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Cambridge Tracts in Mathematics
and Mathematical Physics

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No. 58

STOCHASTIC APPROXIMATION

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STOCHASTIC APPROXIMATION

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AT THE UNIVERSITY PRESS
1969

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PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York NY 10011–4211, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa

http://www.cambridge.org

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First published 1969
First paperback edition 2004

A catalogue record for this book is available from the British Library

Library of Congress Catalogue Card Number: 69–11150

ISBN 0 521 07368 5 hardback
ISBN 0 521 60485 0 paperback

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PREFACE

The subject of stochastic approximation is of recent origin, but the number of research papers which have appeared in theoretical and applied journals over the last fifteen years, speaks for its practical utility and theoretical importance. I had the following motives in preparing this manuscript.

First, I have collected a number of applications of stochastic approximation to problems in engineering and medicine and have shown how one can make use of these techniques in practice. Thus the manuscript can have practical values.

Secondly, since the material of the manuscript is not available in text-book form except for a chapter in a sequential methodology book in statistics or mathematics, this can be used for one semester course for graduate students in mathematics, computer sciences and mathematical statistics.

Finally, this can serve as a reference text for research workers in science, engineering, mathematics, statistics and many other fields.

I cannot precisely state the prerequisites for studying this subject, but a reader should know the subject matter of the appendices. Usually a proof of a result is carried out extensively, but in order to avoid monotony, in a few places proofs are kept concise where they are already fairly long. Appendices 2 and 3 are of great help in these cases.

I am grateful to Professor D. L. Burkholder who introduced me to the field of stochastic approximation by his interesting course conducted in the Fall semester of 1956 at University of Illinois. Since then I have kept my interest alive by application of these techniques to industrial problems, doing research and conducting a course at Queen's University.

It is a great pleasure to extend my thanks to Professor R. A. Bradley who encouraged me to convert my lecture notes into a text during his visit to Queen's University under the IMS visiting lecturer programme. I also thank Professors R. H. Farrell, M. M. Rao and many others for their helpful sug-

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gestions and comments in preparation of the manuscript. I am grateful to Professor V. Fabian for reading the manuscript and making several valuable suggestions, and I extend my appreciation to Professor J. F. C. Kingman who edited this manuscript with patience and great care, which improved the manuscript, particularly, its presentation.

I am grateful to the Queen's Art Research Committee for the grant and the University of Bombay for the facilities. Finally I express my thanks to Mr H. R. Chavan and Mr M. B. Joshi for typing the first draft of such a difficult manuscript, and to Mrs E. M. Wight for the final draft.

M. T. W.

January 1968

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