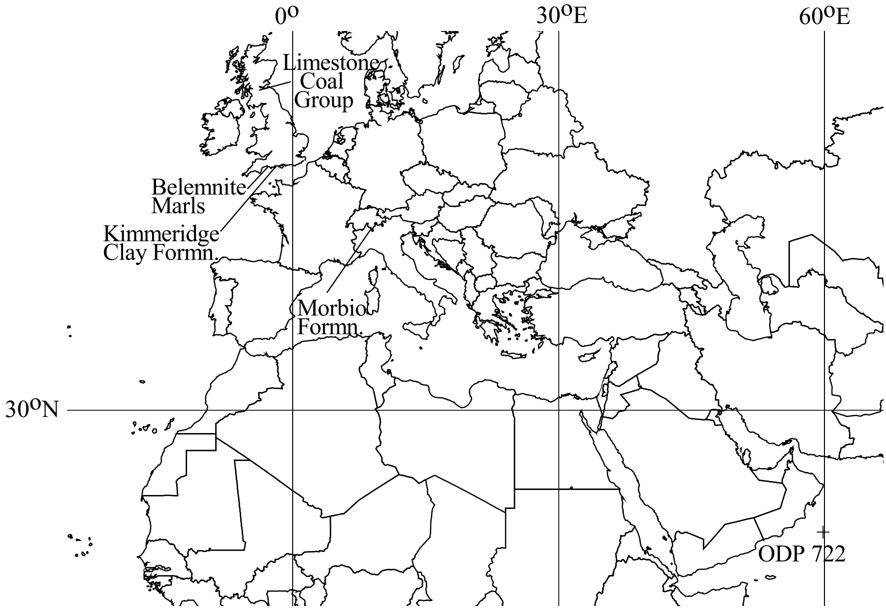

Fig. 1.8 Location map for the cyclostratigraphic records illustrated in Chapters 1 to 5. Formn. denotes formation.

Observational time series usually consist of the addition of tens or hundreds of regular sine and cosine components. This means that every possible regular frequency component has a non-zero amplitude. Consequently, spectral analysis of stratigraphic time series is used to look for spectral peaks that emerge from a background of spectral values.

This is an appropriate point to introduce the data sets used to illustrate this book. They are listed, according to the age of the strata from which they were obtained, in Table 1.1. Figures 1.8 and 6.1 show where these cyclostratigraphic records were obtained. Many of these records are based on oxygen-isotope records (see Faure, 1986 for an introduction to the determination and uses of δ^{18}O and an explanation of the delta notation). One of the records has been selected to help illustrate the various time series methods described in the book. The information comes from the Early Jurassic hemipelagic formation called the Belemnite Marls, which is approximately 190 million years old and exposed on the coast of Dorset, England (Fig. 1.8, Table 1.1, Weedon and Jenkyns, 1990, 1999). In the field much of this unit consists of interbedded light-grey marls, dark-grey marls and brown-black laminated shales. These lithologies form decimetre-scale bedding couplets that are grouped into metre-scale bundles (Fig. 1.9). In several intervals at the base of the formation, the bedding is barely visible. Towards the top the couplets and bundles are noticeably thinner (Fig. 1.9).

As for all stratigraphic records obtained from the Jurassic system, the absolute dating uncertainties create difficulties when assessing the duration of processes lasting less

