This book is an introductory textbook on the physical processes occurring in the Earth's radiation belts. The presentation is at the senior or first year graduate level, and it is appropriate for students who intend to work in some aspect of magnetospheric physics.

The treatment is quantitative and provides the mathematical basis for original work in this subject. The equations describing the motion of energetic ions and electrons in the geomagnetic field are derived from basic principles, and concepts such as magnetic field representations, guiding center motion, adiabatic invariance, and particle distribution functions are presented in a detailed and accessible manner. Relevant experimental techniques are reviewed and a summary is given of the intensity and energy spectra of the particle populations in the Earth's radiation belts.

Problem sets are included as well as appendices of tables, graphs and frequently used formulas.

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Introduction to Geomagnetically Trapped Radiation

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Preface

This book is the outgrowth of teaching an introductory course in geomagnetically trapped radiation to graduate and undergraduate students at Stanford University. Because this material is well-plowed ground, the topics are presented in what is hoped to be a logical sequence, and the historical chronology is not followed. The emphasis is on the basic physical processes treated in a tutorial manner, rather than on the descriptive aspects of the magnetosphere. Thus the principles developed here should be useful regardless of further evolution in our understanding of trapped radiation. Items such as frequently used formulas and graphs of trapped particle parameters are collected in appendices for ready access and for future reference.

The rationalized MKS system of electromagnetic units is used throughout the book. Although much of the published research in this field is in Gaussian units, the author believes that the MKS system is preferable and will eventually prevail. Some exceptions to SI units are allowed, such as particle fluxes, which are expressed in $cm^{-2}\,s^{-1}$ rather than $m^{-2}\,s^{-1}$ because of convention and because detector apertures are more readily visualized in cm^2.

Inasmuch as this work is written to be a textbook rather than a review of current research, references to original works are limited. Undergraduates and first-year graduate students are more interested in understanding the material than in learning the source. The references listed are primarily those articles and books which will be useful to the student who wishes to dig deeper into any of the topics covered. The numbers listed after each of the references indicate the chapter for which that reference is particularly applicable. I apologize to my many colleagues whose work is utilized here but is not explicitly acknowledged.

Special thanks go to many of my friends who helped with various stages

of the book and who provided many stimulating and informative discussions. My colleagues John Cladis, Ted Northrop, David Stern, Tim Bell and Michael Schulz were particularly helpful and patient, both in reviewing portions of the text and in working the problem sets. I must also thank the students who ferreted out inconsistencies in the lecture notes and who provided fresh perspectives on the subject.

I am grateful to the Lockheed Missiles and Space Company for authorizing my teaching efforts at Stanford University and to Stanford for providing the opportunity.

The endless number of drafts with redundant and repetitive alterations were patiently processed by my secretary, Mrs Glenda Roberts, whose competence and good nature were essential to convert rough notes into legible text.

The support and encouragement of my wife during the years involved in this book is acknowledged and appreciated.

Symbols

Symbols in bold face denote vector quantities. When only the magnitude of the quantity is used, the same symbol is used in standard type.

$\mathbf{A}$ Magnetic vector potential
A Area
$A(t)$ Asymmetric part of the time variation of the Earth's magnetic field
$\mathbf{B}$ Magnetic field
$\mathbf{B}_\mathrm{d}$ Dipole magnetic moment
B_0 Mean value of geomagnetic field on the equator at the Earth's surface
B_eq Value of B in the equatorial plane not at the Earth's surface
B_m Value of B at the mirroring point of a particle
B_r, B_θ, B_ϕ Components of geomagnetic field in spherical coordinates
B_max Maximum value of B above the atmosphere for a given drift shell
$\mathbf{b}$ Magnetic field of a wave or perturbation on the Earth's magnetic field
c Velocity of light in vacuum
D, D_{xx}, $D_{\alpha\alpha}$, D_{LL} Diffusion coefficients for particle transport
D_{LL}^{M} Radial diffusion coefficient produced by magnetic field perturbations
D_{LL}^{E} Radial diffusion coefficient produced by electric field perturbations
$\mathcal{D}_\mathrm{v}$ Differential operator defined by equation (7.46)
$\mathbf{E}$, $\mathbf{E}_\perp$, $\mathbf{E}_\parallel$ Electric field intensity, perpendicular and parallel components
E_θ, E_r, E_ϕ Components of electric field in spherical polar coordinates
$E_{\phi n}$ nth Fourier coefficient in expansion of the Earth's electric field fluctuations
E Particle kinetic energy
$\hat{\mathbf{e}}_1$, $\hat{\mathbf{e}}_2$, $\hat{\mathbf{e}}_3$; $\hat{\mathbf{e}}_x$, $\hat{\mathbf{e}}_y$, $\hat{\mathbf{e}}_z$ and $\hat{\mathbf{e}}_\mathrm{r}$, $\hat{\mathbf{e}}_\theta$, $\hat{\mathbf{e}}_\phi$ Unit orthogonal vectors
ε, $\bar{\varepsilon}$ Kinetic plus potential energy, average kinetic plus potential energy
ε', $\bar{\varepsilon}'$ Constant of integration equal to $\frac{1}{2}mv_\parallel^2 + \mu B$, average value
$\mathbf{F}$, F_x, F_y, F_z Force, orthogonal components of force

