SMP AS/A2 Mathematics

# AQA
# Core 1

CAMBRIDGE
UNIVERSITY PRESS

The School Mathematics Project

**SMP AS/A2 Mathematics writing team** David Cassell, Spencer Instone, John Ling, Paul Scruton, Susan Shilton, Heather West

**SMP design and administration** Melanie Bull, Carol Cole, Pam Keetch, Nicky Lake, Jane Seaton, Cathy Syred, Ann White

The authors thank Sue Glover for the technical advice she gave when this AS/A2 project began and for her detailed editorial contribution to this book. The authors are also very grateful to those teachers who advised on the book at the planning stage and commented in detail on draft chapters.

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York NY 10011–4211, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa

http://www.cambridge.org/


First published 2004

Printed in the United Kingdom at the University Press, Cambridge

*Typeface* Minion *System* QuarkXPress®

*A catalogue record for this book is available from the British Library*

ISBN 0 521 60525 3 paperback

Typesetting and technical illustrations by The School Mathematics Project

The authors and publisher are grateful to the Assessment and Qualifications Alliance for permission to reproduce questions from past examination papers. Individual questions are marked AQA.

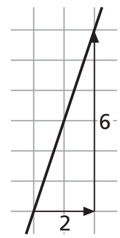
# Contents

# 1 Linear graphs and equations

In this chapter you will
- revise work on linear graphs and their equations
- change a given linear equation to a different form
- solve problems by using intersecting linear graphs
- revise and extend work on simultaneous equations

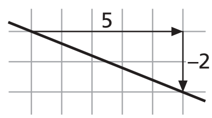
## A Linear graphs (answers p 153)

K On any part of a straight line graph, the gradient is the ratio $\dfrac{\text{increase in } y}{\text{increase in } x}$.




$\frac{6}{2} = 3$

A gradient can be a fraction. It is negative if the line goes down as it goes to the right (since the increase in $y$ is negative).



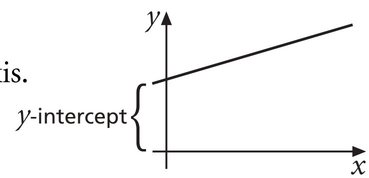

$\frac{-2}{5} = -\frac{2}{5}$ (or –0.4)

K Lines that have the same gradient are parallel.

**A1** Give the gradients of each of these straight lines. Draw sketches if you need to. Are any of the lines parallel?

**A** The line from (2, 1) to (4, 6)
**B** The line from (–4, –2) to (–1, –5)
**C** The line from (–1, 0) to (2, –1)
**D** The line from (–3, –3) to (1, 7)

K An equation of the form $y = mx + c$ is a straight line.
$m$ is the gradient and $c$ is the $y$-intercept.
The $y$-intercept is the value of $y$ where the line cuts the $y$-axis.




**A2** Draw these straight lines on squared paper.

**(a)** $y = 2x + 4$ **(b)** $s = -2t + 7$ **(c)** $y = 3x - 5$ **(d)** $y = 5$

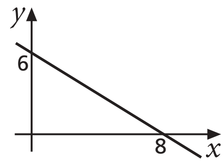
**A3** A graph has the equation $3x + 4y - 24 = 0$.

**(a)** Substitute 0 for $x$ in the equation.
Solve the equation you get.
What point does this tell you the graph goes through?

**(b)** Substitute 0 for $y$ in the equation.
Solve the equation you get.
What point does this tell you the graph goes through?

The results from question A3 tell you where the graph cuts the axes, enabling you to sketch the graph.

A sketch graph is not drawn on graph paper, but key points are labelled.




**A4** Use the above method to sketch a graph of each of these equations.

**(a)** $2x + 5y - 20 = 0$ **(b)** $7x + 4y - 28 = 0$ **(c)** $2x - 4y + 8 = 0$

**(d)** $4x - 3y - 12 = 0$ **(e)** $6x + 5y + 30 = 0$ **(f)** $x + 5y + 5 = 0$

**A5** Sketch graphs of these.

**(a)** $3x + 7y = 21$ **(b)** $5x + y = -5$ **(c)** $2x - 2y = -9$

**A6** A straight line goes through (0, 8) and (6, 0). Write its equation in the form $ax + by = c$, where $a$, $b$ and $c$ are constants.

You will often find it useful to change the equation of a linear graph into an equivalent form, as in the following example.

---

**Example 1**

Find the gradient of the straight line graph $4x + 5y - 3 = 0$.

**Solution**

*Make y the subject of the equation.* $5y = -4x + 3$

$y = -\frac{4}{5}x + \frac{3}{5}$

*Look at the coefficient of x.* So the gradient is $-\frac{4}{5}$.