Table 1.1. *Cyclostratigraphic time series used to illustrate the book.*

Period (time interval)	Borehole site or formation, location	Number of points	Sample interval	Variable	Time series description	Chapter
Recent (1840–1994)	Maiana Atoll, W. Pacific Ocean	928	0.166 years	$\delta^{18}O_{CORAL}$	Urban *et al.*, 2000	6
Recent (1936–1982)	Galapagos Islands, E. Pacific Ocean	183	0.5 years	Ba/Ca_{CORAL}	Shen *et al.*, 1992	6
Recent (1967–1711)	GISP2, Greenland	2047	0.125 years	$\delta^{18}O_{ICE}$	Section 6.2	6
Recent (1987–818)	GISP2, Greenland	1170	1.0 year	$\delta^{18}O_{ICE}$	Section 6.2	6
Recent (0.0–10.0 ka BP)	GISP2, Greenland	503	20 years	$\delta^{18}O_{ICE}$	Section 6.2	6
Late Pleistocene–Recent (0–80.8 ka BP)	GISP2, Greenland	405	200 years	$\delta^{18}O_{ICE}$	Section 6.2	6
Late Pleistocene–Recent (0–330 ka BP)	ODP 980, North Atlantic	928	Variable	$\delta^{18}O_{PF}$	McManus *et al.*, 1999	6
Late Pleistocene–Recent (0–370 ka BP)	ODP 722, N.W. Indian Ocean	147	Variable	Ba/Al, Ti/Al	Weedon and Shimmield, 1991	5
Late Miocene–Recent (0–6 Ma BP)	ODP 677 and ODP 846, E. Pacific Ocean	2001	3000 years	$\delta^{18}O_{BF}$	Shackleton *et al.*, 1990, 1995b	6
Early Miocene	Marine Molasse, Auribeau, S. France	154	Bundle thickness	Tidal bundle thickness	Archer, 1996	6
Late Jurassic	Kimmeridge Clay Formation, S. England	360	0.05 m	Magnetic susceptibility	Morgans-Bell *et al.*, 2001; Weedon *et al.*, 1999	5
Late Jurassic	Kimmeridge Clay Formation, S. England	117	0.1524 m	Photoelectric factor	Gallois, 2000; Morgans-Bell *et al.*, 2001	5
Late Jurassic	Kimmeridge Clay Formation, S. England	90	0.2 m	%TOC	Morgans-Bell *et al.*, 2001; Weedon *et al.*, 1999	5
Early Jurassic	Morbio Formation, S. Switzerland	1024	0.01 m	Rock type code	Weedon, 1989	3
Early Jurassic	Belemnite Marls, S. England	798	0.03 m	$\%CaCO_3$, %TOC	Weedon and Jenkyns, 1999	1–6
Late Carboniferous	Abbott Formation, S. Illinois, USA	208	Bed thickness	Tidal bed thickness	Archer, 1996	6
Early Carboniferous	Limestone Coal Group, C. Scotland	5236	0.01 m	Rock type code	Weedon and Read, 1995	5

Abbreviations: $\delta^{18}O_{PF}$, $\delta^{18}O$ in planktonic foraminifera; $\delta^{18}O_{BF}$, $\delta^{18}O$ in benthic foraminifera; ODP, Ocean Drilling Program; %TOC = percentage total organic carbon; ka, thousands of years; Ma, millions of years; BP, before present. Figures 1.8 and 6.1 provide location maps for these records.

Fig. 1.9 Photograph of the Belemnite Marls as exposed below Stonebarrow near Charmouth in Dorset, England. The whole formation is close to 24 m thick. The majority of the formation consists of beds of light-grey marl alternating with dark-grey marl and brown-black laminated shales. The alternations form couplets that are grouped into bundles. Both couplets and bundles become much thinner towards the top of the formation.

than about 10 million years (Gradstein *et al.*, 1994). Nevertheless, it is clear from time-series analysis that the decimetre-scale bedding couplets relate to the 20,000 orbital-precession cycle (Weedon and Jenkyns, 1999). Samples were collected throughout the Belemnite Marls at fixed 3-cm intervals and analysed for weight percent calcium carbonate and total organic carbon (or TOC, Fig. 1.10). Weedon and Jenkyns (1999) give instructions for obtaining a listing of these data. The results show that the light-grey marls have higher carbonate contents and less organic carbon than the dark-grey marls and laminated shales. Additionally, in the visually almost homogeneous interval towards the base (bed 110), there are the same types of compositional variations found elsewhere.

In Fig. 1.11 part of the time series of weight percent $CaCO_3$ from the Belemnite Marls is illustrated with the corresponding power spectrum. Although a large number of frequency components appear, the spectrum of the Belemnite Marls carbonate contents is clearly dominated by three main spectral peaks which emerge from the spectral background. These large spectral peaks are described as relating to regular sedimentary cycles, even though mathematically all the spectral values relate to regular components. Thus it is the geological interpretation of the spectrum that leads to the data being regarded as composed of a three-component regular cyclic ‘signal’ plus the irregular ‘noise’ accounting for the spectral background. The spectral peak