$f(\mathbf{x}, t)$ Number of particles at coordinate $\mathbf{x}$ per unit $d\mathbf{x}$ at time t.
$F(\mathbf{p}, \mathbf{q}, t)$, $F(\mu, J, \phi, t)$ Distribution function of particles in phase space or adiabatic invariant space at time t.
g_n^m, h_n^m Harmonic expansion coefficients describing the core geomagnetic field
$\bar{g}_n^m$, $\bar{h}_n^m$ Harmonic expansion coefficients describing the magnetic field produced by magnetopause current systems
g_m Spatial eigenfunctions of the diffusion equation
G Geometric factor of a detector
$\mathbf{i}$ Electric current density
I Integral invariant function
$\mathbf{I}$ Electric current
$j(E, \alpha)$ Differential, directional particle flux
$j(E)$ Omnidirectional particle flux
$j(\alpha, E > E_0)$ Integral particle flux above an energy E_0
J Action integral, adiabatic invariant
$J_1 = \mu$, $J_2 = J$, $J_3 = \Phi$ First, second and third adiabatic invariants or action integrals
$\mathcal{J} = \dfrac{\partial(x_1, x_2, x_3)}{\partial(y_1, y_2, y_3)}$ Jacobian relating two sets of variables or coordinates
L Magnetic shell parameter – approximate distance from center of Earth to equatorial crossing of the field line in Earth radii)
m Mass of a particle
m_e, m_p Mass of electron, mass of proton
m_0 Rest mass of a particle
μ First adiabatic invariant, magnetic moment of a charged particle
$\mathcal{M}_E$ Magnetic moment of a dipole field
N Number density of scattering centers, number of particles
$N_1(x)$, $N_2(x)$ Geometric function of geomagnetic field defined by equations (6.28) and (6.31)
$\mathbf{n}$ Unit vector normal to surface or parallel to the radius of curvature of line
$\mathbf{p}$, $\mathbf{p}_\perp$, $\mathbf{p}_\parallel$ Particle total momentum, perpendicular and parallel components
$P_A(\Omega)$ Power spectrum of $A(t)$ evaluated at angular frequency Ω
$\mathbf{P}$ Canonical momentum of a charged particle
P_n^m Associated Legendre functions, Schmidt normalization
P_{nm} Associated Legendre functions, usual normalization
q Electric charge
q_i Position coordinate conjugate to momentum coordinate p_i
Q Source intensity for a particle distribution
r, θ, ϕ Spherical polar coordinates
$\mathbf{r}$ Position vector