---

**A7** Write each of these equations in the form $y = mx + c$.

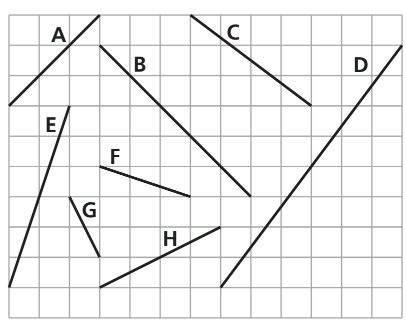
**(a)** $3x + y - 2 = 0$ **(b)** $x - 2y + 6 = 0$ **(c)** $3x + 5y - 2 = 0$

**A8** Sort these into three sets of parallel lines, giving the gradient for each set.

$3x - y - 4 = 0$ $y = -\frac{1}{2}x + 2$ $y = 3x + \frac{1}{2}$ $y = -3x + 4$

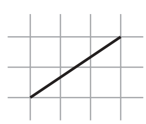
$y = 7 - \frac{1}{2}x$ $3x + y - 7 = 0$ $y = -\frac{1}{2} + 3x$ $y = -2 - 3x$

**A9** There are some pairs of **perpendicular** lines here (one line at right angles to the other).




**(a)** Record the gradients of each pair of perpendicular lines. Describe how the gradients in any pair are related.

**(b)** What is the gradient of any line perpendicular to a line that has gradient $\frac{2}{3}$?



**(c)** What is the gradient of any line perpendicular to a line with gradient $-4$?

**(d)** If two lines with gradients $p$ and $q$ are perpendicular, what is the value of $pq$?

K If two lines with gradients $m_1$ and $m_2$ are perpendicular,
$m_2 = -\frac{1}{m_1}$ or $m_1 m_2 = -1$.

**A10** Which of these lines are perpendicular to one another?

**A** The line from (1, 3) to (7, 4) **B** The line from (0, 0) to (5, 4)
**C** The line from (4, 2) to (8, –3) **D** The line from (0, 0) to (12, 2)

**A11** Which of these lines are perpendicular to one another?

**A** $y = 2x + 4$ **B** $5x + y = 2$ **C** $y = -\frac{1}{2}x - 3$ **D** $y = \frac{1}{5}x - 2$
**E** $y = \frac{3}{4}x$ **F** $y = 3 - 2x$ **G** $y = \frac{1}{2}x + 5$ **H** $3x + 4y - 1 = 0$

**A12** Can you think of any perpendicular lines for which the rule $m_1 m_2 = -1$ will not work?

---

**Example 2**

Identify any parallel or perpendicular lines among these.

**A** $y = 3x + 2$ **B** $3y = 2x$ **C** $3x - y - 5 = 0$ **D** $3x + 2y = 5$

**Solution**

*Write the equations in the form $y = mx + c$.*

A $y = 3x + 2$
B $y = \frac{2}{3}x$
C $y = 3x - 5$
D $y = -\frac{3}{2}x + \frac{5}{2}$

*Examine the x-coefficients (gradients).*

A and C are parallel (with gradient 3).
B and D are perpendicular because $\frac{2}{3} \times -\frac{3}{2} = -1$.

---

**Example 3**

Sketch the graph of $\frac{x}{2} + \frac{y}{3} = 5$.

**Solution**

*Multiply through by 6 to avoid the fractions.* $3x + 2y = 30$

*Find where the graph cuts the x-axis (where $y = 0$) by substituting 0 for y.*

$3x + 0 = 30$
$x = 10$

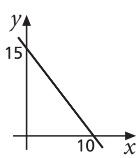
So the graph cuts the $x$-axis at (10, 0).

*Similarly for the y-intercept substitute 0 for x.*

$0 + 2y = 30$
$y = 15$

So the graph cuts the $y$-axis at (0, 15).

*Sketch the graph and label key points.*




---

## Exercise A (answers p 153)

**1** Which of the following graphs are parallel to the graph of $5x + 6y = 15$?

$y = \frac{5}{6}x - 15$ $\qquad \frac{x}{6} + \frac{y}{5} = 1$ $\qquad y = -\frac{5}{6}x + 4$ $\qquad y = 30 + \frac{5}{6}x$

**2** Draw a sketch of each of these graphs.

**(a)** $y = 2x - 7$ **(b)** $7x + 6y - 42 = 0$ **(c)** $2x - 3y = 12$

**3** There are three sets of parallel lines here.
Match them up and say what the gradient is for each set.

**A** $x + 7y = 1$ **B** $y = -\frac{2}{7}x + 3$ **C** $\frac{x}{7} + y = 3$ **D** $2y = 7x - 3$

**E** $2x + 7y = 1$ **F** $2y - 7x - 1 = 0$ **G** $x + 7y + 2 = 0$ **H** $\frac{x}{2} - \frac{y}{7} + 3 = 0$

**4** For each of these equations,

**(i)** rearrange it into the form $y = mx + c$

**(ii)** give the gradient

**(iii)** give the intercept on the $y$-axis

**(a)** $3x + y + 7 = 0$ **(b)** $x + 2y - 8 = 0$ **(c)** $4x + 5y + 1 = 0$

**(d)** $3x - 2y - 6 = 0$ **(e)** $-7x + 2y + 3 = 0$ **(f)** $-4x - 6y + 9 = 0$

**5** What is the gradient of a straight line of the form $ax + by + c = 0$,
where $a$, $b$ and $c$ are constants?

**6** Which of these lines are parallel to the line $y = 4x - 2$?

$y = 2 - 4x$ $\qquad y = 4x + 8$ $\qquad 4x + y + 6 = 0$ $\qquad -8x + 2y - 7 = 0$

**7** Which of these lines are parallel to the line $2x + 3y = 4$?

$3x - 2y + 1 = 0$ $\qquad y = \frac{2}{3}x + 6$ $\qquad 4x + 6y + 3 = 0$ $\qquad y = -\frac{2}{3}x$