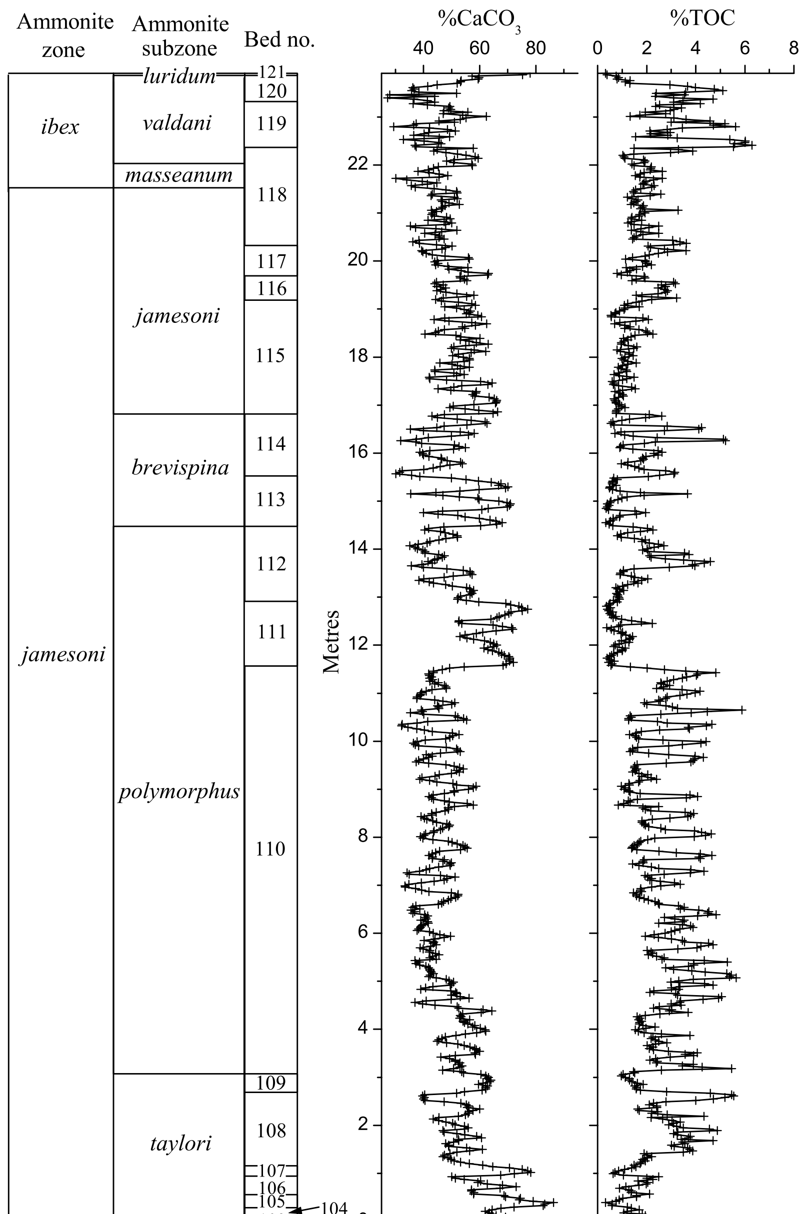

Fig. 1.10 Calcium carbonate ($CaCO_3$) and total organic carbon (TOC) time series from the whole of the Belemnite Marls. The top and base of the formation are marked by thin early diagenetic limestones that are associated with stratigraphic gaps (Weedon and Jenkyns, 1999). The formation covers the first two ammonite zones of the Pliensbachian Stage (Lower Jurassic). Bed numbers follow Lang *et al.* (1928). Note the persistence of couplet-like carbonate and organic carbon oscillations throughout bed 110, even though in the field the bedding is barely visible (Fig. 1.9).

