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Contents

Preface

This book was born out of my fascination with applied mathematics as a place where the physical world meets the mathematical structures and techniques that are the cornerstones of most applied mathematics courses. I am interested largely in human-sized theatres of interaction, leaving cosmology and particle physics to others. Much of my research has been motivated by interactions with industry or by contact with scientists in other disciplines. One immediate lesson from these contacts is that it is a great asset to an interactive applied mathematician to be open to ideas from any direction at all. Almost any physical situation has some mathematical interest, but the kind of mathematics may vary from case to case. We need a strong generalist streak to go with our areas of technical expertise.

Another thing we need is some expertise in numerical methods. To be honest, this is not my strong point. That is one reason why the book does not contain much about these methods. (Another is that if it had then it would have been half as long again and would have taken five more years to write.) In the modern world, with its fast computers and plethora of easy-to-use packages, any applied mathematician has to be able to switch into numerical mode as required. At the very least, you should learn to use packages such as Maple and Matlab for their data display and plotting capabilities and for the built-in software routines for solving standard problems such as ordinary differential equations. With more confidence, you can write your own programs. In many cases, a quick and dirty first try can provide valuable information, even if this is not the finished product. Explicit finite differences (remember to use upwind differencing for first derivatives) and tiny time steps will get you a long way.

Who should read this book? Many people, I hope, but there are some prerequisites. I assume that readers have a good background in calculus up to vector calculus (grad, div, curl) and the elementary mechanics of particles. I also assume that they have done an introductory (inviscid) fluid mechanics course and a first course in partial differential equations, enough to know the basics of the heat, wave and Laplace equations (where they come from, and how to solve them in simple geometries). Linear algebra, complex analysis and probability put in an occasional appearance. High-school physics is an advantage. But the most important prerequisite is an attitude: to go out and apply your mathematics, to see it in action in the world around you, and not to worry too much about the technical aspects, focusing instead on the big picture.

Another way to assess the technical level of the book is to position it relative to the competition. From that point of view it can be thought of as a precursor to the books by Tayler [55] and Fowler [18], while being more difficult than, say, Fowkes & Mahoney [17] or Fulford & Broadbridge [21]. The edited collections [9, 40] are at the same general level, but they are organised along different lines. The books [40, 56] cover similar material but with a less industrial slant.

Organisation. The book is organised, roughly, along mathematical lines. Chapters are devoted to mathematical techniques, starting in Part I with some ideas about modelling, moving on in Part II to differential equations and distributions, and concluding with asymptotic (systematic approximation) methods in Part III. Interspersed among the chapters are case studies, descriptions of problems that illustrate the techniques; they are necessarily rather open-ended and invite you to develop your own ideas. The case studies run as strands through the book. You can ignore any of them without much impact on the rest of the book, although the more you ignore the less you will benefit from the remainder. There are long sections of exercises at the ends of the chapters; they should be regarded as an integral part of the book and at least should be read through if not attempted.

Conventions. I use 'we', as in 'we can solve this by a Laplace transform', to signal the usual polite fiction that you, the reader, and I, the author, are engaged on a joint voyage of discovery; 'we' also signifies that I am presenting ideas within a whole tradition of thought. 'You' is mostly used to suggest that *you* should get your pen out and work though some of the 'we' stuff, a good idea in view of my fallible arithmetic, or do an exercise to fill in some details. 'I' is associated with authorial opinions and can mostly be ignored if you like.

I have tried to draw together a lot of threads in this book, and in writing it I have constantly wanted to point out connections with something else or make a peripheral remark. However, I don't want to lose track of the argument. As a compromise, I have used marginal notes and footnotes[1] with slightly different purposes in mind.

Marginal notes are usually directly relevant to the current discussion, often being used to fill in details or point out a feature of a calculation. This is a book to work through: feel free to use the empty margin spaces for calculations.

[1] Footnotes are more digressional and can be ignored by readers who just want to follow the main line of argument.

Acknowledgements. I have taken examples from many sources. Some examples are very familiar and I do not apologise for this: the old ones are often the best. Much the same goes for the influence of books; if you teach a course using other people's books and then write your own, some impact is inevitable. Among the books that have been especially influential are those by Tayler [55], Fowler [18], Hinch [26] and Keener [32]. Even more influential has been the contribution of colleagues and students. Many a way of looking at a problem can be traced back to a coffee-time conversation or a Study Group meeting.[2] There are far too many of these collaborators for me to attempt the invidious task of thanking them individually. Their influence is pervasive. At a more local level, I am immensely grateful to the OCIAM students who got me out of computer trouble on various occasions and found a number of errors in drafts of the book. Any remaining errors are quite likely to have been caused by cosmic ray impact on the computer memory, or perhaps by cyber-terrorists. I will be happy to hear about them.

The book began when I was asked to give some lectures at a summer school in Siena and was continued through a similar event a year later in Pisa. I am most grateful for the hospitality extended to me during these visits. I would like to thank the editors and technical staff at Cambridge University Press for their assistance in the production of the book. In particular, I am extremely grateful to Susan Parkinson for her careful, constructive and thoughtful copy-editing of the manuscript. Lastly I would like to thank my family for their forbearance, love and support while I was locked away typing. This book is dedicated to them.

Colemanballs. At the end of each section of exercises is what would normally be a wasted space. Into each of these I have put two things. One is a depiction of a wave form and is explained on p. 212. The other is a statement made by a real live applied mathematician in full flow. In the spirit of scientific accuracy, they are wholly unedited. They are mostly there for their intrinsic qualities (and it would be a miserable publisher who would deny me that extra ink), but they make a point: interdisciplinary mathematics is a collaborative affair; it involves discussions and

[2] Study Groups are week-long intensive meetings at which academics and industrial researchers get together to work on open problems from industry, proposed by the industrial participants. Over the week, heated discussions take place involving anybody who is interested in the problem, and a short report is produced at the end. The first UK Study Group was held in Oxford in 1968, and they have been held every year since, in Oxford and other UK universities. The idea has now spread to more than 15 countries on all the habitable continents of the world. Details of forthcoming events, and reports of problems studied at past meetings, can be found on their dedicated website `www.mathematics-in-industry.org`.

arguments, the less inhibited the better. We all have to go out on a limb, in the interests of pushing the science forwards. If we are wrong, we try again. And if the mind runs ahead of the voice, our colleagues won't take it too seriously (nor will they let us forget it). Here is one to be going on with, from the collection [28] of the same title:

'If I remember rightly, $\cos \pi/2 = 1$.'