**8** Which of these lines are perpendicular to the line $y = 3x + 1$?

$y = \frac{1}{3}x - 1$ $\qquad 6x - 2y + 3 = 0$ $\qquad y = -\frac{1}{3}x + 2$ $\qquad x + 3y = 1$

**9** Which of these lines are perpendicular to the line $5x - 3y + 2 = 0$?

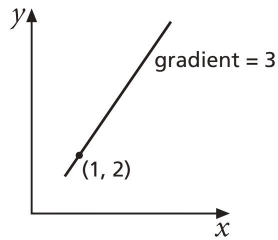
$-3x - 5y + 1 = 0$ $\qquad 3x - 5y - 10 = 0$ $\qquad y = 6 - \frac{3}{5}x$ $\qquad y = \frac{3}{5}x + 4$

**10** A triangle has vertices $A$ (–2, 1), $B$ (–1, –4) and $C$ (9, –2).
Find the gradients of its sides $AB$, $BC$ and $CA$.
What does this tell you about the triangle?

**11** A quadrilateral has vertices $A$ (1, 4), $B$ (4, 2), $C$ (9, 3) and $D$ (3, 7).
Find the gradients of its sides $AB$, $DC$, $AD$ and $BC$.
What special kind of quadrilateral is it?

## B Finding the equation of a linear graph (answers p 154)

Here a straight line with gradient 3 goes through the point (1, 2).




D

**B1** Give the coordinates of three other points on the line.

**B2** What do you think the line's equation is?

Although you can probably spot the equation mentally in a simple case like this, it is a good idea to have a written method that also works for less obvious equations.

One such method is as follows.

Consider the standard form of a straight line $y = mx + c$.

Substitute the values given above.

$$2 = 3 \times 1 + c$$
$$2 = 3 + c$$
$$c = -1$$

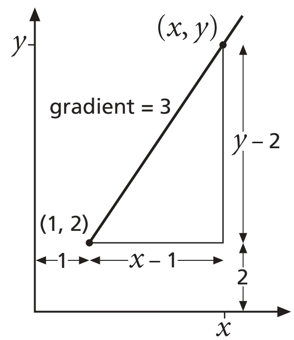
So the line's equation is $y = 3x - 1$.

This is a satisfactory method. However the following approach has the advantage of leading to a general formula that allows you to deal with some problems in a more direct way.

Here again is the line with gradient 3 going through (1, 2).

The point labelled $(x, y)$ is any point (a 'general point') on the line.

Whatever the value of $y$, the height of the triangle is $y - 2$.
Similarly the base of the triangle is $x - 1$.




*Since the gradient is* 3, $\dfrac{y-2}{x-1} = 3$

*Multiply both sides by* $(x - 1)$. 
$$y - 2 = 3(x - 1)$$
$$y - 2 = 3x - 3$$
$$y = 3x - 1$$

K

In general, a straight line with gradient $m$ passing through the point $(x_1, y_1)$ has the equation $\dfrac{y - y_1}{x - x_1} = m$ or $y - y_1 = m(x - x_1)$.

**B3** Use this formula to find the equation of each of these straight lines, in the form $y = mx + c$.

**(a)** Passing through (3, 2) with gradient 4

**(b)** Passing through (2, 4) with gradient –1

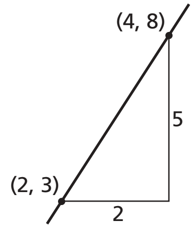
**(c)** Passing through (–2, –5) with gradient 3

**(d)** Passing through (–6, 1) with gradient $\frac{1}{4}$

**(e)** Passing through (–3, 2) with gradient $-\frac{1}{2}$

## Example 4

Find the equation of the line that passes through (2, 3) and (4, 8).




### Solution

*Find the height and width of the triangle made between the points.*

$$\text{gradient} = \tfrac{5}{2}$$

*Substitute this gradient and one of the points in the formula* $y - y_1 = m(x - x_1)$;
*here the point* (2, 3) *has been chosen.*

$$y - 3 = \tfrac{5}{2}(x - 2)$$
$$y - 3 = \tfrac{5}{2}x - 5$$
$$y = \tfrac{5}{2}x - 2$$

**Alternative solution** (without use of the formula above)

*Again, from the dimensions of the triangle,* gradient $= \tfrac{5}{2}$

So the required line has the form $y = \tfrac{5}{2}x + c$.

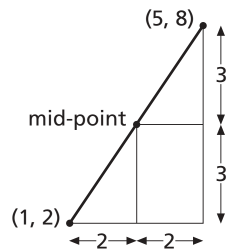
*Substitute the values of the point* (2, 3).

$$3 = \tfrac{5}{2} \times 2 + c$$
$$3 = 5 + c$$
$$c = -2$$

So the required line is $y = \tfrac{5}{2}\,x - 2$.

### Mid-points

If a straight line is drawn between the points (1, 2) and (5, 8) it is easy to see that their mid-point (the point halfway between them) is (3, 5).




Adding the two given $x$-coordinates and dividing by 2 gives the $x$-coordinate of the mid-point; similarly with the $y$-coordinates.

The mid-point of the points $(x_1, y_1)$ and $(x_2, y_2)$ is $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.