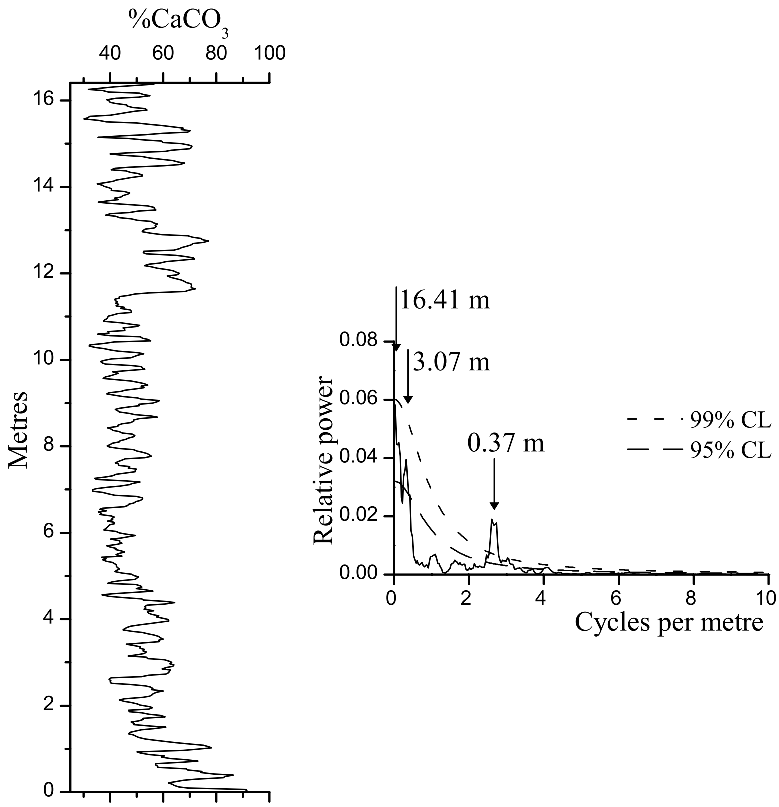

Fig. 1.11 A time series of calcium carbonate contents from the lower two-thirds of the Belemnite Marls. In real data there is some variation in the measured variable at every scale or wavelength. In general, lower frequency variations have a larger average amplitude than higher frequency variations. This produces a sloping continuum in the spectrum that rises towards the lowest frequency end. In the Belemnite Marls there are three main scales of variation in carbonate contents (Weedon and Jenkyns, 1999) that account for the labelled spectral peaks emerging from the sloping background continuum. The dashed lines indicate confidence levels (CL) and are used to distinguish peaks from the spectral background (Section 3.5).

For completeness the following describes the methods used to generate this spectrum (see Chapter 3 for explanations): (a) linear detrending of 548 data points, (b) zero-padding to 1024 points, (c) multi-taper spectral estimation using six data tapers. The 95% and 99% confidence levels (e.g. 99% CL) are used to identify those spectral peaks that cannot be attributed to the background noise.

labelled with a wavelength of 0.37 m relates to the bedding couplets and the peak labelled 3.07 m relates to the metre-scale bundles of couplets. In this case the noise in the time series can be regarded as the product of irregular oscillations in the environment plus measurement errors, which together partly explain the continuous spectral background.

Details of the methods needed to generate power spectra are given in Chapter 3. Many other methods of time-series analysis are available and some of the standard procedures are described in Chapter 4. The three most important methods are used to: (a) isolate regular cycles from the time series (filtering); (b) examine variations in cycle amplitude (amplitude demodulation); and (c) study the phase and amplitude relationships of pairs of variables obtained simultaneously from the same samples or time intervals (i.e. cross-spectral analysis). Chapter 5 is concerned with distortions of these environmental signals as recorded stratigraphically, as well as practical considerations in conducting time-series analyses of real data. Chapter 6 discusses the many environmental origins for the regular cycles that have been observed in stratigraphic records (from tidal to Milankovitch cycles).

In the context of cyclostratigraphic data sets, spectral analysis is usually the first procedure used, because it allows the detection of regular cyclicity and determination of wavelengths and average amplitudes. However, before the difficulties of generating and interpreting power spectra can be considered, it is essential that one is aware of the many issues concerning the construction of time series (Chapter 2).

1.4 Chapter overview

- Time-series analysis provides procedures for examining quantitative records of environmental variability. It was widely adopted in cyclostratigraphic studies following the vindication of the orbital-climatic theory (Milankovitch theory) in the early 1980s.
- Spectral analysis allows the detection of multiple regular cycles in a time series. Each regular component is characterized in terms of its frequency (= 1/period or 1/wavelength) and *average* power (= squared average amplitude).
- Regular amplitude modulation in a time series does *not* generate a spectral peak at the modulation frequency.