R Position vector of the guiding center of a charged particle
$R(r), \Theta(\theta), \Phi(\phi)$ Separation functions for solution of Laplace's equation in spherical coordinates
R_c Radius of curvature of a magnetic field line
R_E Radius of the Earth
R_0 Distance from the center of the Earth to the equatorial crossing point of a magnetic field line
s Distance measured along a geomagnetic field line
s_m, s'_m Conjugate mirroring points on a magnetic field line
Δs Increment of distance along a geomagnetic field line
S Area
$S(t)$ Symmetric part of the time variation of the Earth's magnetic field
S_b Helical distance traveled by a particle during a complete bounce
t Time
$T = \dfrac{E}{m_0c^2}$ Kinetic energy in rest mass units
$\mathbf{v}, \mathbf{v}_\perp, \mathbf{v}_\parallel$ Particle velocity, perpendicular and parallel components
v_x, v_y, v_z Orthogonal components of velocity
$\mathbf{v}_f$ Apparent velocity of magnetic field line motion
$\mathbf{v}_g$ Group velocity of a wave
$\mathbf{v}_{ph}$ Phase velocity of a wave
$V(x)$ Potential energy
$\mathbf{V}$ Velocity of moving reference frame
$\mathbf{V}_E$ Guiding center drift velocity of a particle caused by an electric field perpendicular to the magnetic field
$\mathbf{V}_G$ Gradient drift velocity
$\mathbf{V}_c$ Curvature drift velocity
$\mathbf{V}_\perp$ Guiding center drift velocity perpendicular to the magnetic field
W Relativistic total energy of a particle (kinetic energy + rest energy)
x, y, z Rectangular coordinate axes
x $\cos\alpha_{eq}$ = cosine of equatorial pitch angle of a trapped particle
$\mathbf{x}$ Generalized vector coordinate
z Atomic number of an atom
α Pitch angle of particle in a magnetic field $= \tan^{-1}(v_\perp/v_\parallel)$
α_{eq} Pitch angle of a particle measured at the equatorial plane
α_{LC} Bounce loss cone pitch angle
α^d_{LC} Drift loss cone pitch angle
β v/c
γ Relativistic factor $(1 - \beta^2)^{-1/2}$, wave growth rate
ε_0 Permitivity of free space

$\Gamma(\alpha_{eq})$	Latitude-dependent factor of radial diffusion coefficient
η	Scattering angle, longitude
η_{min}	Minimum Coulomb scattering angle determined by shielding of nuclear charge
λ	Geomagnetic latitude
λ_m	Latitude of a particle mirroring point
λ_n, λ_m	Eigenvalues of the spatial part of the diffusion equation
ν	Frequency of global magnetic or electric field fluctuations
ν_{drift}	Longitudinal drift frequency of trapped particles
μ	First adiabatic invariant = $p_\perp^2/2m_0B$ (magnetic moment)
μ_0	Permeability of free space
$\boldsymbol{\rho}$	Gyroradius
ρ	Charge density, mass density
$\sigma(\eta)$	Cross-section for Coulomb scattering of an electron through angle η
τ_b	Bounce period of a trapped particle
τ_d	Longitudinal drift period of a trapped particle
τ_g	Gyration period of a trapped particle
ϕ	Phase angle between $\mathbf{v}_\perp$ and the magnetic field vector of a wave
Φ	Magnetic flux, third adiabatic invariant
Ψ	Probability function, scalar potential
ψ	Azimuthal angle
Ω	Solid angle
Ω, Ω_e	Angular gyration frequency, electron gyration frequency
Ω_D	Longitudinal angular drift frequency of a trapped particle
ω	Wave angular frequency
ω_d	Doppler shifted wave frequency
ω_p	Plasma frequency
ξ, ξ', ξ'', ζ	Dummy variables of integration
θ	Colatitude, polar angle
χ	$\cos\theta$ where θ is colatitude

Useful constants

e	Elementary charge	1.602×10^{-19} coulombs
m_e	Rest mass of electron	9.11×10^{-31} kg
m_p	Rest mass of proton	1.673×10^{-27} kg
c	Speed of light in vacuum	2.998×10^{8} m s^{-1}
μ_0	Permeability of free space	$4\pi \times 10^{-7}$ newton s^{2} coulomb^{-2}
ε_0	Permittivity of free space	8.85×10^{-12} coulomb^{2} newton^{-1} meter^{-2}

Geophysical quantities

R_E	Mean radius of Earth	6.37×10^3 km
B_0	Mean magnetic field on equator at Earth's surface	3.12×10^{-5} T
$\mathscr{M}_E$	Magnetic dipole moment of the Earth	8.07×10^{22} A m^2
$\dfrac{\mu_0 \mathscr{M}_E}{4\pi}$	Magnetic dipole parameter of the Earth	8.07×10^{15} T m^3

Energy equivalents

$m_p c^2$	938.3 MeV
$m_e c^2$	0.511 MeV
1 eV	1.6×10^{-19} joules