## Example 5

Find the equation of the perpendicular bisector of the line segment between $P$ (12, 9) and $Q$ (16, 15).

### Solution

*The perpendicular bisector is the line perpendicular to PQ going through the mid-point of PQ.*

The mid-point is $\left(\dfrac{12+16}{2}, \dfrac{9+15}{2}\right)$, which is (14, 12).

The gradient of $PQ$ is $\tfrac{6}{4}$, which is $\tfrac{3}{2}$.
So the perpendicular bisector has gradient $-\tfrac{2}{3}$.

*Use the gradient and point P in the formula* $y - y_1 = m(x - x_1)$.

$$y - 14 = -\tfrac{2}{3}(x - 12)$$
$$y - 14 = -\tfrac{2}{3}x + 8$$

*This gives the equation of the required line.* $y = -\tfrac{2}{3}x + 22$

**Exercise B** (answers p 154)

**1** Find the equation of each of these straight lines in the form $y = mx + c$.

**(a)** Passing through (7, 2) with gradient 3

**(b)** Passing through (3, 0) with gradient $\frac{1}{2}$

**(c)** Passing through (–1, –1) and parallel to $y = 2x + 5$

**(d)** Passing through (–2, 4) and parallel to $3x + 2y + 7 = 0$

**2** Find the equation of each of these straight lines in the form $y = mx + c$.

**(a)** Passing through (6, 4) and (12, 6) **(b)** Passing through (–2, 5) and (4, –1)

**3** Find the mid-point of each of these line segments.

**(a)** From (1, 6) to (7, 2) **(b)** From (–3, 1) to (5, 3) **(c)** From (–3, –5) to (4, –1)

**4** Find the equation of each of these straight lines in the form $y = mx + c$.

**(a)** Perpendicular to the line $y = 4 - 2x$, passing through (0, 0)

**(b)** Perpendicular to the line $3x + 4y = 5$, passing through (3, –2)

**(c)** Perpendicular to the line $y = 2.5x + 0.5$, passing through (–1, –2)

**5** For each pair of points,

**(i)** find their mid-point

**(ii)** find the equation of the line passing through them

**(iii)** use Pythagoras's theorem to find the length of the line segment between them, leaving as exact values any square roots that don't 'work out'

**(a)** (4, 4) and (6, 10) **(b)** (4, –2) and (8, 4) **(c)** (–4, 5) and (–1, 2)

**6** **(a)** Find the equation of the line passing through (–4, –3) and (2, 1).

**(b)** Given that this line also passes through the point $(a, 5)$, find $a$.

**7** (–1, 2) is the mid-point of line segment $AB$. $B$ is the point (2, –0.5). What is point $A$?

**8** A line segment is drawn from (1, 2) to (6, 4). Find the equation of a line that goes through its mid-point and is perpendicular to it.

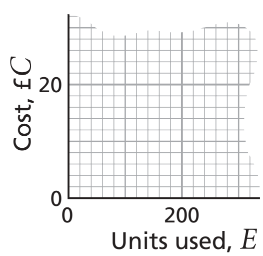
## C Problem solving with linear graphs (answers p 154)

Electricity companies send out bills every three months (every quarter).

**C1** Company P simply charges £0.09 per unit of electricity used.

**(a)** Write a formula for $C$, the cost in £, in terms of $E$, the number of units used in the quarter.

**(b)** Draw axes like these. Go up to 1000 units and £100. Draw a graph of company P's formula and label it.



D

**C2** Company Q has a different way of charging each quarter. The table shows examples of amounts charged.

| Units used, $E$ | 100 | 500 | 700 | 1000 |
|---|---|---|---|---|
| Cost, £$C$ | 22 | 50 | 64 | 85 |

**(a)** Draw a graph of this data on the same grid and label it Q.

**(b)** Write a formula for $C$ in terms of $E$ that fits this information.

**(c)** If you had to describe company Q's way of charging each quarter using words rather than a formula, what would you say?

Linear graphs like those you have drawn for C1 and C2 can help you see real-life problems clearly and make comparisons.

**C3** For each of the following customers, use your graphs to decide which of the two electricity companies would give better value and what would it charge.

**(a)** A customer who uses 300 units a quarter

**(b)** One who uses 750 units a quarter

**(c)** One who uses 900 units a quarter

With some website research, comparing different suppliers' costs this way can save a consumer a lot of money.

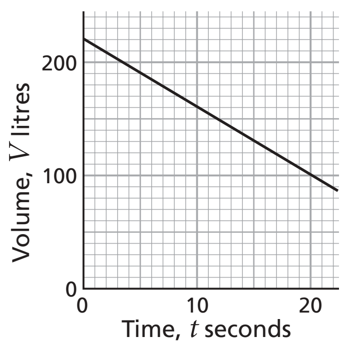
## Exercise C (answers p 154)

**1** A couple opening a restaurant want some publicity leaflets printed.
Printer R quotes '£50 one-off charge plus £0.02 per leaflet'.
Printer S uses the formula $C = 0.05n + 40$, where $C$ is the cost in £s and $n$ is the number of leaflets.

**(a)** On a grid with $n$ on the horizontal axis, draw graphs of what the printers charge.

**(b)** The couple plan to spend £65 on as many leaflets as possible.
Which printer should they choose and how many leaflets should they order?

