

SMP AS/A2 Mathematics

Statistics 2

for AQA



The School Mathematics Project

SMP AS/A2 Mathematics writing team John Ling, Paul Scruton,
Susan Shilton, Heather West

SMP design and administration Melanie Bull, Pam Keetch, Nicky Lake,
Cathy Syred, Ann White

The authors thank Sue Glover for the technical advice she gave when this AS/A2 project began and for her detailed editorial contribution to this book. The authors are also very grateful to those teachers who advised on the book at the planning stage and commented in detail on draft chapters.

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo
Cambridge University Press
The Edinburgh Building, Cambridge CB2 2RU, UK
www.cambridge.org
Information on this title: www.cambridge.org/9780521605311

© The School Mathematics Project 2005

First published 2005

Printed in the United Kingdom at the University Press, Cambridge

A catalogue record for this publication is available from the British Library

ISBN-13 978-0-521-60531-1 paperback

ISBN-10 0-521-60531-8 paperback

Typesetting and technical illustrations by The School Mathematics Project

The authors and publisher are grateful to the Assessment and Qualifications Alliance for permission to reproduce questions from past examination papers and the tables on pages 110–115. Individual questions are marked AQA.

NOTICE TO TEACHERS

It is illegal to reproduce any part of this work in material form (including photocopying and electronic storage) except under the following circumstances:

- (i) where you are abiding by a licence granted to your school or institution by the Copyright Licensing Agency;
- (ii) where no such licence exists, or where you wish to exceed the terms of a licence, and you have gained the written permission of Cambridge University Press;
- (iii) where you are allowed to reproduce without permission under the provisions of Chapter 3 of the Copyright, Designs and Patents Act 1988, which covers, for example, the reproduction of short passages within certain types of educational anthology and reproduction for the purposes of setting examination questions.

Using this book

Each chapter begins with a **summary** of what the student is expected to learn. The chapter then has sections lettered A, B, C, ... (see the contents overleaf). In most cases a section consists of development material, worked examples and an exercise.

The **development material** interweaves explanation with questions that involve the student in making sense of ideas and techniques. Development questions are labelled according to their section letter (A1, A2, ..., B1, B2, ...) and answers to them are provided.

D Some development questions are particularly suitable for discussion – either by the whole class or by smaller groups – because they have the potential to bring out a key issue or clarify a technique. Such **discussion questions** are marked with a bar, as here.

K **Key points** established in the development material are marked with a bar as here, so the student may readily refer to them during later work or revision. Each chapter's key points are also gathered together in a panel after the last lettered section.

The **worked examples** have been chosen to clarify ideas and techniques, and as models for students to follow in setting out their own work. Guidance for the student is in italic.

The **exercise** at the end of each lettered section is designed to consolidate the skills and understanding acquired earlier in the section. Unlike those in the development material, questions in the exercise are denoted by a number only.

Starred questions are more demanding.

After the lettered sections and the key points panel there may be a set of **mixed questions**, combining ideas from several sections in the chapter; these may also involve topics from earlier chapters.

Every chapter ends with a selection of **questions for self-assessment** ('Test yourself').

Included in the mixed questions and 'Test yourself' are **past AQA exam questions**, to give the student an idea of the style and standard that may be expected, and to build confidence.

Contents

1 Discrete random variables 6

- A Probability distribution 6**
specifying by a simple function, for finite number of possible outcomes
 - B Mean or expected value 9**
of discrete random variable defined by table
 - C Expectation of a function of a discrete random variable 12**
linear and simple non-linear functions of discrete random variable defined by table, sum of two functions, multiple of a function
 - D Variance and standard deviation 16**
discrete random variable defined by table
 - E Variance of a function of a discrete random variable 18**
linear and non-linear functions of discrete random variable defined by table
- Mixed questions 22

2 Poisson distribution 25

- A The Poisson model 25**
conditions for use, finding probability of a given number of events occurring using formula and values of $e^{-\lambda}$ from a calculator, shape of distribution as λ increases, Poisson distribution as limiting case of binomial distribution
 - B The Poisson table 29**
cumulative probabilities
 - C Mean, variance and standard deviation 31**
 - D Independent Poisson distributions 32**
parameter for $X + Y$ and for $X_1 + X_2 + X_3 + \dots$, checking conditions for use of Poisson model
- Mixed questions 35

3 Continuous random variables 38

- A Probability density function 38**
conditions for a function to be a probability density function, probability that an observation lies in a given interval
 - B Distribution function 43**
relationship to probability density function
 - C Median, quartiles and percentiles 45**
 - D Mean or expected value 49**
 - E Expectation of a function of a continuous random variable 52**
linear and simple non-linear function
 - F Variance and standard deviation 54**
including variance of a linear or simple non-linear function of a continuous random variable
 - G Rectangular distribution 57**
probability that an observation lies in a given interval; proof of mean, variance, standard deviation
- Mixed questions 60

4 Estimation 62

- A Normal distribution: review 62**
use of standardised variable
- B Unbiased estimators and estimates: review 64**
estimating μ and σ^2 , degrees of freedom
- C Sampling distribution: review 66**
standard error, from central limit theorem
sample mean for any population distribution treated as normally distributed
- D Confidence intervals: review 68**
for population mean of normal distribution
- E Confidence intervals: variance unknown 70**
 t -distribution

5 Hypothesis testing 76

- A Basic ideas of hypothesis testing 76**
 null hypothesis that a parameter takes a specified value, alternative hypothesis, level of significance
- B Mean of a normal distribution with known variance 78**
 test statistic for normally distributed continuous random variable, critical value, critical region, acceptance region, one-tailed and two-tailed tests
- C Mean of a normal distribution with unknown variance 85**
 t -distribution, t -statistic
- D Using a normal approximation 89**
 from central limit theorem sample mean for any population distribution treated as normally distributed
- E Type I and Type II errors 91**
 size of Type I error
 Mixed questions 92

6 Chi-squared tests 94

- A The chi-squared distribution 94**
 relationship to standard normal distribution, degrees of freedom, use of table
- B Testing goodness of fit 96**
 χ^2 -statistic, condition for this approximation to be valid
- C Contingency tables 101**
 χ^2 -test for association between two variables, degrees of freedom, Yates's correction, low values of the χ^2 -statistic
 Mixed questions 106

Tables 110

- Cumulative Poisson distribution function 110
- Normal distribution function 112
- Percentage points of the normal distribution 113
- Percentage points of the Student's t -distribution 114
- Percentage points of the χ^2 -distribution 115

Answers 116

Index 135