**2** Oil is being pumped out of a tank at a constant rate.
This graph shows the volume, $V$ litres, of oil in the tank at a time $t$ seconds from when the pumping starts.




**(a)** At what rate, in litres per second, is oil leaving the tank?

**(b)** Write a formula for $V$ in terms of $t$.
Substitute $V = 0$ into your formula and solve the equation to find when the tank will be empty.

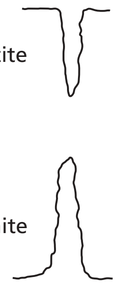
**(c)** Use this information to complete the graph accurately on graph paper.

**(d)** A second tank is having oil pumped steadily out of it at the same time. The formula for the volume of oil it holds is $V = 250 - 8t$. Work out when it will be empty and draw its graph on the same grid.

**(e)** When will the two tanks hold the same amount?
What is this amount?

**3** At a certain place in a cave, a stalactite and a stalagmite start to form.

The stalagmite grows up from the floor at 0.3 cm per year.




**(a)** Write a formula for the height $H$ cm of its tip above the ground after $t$ years.

The stalactite grows down from the ceiling at 0.2 cm per year.
The ceiling is 320 cm above the floor at this place.

**(b)** Write a formula for the height $H$ cm of the stalactite's tip above the ground after $t$ years.

**(c)** Draw graphs of these formulas on the same grid.
(You will need several hundred years on the horizontal axis.)

**(d)** From the graphs, when will the two tips be 100 cm apart?

**(e)** When will the stalactite and stalagmite touch?
How high above the floor will the point of contact be?

**4** A school band makes a CD and investigates the cost of having copies made with a coloured design printed on them.

Company X quotes £110 for 20 copies or £190 for 100 copies.

Company Y quotes £50 for 10 copies plus £2 for each additional copy.

**(a)** For each company, find a linear formula for the cost £$C$ of $n$ copies.
Draw or sketch graphs if it helps.

**(b)** Use the formulas to decide which company is cheaper for

**(i)** 50 copies **(ii)** 80 copies

## D Solving simultaneous linear equations

Problems that involve finding where the graphs of two linear equations intersect can be dealt with by solving a pair of simultaneous equations. This can be quicker and is more accurate than using the graphs.

There are several methods for solving a pair of simultaneous equations. Which is best depends on the form of the equations.

The symbol $\Rightarrow$ appears in the examples on the opposite page.
It is used to connect two mathematical statements when the second one follows mathematically from the first (that is, the first statement **implies** the second).
In practice you can think of it as meaning 'So …'.

The symbol $\therefore$ ('therefore') is used in a similar way.

**Example 6**

Find where these two linear graphs intersect. $3x + 4y = 26$
$7x - y = 9$

**Solution** (by equating coefficients)

*Note the coefficient of y in the first equation.* $3x + 4y = 26$
*Multiply the second equation by* 4 *to get the 'same size' coefficient.* $28x - 4y = 36$

*Add the previous two equations together.* $31x = 62$
$\Rightarrow \quad x = 2$

*Substitute this in the simpler equation.* $14 - y = 9$
$\Rightarrow \quad y = 5$

So the point of intersection is (2, 5).

*Check by substituting your values for x and y into the original equations.*

LHS = $3\times2 + 4\times5 = 26$, which equals RHS
LHS = $7\times2 - 5 = 9$, which equals RHS

**Example 7**

Find where these two graphs intersect. $y = 7 - 3x$
$2x + 5y = 9$

**Solution** (by equating coefficients)

*In the first equation get the x and y terms on the left-hand side.* $3x + y = 7$
*Multiply this equation by* 5. $15x + 5y = 35$
*Here is the second equation.* $2x + 5y = 9$
*Subtract it from the multiplied first equation.* $13x = 26$
$\Rightarrow \quad x = 2$

*Substitute into the first given equation.* $y = 7 - 6 = 1$
So the graphs intersect at (2, 1).

*Check by substitution into the formulas for the graphs.*

**Solution** (by substitution)

*Here are the two given equations again.* $y = 7 - 3x$
$2x + 5y = 9$

*The first one states that the expression 7 – 3x is equal to y.*
*So replace y in the second equation by this expression.* $2x + 5(7 - 3x) = 9$
*Simplify.* $2x + 35 - 15x = 9$
$\Rightarrow \quad -13x = -26$
$\Rightarrow \quad x = 2$

*Then find y as in the previous method.*

You have just seen two methods for solving simultaneous equations. There is a third method that is particularly suited to intersecting graphs that are in the form $y = \ldots$

---

**Example 8**

Find the point of intersection of these graphs. $y = 2x - 1$
$y = 4 - x$

**Solution** (by equating the expressions for $y$)

*At the point of intersection, both graphs have the same y value. Therefore the expression for y in the first graph must equal the expression for y in the second graph.*

$$2x - 1 = 4 - x$$
$$\Rightarrow \quad 3x = 5$$
$$\Rightarrow \quad x = \tfrac{5}{3}$$

*Substitute into the simpler equation.* $y = 4 - \frac{5}{3} = \frac{7}{3}$

So the point of intersection is $(\frac{5}{3}, \frac{7}{3})$.

---

The following example is not about graphs (though it could be solved by drawing a pair of intersecting graphs). It is still suited to the 'equating the expressions for $y$' approach (though here expressions for $h$ are equated).

---

**Example 9**

A snail climbs up from the bottom of a garden wall at 0.5 cm per minute. Starting at the same time from a point 3.24 metres up the wall, a millipede walks down the wall at a steady rate of 4 cm per minute.
When are the two at the same level, and what is that level?

**Solution**

*Define the 'unknowns'.* Let $h$ be the height up the wall in centimetres.
Let $t$ be the time in minutes from the start.

*Express each creature's height in terms of time.* $h = 0.5t$ (snail)
$h = 324 - 4t$ (millipede)

When they are at the same height,

$$0.5t = 324 - 4t \qquad \text{(snail's } h = \text{millipede's } h\text{)}$$
$$\Rightarrow \quad 4.5t = 324$$
$$\Rightarrow \quad t = 72$$

*Substitute in the snail's formula.* $h = 0.5 \times 72 = 36$

After 72 minutes they are both 36 cm from the bottom of the wall.

---

**Exercise D** (answers p 155)

**1** Solve the following pairs of equations using the 'equating coefficients' method.

**(a)** $3p + q = 19$
$5p + 2q = 32$

**(b)** $5a + 3b = 8$
$3a - b = 9$

**(c)** $2h + 3j = 1$
$7h + 4j = -16$

**2** Solve the following pairs of equations using the 'substitution' method.

**(a)** $y = 2x$
$3x + y = 15$

**(b)** $3x + 2y + 12 = 0$
$y = x - 1$

**(c)** $q = 2p - 6$
$3p - 2q = 11$

**3** Solve the following pairs of equations using the 'substitution' method.
Where necessary, first make $y$ the subject of one of the equations.

**(a)** $y = 5x$
$7x - 3y + 4 = 0$

**(b)** $x + y + 1 = 0$
$2x + 3y - 1 = 0$

**(c)** $y - x + 8 = 0$
$5x + 4y + 5 = 0$

**4** Solve these pairs of equations by 'equating the expressions for $y$'.

**(a)** $y = 2 - \frac{1}{3}x$
$y = 3(x - 1)$

**(b)** $y = \frac{2}{3}x$
$y = 2(2 - x)$

**(c)** $s = 2t - 3$
$s = 1 + \frac{1}{2}t$

**5** Solve these pairs of equations by 'equating the expressions for $y$'.
Where necessary, first make $y$ the subject of one of the equations.

**(a)** $x + y = 1$
$y = 2x - 14$

**(b)** $y = 1 - 3x$
$y = 5(1 + x)$

**(c)** $y - x = 1$
$y = 4x + 10$

**6** Find the point of intersection of each pair of straight lines.
Choose an appropriate method in each case.

**(a)** $y = 2x - 7$
$\frac{x}{3} + \frac{y}{5} - 3 = 0$

**(b)** $3x + 5y = 25$
$2x + 6y = 26$

**(c)** $y = 2x + 1$
$7x + 10y = 64$

**7** What happens when you try to solve the following simultaneous equations?
Give a graphical explanation in each case.

**(a)** $y = 2 - \frac{2}{3}x$
$2x + 3y = 18$

**(b)** $y = -5 - \frac{5}{3}x$
$5x + 3y + 15 = 0$

**(c)** $x - 2 - \frac{1}{2}y = 0$
$y = 2x + 5$

**8** The straight lines represented by these equations form a triangle.

$y = 4x - 17$ $\qquad$ $y = 2x + 4$ $\qquad$ $2x + 3y = 5$

Find the coordinates of the vertices of the triangle.

**9** Plumber A makes a £40 call-out charge and then charges £40 per hour.
Plumber B makes a £29 call-out charge and then charges £44 per hour.

**(a)** For each plumber, write a formula for $C$, the total cost in £s, in terms of $t$, the time worked in hours.

**(b)** Find the length of job for which both plumbers charge the same amount.

**(c)** Which plumber is cheaper for jobs that take longer than the length of time you found for (b)?

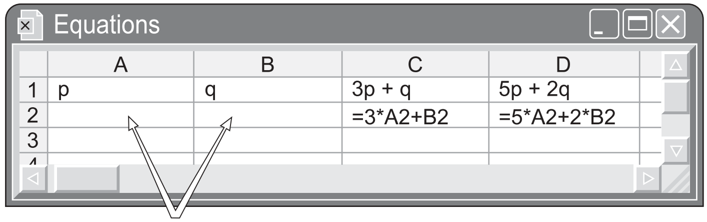
**10** Obtain an exact answer to exercise C question 2 (e) using simultaneous equations.

**11** Obtain an exact answer to exercise C question 3 (e) using simultaneous equations.

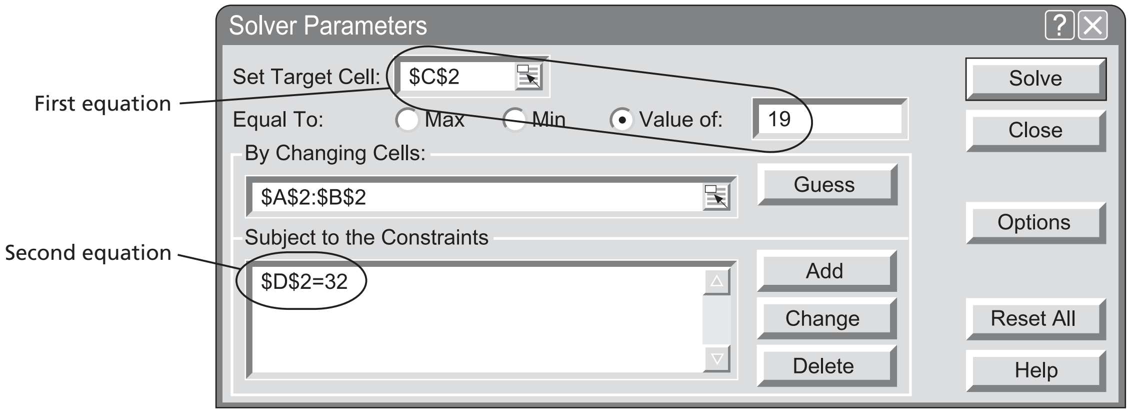
### Using a computer

In AS mathematics you need to solve simultaneous equations on paper. But for real-life problems people often use computers.

You can do this with Microsoft® Excel's Solver tool, which is available as an add-in. Here it has been set up for question 1 (a) of exercise D.




Solver will put the solutions in these blank cells.




Computers can solve larger groups of simultaneous equations, not just pairs. Their immense equation-solving power is put to good use in, for example, weather forecasting.

### Key points

- For any part of a straight line graph, the gradient is the ratio $\dfrac{\text{increase in } y}{\text{increase in } x}$. (p 6)
- An equation of the form $y = mx + c$ is a straight line, where $m$ is the gradient and $c$ is the $y$-intercept. (p 6)
- Lines that have the same gradient are parallel. (p 6)
- If two lines with gradients $m_1$ and $m_2$ are perpendicular, $m_2 = -\dfrac{1}{m_1}$ or $m_1 m_2 = -1$. (p 8)
- A straight line with gradient $m$ passing through the point $(x_1, y_1)$ has the equation $\dfrac{y - y_1}{x - x_1} = m$ or $y - y_1 = m(x - x_1)$. (p 10)
- The mid-point of the points $(x_1, y_1)$ and $(x_2, y_2)$ is $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$. (p 11)
- The point of intersection of two graphs is found by treating their equations as simultaneous equations and solving them. (pp 14–16)

## Mixed questions (answers p 155)

**1** A quadrilateral is formed by the lines $y = 3x - 4$, $y = -\frac{1}{3}x$, $y = 3x + 5$ and $y = 7 - \frac{1}{3}x$.
What special type of quadrilateral is it?

**2** A quadrilateral is formed by the lines $y = 0$, $3x + 5y = 15$, $x = 0$ and $3x + 5y = 30$.

**(a)** What special type of quadrilateral is it?

**(b)** Find its vertices.

**3** The points $A$, $B$ and $C$ have coordinates (1, 7), (5, 5) and (7, 9) respectively.

**(a)** Show that $AB$ and $BC$ are perpendicular.

**(b)** Find an equation of the line $BC$.

**(c)** The equation of the line $AC$ is $3y = x + 20$ and $M$ is the mid-point of $AB$.

**(i)** Find an equation of the line through $M$ parallel to $AC$.

**(ii)** This line intersects $BC$ at the point $T$. Find the coordinates of $T$. AQA 2001

**4** The line $p$ has the equation $4x + 3y - 18 = 0$.
The line $q$ has the equation $2y = 7 - x$.

**(a)** Sketch and label these lines on the same axes, showing clearly where each of them meets the $x$- and $y$-axes.

**(b)** Find the coordinates of the point where $p$ and $q$ intersect.

**(c)** Find the equation of a third line, $r$, which is parallel to $p$ and goes through the point where $q$ meets the $x$-axis.
Give the equation in the form $ax + by = c$, where $a$, $b$ and $c$ are integers.

**5** Line $p$ has the equation $3x + 4y + 8 = 0$.

**(a)** Sketch it, marking values where it crosses the axes.

Line $q$ goes through the point (6, 1) and is perpendicular to line $p$.

**(b)** Find the equation of line $q$, giving it in the form $y = mx + c$.

**(c)** Add line $q$ to your sketch, marking values where it crosses the axes.

**(d)** Find the point of intersection of lines $p$ and $q$, using exact fractions in your answer.

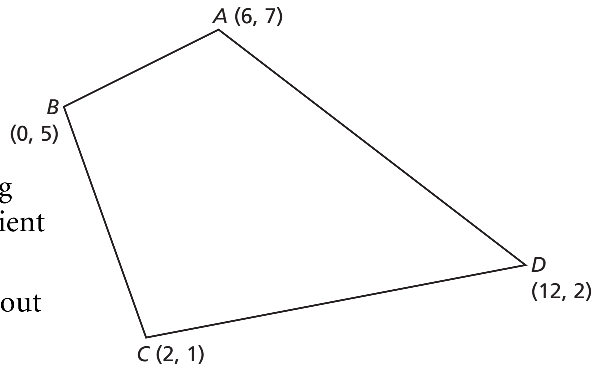
**6** A quadrilateral is drawn with its vertices at the points shown.

**(a)** Find the equations of its diagonals $AC$ and $BD$, and hence find their point of intersection.

**(b)** Give the mid-points of the four sides.

**(c)** A new quadrilateral is formed by joining the four mid-points. Work out the gradient of each side of this new quadrilateral.

**(d)** What do your answers to (c) tell you about this new quadrilateral?



**7** Find the equation of the line that goes through the origin (0, 0) and is perpendicular to the line $8x + 6y - 50 = 0$.

Find where these two lines intersect and hence find the perpendicular distance of the line $8x + 6y - 50 = 0$ from the origin.

**8** Determine, with reasons, whether the following points all lie on the same straight line.

$(-3, -3) \quad (2, -1) \quad (9, 2)$

**9** The line $AB$ has equation $5x - 2y = 7$, and the point $A$ has coordinates $(1, -1)$ and the point $B$ has coordinates $(3, k)$.

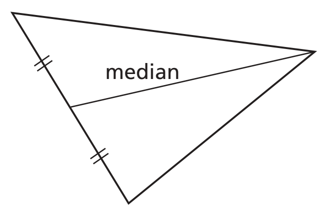
**(a)** **(i)** Find the value of $k$.

**(ii)** Find the gradient of $AB$.

**(b)** Find an equation for the line through $A$ which is perpendicular to $AB$.

**(c)** The point $C$ has coordinates $(-6, -2)$, Show that $AC$ has length $p\sqrt{2}$, stating the value of $p$. AQA 2003

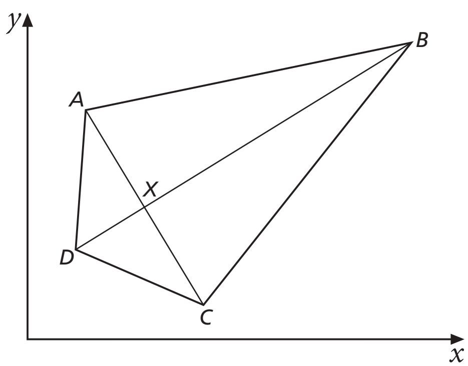
**10** In a triangle, a median is a line from a vertex to the mid-point of the opposite side.




**(a)** Find the equations of the three medians of the triangle with vertices $A$ (11, 1), $B$ (9, 9) and $C$ (1, 5).

**(b)** Show that these medians intersect at a single point, giving its coordinates.

***11** The diagram shows a kite. $A$ is the point (2, 9). $C$ is the point (8, 1).




**(a)** Find the coordinates of $X$, where the diagonals intersect.

**(b)** Find the equation of the diagonal $DB$.

$D$ is the point $(1, p)$ and $B$ is the point $(q, 11)$

**(c)** Find $p$ and $q$.

**(d)** Show that $\angle ADC$ is a right angle.

**(e)** Find the area of the kite.

## Test yourself (answers p 156)

None of these questions requires a calculator.

**1** Write the equations of the following lines in the form $y = mx + c$.

**(a)** $5x + y = 4$ **(b)** $x - 2y + 6 = 0$ **(c)** $x + 2y = 3$ **(d)** $y - 6 = 2(x + 4)$

**2** The line $y = 3x + 4$ is drawn. State whether each of the following lines is parallel to it, perpendicular to it, or neither.

**(a)** $3x + y = 2$ **(b)** $y = 2 - \frac{1}{3}x$ **(c)** $3x - y - 7 = 0$ **(d)** $x + 3y - 2 = 0$

**3** A straight line goes through the points (2, 3) and (6, 2).

**(a)** What is its gradient?

**(b)** Give its equation in the form $y = mx + c$.

**4** Give the equation of the line joining the origin to the mid-point of (5, 3) and (–1, 7).

**5** Find the length of the line joining (1, –2) and (–1, 1), leaving your answer as an exact value.

**6** Line $l$ has the equation $y = 6x + 1$.

**(a)** Give the equation of the line through (1, 0) parallel to $l$.

**(b)** Give the equation of the line through (3, –1) perpendicular to $l$.

**7** Solve the following pairs of simultaneous equations by an appropriate method.

**(a)** $4x + 5y - 6 = 0$, $y = x + 3$ **(b)** $y = x + 1$, $7x + 3y = 0$ **(c)** $y = 3 - x$, $3x + 5y - 12 = 0$

**(d)** $y = 2x + 5$, $y = -3x$ **(e)** $3x + 5y = 30$, $5x + 3y = 42$ **(f)** $y = \frac{2}{3}x - 1$, $y = \frac{3}{2}x + 4$

**8** The point $A$ has coordinates (2, 3) and $O$ is the origin.

**(a)** Write down the gradient of $OA$ and hence find the equation of the line $OA$.

**(b)** Show that the line which has equation

$$4x + 6y = 13:$$

**(i)** is perpendicular to $OA$;

**(ii)** passes through the mid-point of $OA$. AQA 2003

**9** The equation of the line $AB$ is $5x - 3y = 26$.

**(a)** Find the gradient of $AB$.

**(b)** The point $A$ has coordinates (4, –2) and a point $C$ has coordinates (–6, 4).

**(i)** Prove that $AC$ is perpendicular to $AB$.

**(ii)** Find an equation of the line $AC$, expressing your answer in the form $px + qy = r$, where $p$, $q$ and $r$ are integers.

**(c)** The line with equation $x + 2y = 13$ also passes through the point $B$. Find the coordinates of $B$. AQA 2002